Appendix 2

Proof of Lemma 1

Both players are asked to simultaneously submit their voluntary contributions to the public good. Fixing subject $j$’s contribution, $g_j$, we obtain player $i$’s best response function

$$g_i(g_j) = \begin{cases} 
1 & \text{if } g_j = 0 \\
1 + \frac{\alpha_i - m}{\alpha_i + m} g_j & \text{if } g_j \in \left(0, \frac{m + \alpha_i}{m - \alpha_i}\right) \\
0 & \text{if } g_j \in \left(\frac{m + \alpha_i}{m - \alpha_i}, +\infty\right)
\end{cases}$$

if $\alpha_i < m$. Note that $0 \geq 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j$ holds if, after solving for $g_j$, $g_j \geq \frac{m + \alpha_i}{m - \alpha_i}$. Threshold $\frac{m + \alpha_i}{m - \alpha_i}$ is positive if $\alpha_i < m$; see figure 1(a) in the paper. In contrast, when $\alpha_i > m$ this threshold is never binding for any positive $g_j$, i.e., $g_i$ does not become zero or negative for any positive value of $g_j$, see figure 1(b).

Then, the corresponding best response function for player $i$ in this case is

$$g_i(g_j) = \begin{cases} 
1 & \text{if } g_j = 0 \\
1 + \frac{\alpha_i - m}{\alpha_i + m} g_j & \text{if } g_j > 0
\end{cases}$$

\[\square\]

Proof of Proposition 1

First, take a given player $i$’s best response function, $g_i(g_j)$. Then, $g_i^{Sm} = 1$ only when: (1) the slope of player $j$’s best response function, $g_j(g_i)$, is smaller than -1, and (2) the horizontal intercept of
player \(i\)'s best response function, \(g_i(g_j)\), is higher than 1. Otherwise, both players’ best response functions would cross each other in an interior point. That is, \(g_i^{Sm} = 1\) if and only if \(\frac{\alpha_j - m}{\alpha_i + \alpha_j + m} \leq -1\), which holds if \(\alpha_j = 0\). And \(\frac{m + \alpha_i}{\alpha_i - \alpha_j} \geq 1\) if and only if \(\alpha_i > 0\).

Since \(\alpha_i, \alpha_j \geq 0\), the above conditions on player \(i\) and \(j\)'s concerns about status are \(\alpha_i \geq 0\) and \(\alpha_j = 0\). Hence, \(g_i^{Sm} = 1\) if and only if \(\alpha_i \geq 0\) and \(\alpha_j = 0\). Secondly, \(g_i^{Sm} = 0\) only when the opposite happens. That is, when \(\alpha_i = 0\) and \(\alpha_j \geq 0\). Finally, when none of the above cases is satisfied, i.e., when \(\alpha_i > 0\) and \(\alpha_j > 0\), then we have an interior solution. Solving for \(g_i\) and \(g_j\) in a system of two equations, we obtain \(g_i^{Sm} = \frac{\alpha_j}{(\alpha_i + \alpha_j + m)}\), as the interior Nash equilibrium contribution level.

**Sufficiency.** Let us now check that the second order conditions of incentive compatibility are satisfied. Suppose all but player \(i\) submit a contribution to the public good according to the above equilibrium prediction. I next show that, for any \(\alpha_i\), contributor \(i\) maximizes his utility by following \(g_i^{Sm}\). Let

\[
U (g, \alpha_i) = w - g_i + \ln \left[ m (g_i + g_j^{Sm}) + \alpha_i (g_i - g_j^{Sm}) \right]
\]

be the utility level of player \(i\) when contributing \(g\) to the public good, and having a concern \(\alpha_i\) about status acquisition. We must now show that the derivative \(U_g (g, \alpha_i) \geq 0\) for all \(g < g_i^{Sm}\), and \(U_g (g, \alpha_i) \leq 0\) for all \(g > g_i^{Sm}\), which imply that \(U (g, \alpha_i)\) is indeed maximized at exactly \(g = g_i^{Sm}\). Differentiating \(U (g, \alpha_i)\) with respect to \(g\),

\[
U_g (g, \alpha_i) = -1 + \frac{\alpha_i + m}{\alpha_i (g - g_j^{Sm}) + m (g + g_j^{Sm})}
\]

Let us now suppose that \(g < g_i^{Sm} (\alpha_i)\), and denote \(\bar{\alpha}_i\) to be the concern about status for which the equilibrium contribution is exactly \(g\), i.e., \(g_i^{Sm} (\bar{\alpha}_i) = g\). Since \(g_i^{Sm} (\alpha_i)\) is strictly increasing in \(\alpha_i\) (as one can check from the suggested equilibrium contribution \(g_i^{Sm}\), and confirmed in lemma 4) this implies that \(g_i^{Sm} (\alpha_i) > g_i^{Sm} (\bar{\alpha}_i)\) if and only \(\alpha_i > \bar{\alpha}_i\). Then, \(U_g (g, \bar{\alpha}_i) \leq U_g (g, \alpha_i)\). Since by definition, \(g_i^{Sm} (\bar{\alpha}_i) = g\), it implies that \(U_g (g, \bar{\alpha}_i) = 0\). Hence, \(U_g (g, \alpha_i) \geq 0\) for all \(g < g_i^{Sm}\). By a similar argument, \(U_g (g, \alpha_i) \leq 0\) for all \(g > g_i^{Sm}\). Therefore, \(U (g, \alpha_i)\) is maximized at \(g = g_i^{Sm}\). \(\Box\)

**Proof of Lemma 2**

Differentiating \(g_i^{Sm}\) with respect to \(\alpha_i\), we obtain

\[
\frac{\partial g_i^{Sm}}{\partial \alpha_i} = \begin{cases} 
0 & \text{if } \alpha_i > 0 \text{ and } \alpha_j = 0 \\
\frac{\alpha_j (\alpha_i + \alpha_j + m)}{(\alpha_i + \alpha_j + m)^2} & \text{if } \alpha_i > 0 \text{ and } \alpha_j > 0 \\
0 & \text{if } \alpha_i = 0 \text{ and } \alpha_j > 0 
\end{cases}
\]
which is weakly positive for all parameter values. On the other hand, differentiating $g_i^{Sm}$ with respect to $\alpha_j$, we obtain

$$\frac{\partial g_i^{Sm}}{\partial \alpha_j} = \begin{cases} 0 & \text{if } \alpha_i > 0 \text{ and } \alpha_j = 0 \\ \frac{\alpha_i(\alpha_i - m)}{(\alpha_i + \alpha_j)^2 m} & \text{if } \alpha_i > 0 \text{ and } \alpha_j > 0 \\ 0 & \text{if } \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

which is weakly positive for all parameter values if $\alpha_i \geq m$. Finally, differentiating $g_i^{Sm}$ with respect to $\alpha_j$, we obtain

$$\frac{\partial g_i^{Sm}}{\partial \alpha_j} = \begin{cases} 0 & \text{if } \alpha_i > 0 \text{ and } \alpha_j = 0 \\ -\frac{\alpha_i \alpha_j}{(\alpha_i + \alpha_j)^2 m} & \text{if } \alpha_i > 0 \text{ and } \alpha_j > 0 \\ 0 & \text{if } \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

which is weakly negative for all parameter values.

\textbf{Proof of Lemma 3}

If $\alpha_i > 0$ and $\alpha_j = 0$, then from proposition 1 we know that $g_i^{Sm} = 1$ and $g_j^{Sm} = 0$. Hence, $G^{Sm} = 1$. If, on the contrary, $\alpha_i = 0$ and $\alpha_j \geq 0$, then from proposition 1 we also know that $g_i^{Sm} = 0$ and $g_j^{Sm} = 1$. Hence, $G^{Sm} = 1$ as well. Finally, if $\alpha_i > 0$ and $\alpha_j = 0$, then

$$g_i^{Sm} = \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j) m},$$

and similarly for player $j$, which yields

$$G^{Sm} = 1 + \frac{2\alpha_i \alpha_j}{(\alpha_i + \alpha_j) m}.$$

Note that if status concerns $(\alpha_i, \alpha_j)$ are chosen in order to maximize $G^{Sm}$, \( \max_{\alpha_i, \alpha_j \geq 0} G^{Sm} \), we obtain the following first order condition for every $\alpha_i$,

$$\frac{2\alpha_i^2}{(\alpha_i + \alpha_j)^2 m} \leq 0,$$

and for $\alpha_j$,

$$\frac{2\alpha_j^2}{(\alpha_i + \alpha_j)^2 m} \leq 0.$$

This gives a continuum of $(\alpha_i, \alpha_j)$ pairs for which $G^{Sm}$ is maximal at $\alpha_i = \alpha_j = \alpha$, and increasing both in $\alpha_i$ and in $\alpha_j$. \( \Box \)
Proof of Proposition 2

Operating by sequential rationality, player $i$ inserts the follower’s best response function into his utility function,

$$U_i = w - g_i + \ln \left[ m(g_i + g_j(g_i)) + \alpha_i (g_i - g_j(g_i)) \right],$$

which is maximized at

$$g_{Seq}^i = \begin{cases} 
0 & \text{if } \alpha_i \in [0, \bar{\alpha}_i] \\
\frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)} & \text{if } \alpha_i \in (\bar{\alpha}_i, +\infty) 
\end{cases}$$

where $\bar{\alpha}_i = \frac{m(m - \alpha_j)}{3m + \alpha_j}$. Given the above contribution of the first donor and $g_j(g_i)$ specified above, player $j$ submits

$$g_{Seq}^j = \begin{cases} 
\frac{1}{2} \left( \frac{\alpha_i \alpha_j}{\alpha_i + \alpha_j} + \frac{m}{\alpha_i + \alpha_j} + \frac{4\alpha_j}{\alpha_j + m} - 1 \right) & \text{if } \alpha_i \in [\bar{\alpha}_i, \hat{\alpha}_i) \\
1 & \text{if } \alpha_i \in [0, \bar{\alpha}_i) \\
0 & \text{if } \alpha_i \in [\hat{\alpha}_i, +\infty) 
\end{cases}$$

if $\alpha_j < m$. Clearly, note that when player $j$’s best response function is negative, i.e., $\alpha_j < m$, player $j$ submits no positive contribution if $1 - \frac{\alpha_i - m}{\alpha_i + m} g_j \geq \frac{m + \alpha_i}{m - \alpha_j}$, or in equilibrium, when $\alpha_i \geq \hat{\alpha}_i$, where

$$\hat{\alpha}_i = \frac{m \left( 3\alpha_j^2 + m^2 \right)}{-\alpha_j^2 - 4\alpha_j m + m^2}.$$

On the other hand, if player $j$’s best response function is positive, $\alpha_j > m$, player $j$ submits

$$g_{Seq}^j = \begin{cases} 
\frac{1}{2} \left( \frac{\alpha_i \alpha_j}{\alpha_i + \alpha_j} + \frac{m}{\alpha_i + \alpha_j} + \frac{4\alpha_j}{\alpha_j + m} - 1 \right) & \text{if } \alpha_i \in [\bar{\alpha}_i, +\infty) \\
1 & \text{if } \alpha_i \in [0, \bar{\alpha}_i) \\
0 & \text{if } \alpha_j < m \text{ and } \alpha_i \in [\bar{\alpha}_i, \hat{\alpha}_i), \text{ or if } \alpha_j > m \text{ and } \alpha_i \in [\hat{\alpha}_i, +\infty) 
\end{cases}$$

Clearly, the above two expressions for $g_{Seq}^j$ can be simplified to

$$g_{Seq}^j = \begin{cases} 
1 & \text{if } \alpha_i \in [0, \bar{\alpha}_i) \\
\frac{1}{2} \left( \frac{\alpha_i \alpha_j}{\alpha_i + \alpha_j} + \frac{m}{\alpha_i + \alpha_j} + \frac{4\alpha_j}{\alpha_j + m} - 1 \right) & \text{if } \alpha_j < m \text{ and } \alpha_i \in [\bar{\alpha}_i, \hat{\alpha}_i), \text{ or if } \alpha_j > m \text{ and } \alpha_i \in [\hat{\alpha}_i, +\infty) \\
0 & \text{if } \alpha_j < m \text{ and } \alpha_i \in [\hat{\alpha}_i, +\infty) 
\end{cases}$$

$\square$

Proof of Corollary 1

First result: From proposition 2, we know that player $i$ submits strictly positive contributions if and only if

$$\alpha_i > \frac{m (m - \alpha_j)}{3m + \alpha_j}.$$
Then, if $\alpha_i = 0$, the former condition can only be satisfied if $0 > \frac{m(m-\alpha_i)}{3m+\alpha_j}$, or $\alpha_j > m$.

**Second result:** Since $\bar{\alpha} = \frac{m(m-\alpha_j)}{3m+\alpha_j} < m$, for any $\alpha_j \geq 0$, then if $m < \alpha_i$ we must have $\bar{\alpha} < m < \alpha_i$ for any $\alpha_j \geq 0$. Therefore, $\bar{\alpha} < \alpha_i$, and player $i$ submits a strictly positive contribution for any concern about status player $j$ may have, $\alpha_j \geq 0$. □

**Proof of Lemma 4**

Differentiating $g_i^{\text{Seq}}$ with respect to $\alpha_i$, we obtain

$$\frac{\partial g_i^{\text{Seq}}}{\partial \alpha_i} = \begin{cases} 0 & \text{if } \alpha_i \in [0, \bar{\alpha}_i] \\ \frac{(\alpha_i-m)^2}{2(\alpha_i+\alpha_j)^2} & \text{if } \alpha_i > \bar{\alpha}_i \end{cases}$$

which is weakly positive for any parameter values. On the other hand, differentiating $g_i^{\text{Seq}}$ with respect to $\alpha_j$, we obtain

$$\frac{\partial g_i^{\text{Seq}}}{\partial \alpha_j} = \begin{cases} 0 & \text{if } \alpha_j \in [0, \bar{\alpha}_i] \\ \frac{(\alpha_j-m)^2}{2(\alpha_i+\alpha_j)^2} & \text{if } \alpha_j > \bar{\alpha}_i \end{cases}$$

which is weakly positive for any parameter values. □

**Proof of Lemma 5**

When $\alpha_i < \bar{\alpha}_i$, we know from proposition 2 that player $i$ does not contribute, but player $j$ responds submitting a contribution of $g_j^{\text{Seq}} = 1$. This is valid both when $\alpha_j < m$ and when $\alpha_j < m$. Then, $G^{\text{Seq}} = 1$.

In contrast, when $\alpha_i \in (\bar{\alpha}_i, \alpha_i)$ and $\alpha_j < m$ (or when $\alpha_i \in (\bar{\alpha}_i, \infty)$ and $\alpha_j > m$) from proposition 2 we know that player $i$ submits

$$g_i^{\text{Seq}} = \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)}$$

while player $j$ responds by submitting

$$g_j^{\text{Seq}} = \frac{1}{2} \left( \frac{\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m} + \frac{m}{\alpha_i + \alpha_j} + \frac{4\alpha_j}{\alpha_j + m} - 1 \right)$$

Then, the total contributions when $\alpha_i > \bar{\alpha}_i$ adds up to

$$G^{\text{Seq}} = \frac{2\alpha_j}{\alpha_j + m} + \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m}.$$
Finally, if $\alpha_i \in \overline{\alpha_i}$ and $\alpha_j < m$, from proposition 2 we know that player $i$ submits

$$g_{i}^{\text{seq}} = \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)}$$

and player $j$ does not submit any positive contribution (since his best response function is positively sloped and, for these parameter values, it crosses the $g_i$-axis), what implies

$$G^{\text{Seq}} = \frac{\alpha_i \alpha_j + 3\alpha_i m + \alpha_j m - m^2}{2m(\alpha_i + \alpha_j)}.$$ 

Proof of Lemma 6

Regarding player $i$, the difference between his equilibrium contribution in the simultaneous and sequential game is

$$\frac{(\alpha_i - m)(\alpha_j - m)}{2(\alpha_i + \alpha_j)m}$$

which is positive if either $\alpha_i > m$ and $\alpha_j > m$, or if $\alpha_i < m$ and $\alpha_j < m$. Hence, if $\alpha_i > m$ and $\alpha_j > m$ (or if $\alpha_i < m$ and $\alpha_j < m$), then $g_{i}^{\text{Sm}} > g_{i}^{\text{seq}}$. Regarding player $j$, the difference between his equilibrium contribution in the simultaneous and sequential game is

$$\frac{(\alpha_i - m)(\alpha_j - m)}{2(\alpha_i + \alpha_j)m}$$

which is positive if and only if $\alpha_i > m$. Hence, if $\alpha_i > m$, $g_{j}^{\text{Sm}} > g_{j}^{\text{seq}}$. □

Proof of Proposition 3

Applying proposition 1 of Romano and Yildirim (2001), we know that whenever $1 + \frac{\partial g_j(g_i)}{\partial g_i} > 0$, the sign of $\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i}$ and $G^{\text{Seq}} - G^{\text{Sm}}$ coincide. We first find $1 + \frac{\partial g_j(g_i)}{\partial g_i}$. In particular,

$$1 + \frac{\partial g_j(g_i)}{\partial g_i} = 1 + \frac{\alpha_j - m}{\alpha_j + m}$$

which is positive for any $\alpha_j > 0$. On the other hand, from corollary 1, we know that for any $i, j = \{1, 2\}$ where $j \neq i$

$$\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} = \begin{cases} > 0 & \text{if } \alpha_i < m \text{ and } \alpha_j > m, \\ < 0 & \text{otherwise} \end{cases}$$

Therefore, if $\alpha_i < m$ and $\alpha_j > m$ for all $i, j = \{1, 2\}$ and $j \neq i$, then $\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} > 0$ and $G^{\text{Seq}} > G^{\text{Sm}}$, and if $\alpha_i > m$ and $\alpha_j > m$ (or if $\alpha_i < m$ and $\alpha_j < m$), we have that $\frac{\partial U_i}{\partial g_j} \frac{\partial g_j(g_i)}{\partial g_i} < 0$
and \( G^{Seq} < G^{Sm} \). Finally, note that if \( \alpha_i = \alpha_j = 0 \), then

\[
\frac{\partial g_j(g_i)}{\partial g_i} = 1 + \frac{-m}{m} = 0,
\]

which implies that \( \frac{\partial U_i}{\partial g_j} \frac{\partial g_i(g_j)}{\partial g_i} \) becomes zero, and therefore \( G^{Seq} = G^{Sm} \). This result is consistent with that in Varian (1994) where, for public good games with players without preferences for social status, he shows that \( G^{Seq} \leq G^{Sm} \). □

**Proof of Corollary 2**

For \( N = 2 \) players, simultaneous contribution mechanisms generate a larger total revenue than sequential mechanisms (see proposition 3). For \( N \geq 3 \), note that total contributions in the simultaneous-move game are \( G^{Sm} = \frac{m + \alpha}{m} \) and hence are constant in the population size \( N \). In the sequential game, total donations in the interior equilibrium (when \( \alpha > \bar{\alpha} \)) are

\[
G^{Seq} = \frac{m(N - 1)(4 + N^2)\alpha + (N^2 - 2)\alpha^2 - 2m^2(N - 1)^2}{(N - 2)[m^2(N - 1)^2 - \alpha^2]},
\]

which approach \( \frac{\alpha}{m} \) as \( N \to \infty \), implying that \( G^{Sm} > G^{Seq} \). In the corner solution (when \( \alpha \leq \bar{\alpha} \)), total contributions in the sequential game are

\[
G^{Seq} = \frac{(N - 1)(m + \alpha)}{\alpha + (N - 1)m},
\]

which lie weakly below \( G^{Sm} = \frac{m + \alpha}{m} \) for all \( N \). We can hence conclude that \( G^{Sm} \geq G^{Seq} \). (Note that this corner solution embodies the case in which \( \alpha = 0 \), since \( \alpha \leq \bar{\alpha} \), where total contributions under both the simultaneous and sequential mechanism, \( G^{Sm} \) and \( G^{Seq} \), become \( G^{Sm} = G^{Seq} = 1 \) and are hence constant in the population size \( N \)). □

**Proof of Proposition 4**

First, take a given player \( i \)'s best response function, \( g_i(g_j) \). Then, \( g_i^{Sm,Sen} = 1 - \frac{\alpha_i S_i}{\alpha_i + m} \) only when:

1. The slope of player \( j \)'s best response function, \( g_j(g_i) \), is smaller than -1, and

2. The horizontal intercept of player \( i \)'s best response function, \( g_i(g_j) \), is higher than \( 1 - \frac{\alpha_j S_j}{\alpha_j + m} \).

Otherwise, both players' best response functions would cross each other in an interior point. Therefore, \( g_i^{Sm,Sen} = 1 - \frac{\alpha_i S_i}{\alpha_i + m} \) if and only if

\[
\frac{\alpha_j - m}{\alpha_j + m} \leq -1 \quad \text{implies} \quad \alpha_j \leq 0, \quad \text{and}
\]

\[
\frac{\alpha_i S_i - \alpha_i - m}{\alpha_i - m} \geq 1 - \frac{\alpha_j S_j}{\alpha_j + m} \quad \text{implies} \quad \alpha_i \geq \frac{\alpha_j S_j m}{\alpha_j(S_i + S_j - 2) + (S_i - 2)m}
\]
Since $\alpha_i, \alpha_j \geq 0$, the above conditions on player $i$ and $j$’s concerns about status are

$$\alpha_i \geq \frac{\alpha_j S_{jm}}{\alpha_j (S_i + S_j - 2) + (S_j - 2) m} \quad \text{and} \quad \alpha_j = 0.$$  

Secondly, $g^{Sm,Sen}_i = 0$ when the opposite happens. That is, when $\alpha_i = 0$ and $\alpha_j \geq \frac{\alpha_i S_{jm}}{\alpha_i (S_j + S_i - 2) + (S_j - 2) m}$.  

Finally, when both

$$\alpha_i \geq \frac{\alpha_j S_{jm}}{\alpha_j (S_i + S_j - 2) + (S_j - 2) m} \quad \text{and} \quad \alpha_j \geq \frac{\alpha_i S_{jm}}{\alpha_i (S_j + S_i - 2) + (S_j - 2) m},$$

we have an interior solution. Solving for $g_i$ and $g_j$ in a system of two equations, we obtain interior solutions, and therefore,

$$g^{Sm,Sen}_i = \begin{cases} 
1 - \frac{\alpha_i S_{j}}{\alpha_j + m} & \text{if } \alpha_i \geq \tilde{\alpha}_i \text{ and } \alpha_j \geq \tilde{\alpha}_j \\
\frac{\alpha_i S_{j}m - \alpha_j[\alpha_j(S_i+S_j-2)+m(S_j-2)]}{2(\alpha_i+\alpha_j)m} & \text{if } \alpha_i \geq \tilde{\alpha}_i \text{ and } \alpha_j \geq \tilde{\alpha}_j \\
0 & \text{if } \alpha_i \geq 0 \text{ and } \alpha_j \geq \tilde{\alpha}_j 
\end{cases}$$

where $\tilde{\alpha}_i = \frac{\alpha_j S_{jm}}{\alpha_j (S_i+S_j-2) + (S_j-2)m}$ and $\tilde{\alpha}_j = \frac{\alpha_i S_{jm}}{\alpha_i (S_j+S_i-2) + (S_j-2)m}$. Differentiating $g^{Sm,Sen}_i$ with respect to $S_i$, we find that

$$\frac{\partial g^{Sm,Sen}_i}{\partial S_i} = -\frac{\alpha_i (\alpha_j + m)}{2(\alpha_i + \alpha_j) m}$$

which is negative for all parameter values. Similarly, differentiating $g^{Sm,Sen}_i$ with respect to $S_j$, yields

$$\frac{\partial g^{Sm,Sen}_i}{\partial S_j} = \frac{\alpha_j (m - \alpha_i)}{2(\alpha_i + \alpha_j) m}$$

which is negative if and only if $m < \alpha_i$. Using the second mover’s best response function, $g_j(g_i)$, from lemma 10, we know that

$$g_i(g_j) = \begin{cases} 
1 - \frac{\alpha_j S_{jm}}{\alpha_j + m} & \text{if } g_i = 0 \\
1 - \frac{\alpha_j S_{jm}}{\alpha_j + m} + \frac{\alpha_j - m}{\alpha_j - m} g_j & \text{if } g_i \in \left[0, \frac{\alpha_j S_{jm} - \alpha_j - m}{\alpha_j - m}\right] \\
0 & \text{if } g_i > \frac{\alpha_j S_{jm} - \alpha_i - m}{\alpha_j - m}
\end{cases}$$

Regarding player $i$, we know that he inserts the above best response function into his utility function,

$$U_i = w - g_i + \ln \left[ m(g_i + g_j(g_i)) + \alpha_i( S_i + g_i - g_j(g_i)) \right]$$

and differentiating with respect to $g_i$, and solving for $g_i$ we obtain the following optimal contribution

$$g^{Seq,Sen}_i = \begin{cases} 
0 & \text{if } \alpha_i \in \left[0, \alpha_i^A\right] \\
\frac{(\alpha_j + \alpha_j S_{jm} - m - \alpha_j[\alpha_j(S_i+S_j-1)+(S_i-3)m]}{2(\alpha_j + \alpha_j)m} & \text{if } \alpha_i > \alpha_i^A
\end{cases}$$
where \( \alpha_i^A = \frac{(\alpha_i + \alpha_j S_i - m) m}{\alpha_i (S_i + S_j - 1) + (S_j - 3) m} \). Given the above contribution of the first mover, we can now use \( g_j(g_i) \) to find player \( j \)’s equilibrium contribution.

\[
S_{eq,Sen} \quad g^j_{j} = \begin{cases} 
\frac{m}{\alpha_i + \alpha_j} + \frac{4 \alpha_j}{\alpha_i + \alpha_j} \times \frac{1 - \frac{\alpha_i S_i}{\alpha_i + m}}{(\alpha_i + \alpha_j) m} & \text{if } \alpha_i \in [0, \alpha_i^A] \\
0 & \text{if } \alpha_i > \alpha_i^B 
\end{cases}
\]

where \( \alpha_i^A = \frac{(\alpha_i + \alpha_j S_i - m) m}{\alpha_i (S_i + S_j - 1) + (S_j - 3) m} \) and \( \alpha_i^B = \frac{m (m^2 - (S_j - 3) \alpha_i^2 - \alpha_j S_j m)}{\alpha_i^2 (S_i + S_j - 1) + m \alpha_j (S_j - 4) - m^2 (S_j - 1)} \). Differentiating \( g_{j,SM,Sen} \) and \( g_{j,SM,Sen} \) with respect to \( S_i \) and \( S_j \), respectively, yields

\[
\frac{\partial g_{j,SM,Sen}}{\partial S_i} = \frac{\alpha_i (\alpha_i + m)}{2 (\alpha_i + \alpha_j) m} \quad \text{and} \quad \frac{\partial g_{j,SM,Sen}}{\partial S_j} = -\frac{\alpha_j (\alpha_i + m)}{2 (\alpha_i + \alpha_j) m}
\]

which are negative for all parameter values. Similarly, differentiating \( g_{i,SM,Sen} \) and \( g_{j,SM,Sen} \) with respect to \( S_j \) and \( S_i \), respectively, yields

\[
\frac{\partial g_{i,SM,Sen}}{\partial S_j} = \frac{\alpha_j (m - \alpha_i)}{2 (\alpha_i + \alpha_j) m} \quad \text{and} \quad \frac{\partial g_{j,SM,Sen}}{\partial S_i} = \frac{\alpha_i (m - \alpha_j)}{2 (\alpha_i + \alpha_j) m}
\]

which are negative if and only if \( m < \alpha_i \) and \( m < \alpha_j \) respectively.

Applying Romano and Yildirim (2001), we know that whenever \( 1 + \frac{\partial g_{j}(g_i)}{\partial g_j} > 0 \), the sign of \( \frac{\partial U_i}{\partial g_j} \) and \( G_{eq} - G_{Sm} \) coincide. We first find \( 1 + \frac{\partial g_{j}(g_i)}{\partial g_j} \). In particular,

\[
1 + \frac{\partial g_{j}(g_i)}{\partial g_j} = 1 + \frac{\alpha_j - m}{\alpha_j + m}
\]

which is positive for any \( \alpha_j > 0 \). On the other hand,

\[
\frac{\partial U_i}{\partial g_j} = \frac{-\alpha_i + m}{\alpha_i (S_i + g_i - g_j) + m (g_i + g_j)}
\]

which is negative if and only if \( \alpha_i > m \). Then, from corollary 1, we know that for all \( j \neq i \)

\[
\frac{\partial U_i}{\partial g_j} \frac{\partial g_{j}(g_i)}{\partial g_j} \frac{\partial g_{j}(g_i)}{\partial g_i} = \begin{cases} 
> 0 & \text{if } \alpha_i < m \text{ and } \alpha_j > m, \\
< 0 & \text{otherwise}
\end{cases}
\]

Therefore, if \( \alpha_i < m \) and \( \alpha_j > m \) for all \( j \neq i \), then \( \frac{\partial U_i}{\partial g_j} \frac{\partial g_{j}(g_i)}{\partial g_j} > 0 \) and \( G_{eq} > G_{Sm} \); and if \( \alpha_i > m \) and \( \alpha_j > m \) (or if \( \alpha_i < m \) and \( \alpha_j < m \)), then \( \frac{\partial U_i}{\partial g_j} \frac{\partial g_{j}(g_i)}{\partial g_j} < 0 \) and \( G_{eq} < G_{Sm} \) .