Capacity Constrained Firms and Expansion Subsidies:

Should Governments Avoid Generous Subsidies?[*]

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Abstract

This paper examines entry deterrence and signaling when an incumbent firm experiences capacity constraints. Our results show that if the costs that constrained and unconstrained incumbents incur when expanding their facilities are substantially different, separating equilibria can be supported under large parameter values whereby information is perfectly transmitted to the entrant. If, in contrast, both types of incumbent face similar expansion costs, subsidies that reduce expansion costs can help move the industry from a pooling to a separating equilibrium with associated efficient entry. Nonetheless, our results demonstrate that if subsidies are very generous entry patterns remain unaffected, suggesting a potential disadvantage of policies that significantly reduce firms' expansion costs.

KEYWORDS: Capacity constraints; Business expansions; Signaling; Entry deterrence; Subsidies.
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1. Introduction

Several industries have recently experienced capacity constraints. Examples include, among others, the oil industry, the biopharmaceutical manufacturing industry, and the freight-transportation industry.[1] In this context, firms might consider the expansion of their current facility, in order to alleviate such capacity constraint. In the absence of entry threats, firms' decision on whether to expand is straightforward, since firms expand if the increase in future profits associated to the expansion offset its expansion costs. Under entry threats, however, firms must consider not only this direct benefit but also the indirect effects that such expansion might entail. In particular, the expansion might attract potential entrants to the industry, thus suggesting that constrained incumbents face a tradeoff when considering whether or not to expand their facility. Such a tradeoff is specifically relevant in periods of economic recovery, where several firms start experiencing larger customer traffic and sales, making them more likely to experience capacity constraints.

In this paper we examine this tradeoff by studying entry deterrence in a context where the incumbent is privately informed about her capacity constraint. Specifically, the incumbent is constrained if she cannot produce her profit-maximizing output because she faces a limited plant capacity. By contrast, an unconstrained incumbent can produce her profit-maximizing output. Our model considers a setting of incomplete information in which the incumbent chooses whether to expand her facility, and subsequently, the potential entrant (uninformed about the incumbent's capacity constraint) bases her entry decision on the information he infers from the incumbent's expansion.

We first show that both separating and pooling equilibria can be sustained, where information is either perfectly conveyed to the potential entrant or concealed from him, respectively.[2] In the separating equilibrium, such information allows the entrant to base his entry decision on more accurate information about his post-entry competition. By contrast, in the pooling equilibrium the entrant cannot accurately assess the profitability of the market, and thus may enter a market that is actually unprofitable. Hence, the separating equilibrium supports entry in similar contexts as under complete information. Instead, the pooling equilibrium predicts possible entry in industries that the entrant *would have avoided* under complete information, entailing negative net profits for the entrant and subsequent exit.

The fully informative separating equilibrium can be sustained, specifically, when the constrained incumbent faces relatively lower expansion costs than the unconstrained firm. This difference in expansion costs might arise if, for instance, financial institutions discriminate constrained and

[1] As documented for each of these industries by the U.S. Energy Information Administration, BioPlan Associates Inc. in a survey among European and U.S. firms, and the University of Denver's Intermodal Transportation Institute, respectively.

[2] In addition, both the separating and pooling equilibria survive standard equilibrium refinements in signaling games, i.e., the Cho and Kreps' (1987) Intuitive Criterion, under relatively general parameter conditions.

unconstrained firms, charging different financial costs to each type. In this case, only the separating equilibrium arises where, as suggested above, entry patterns coincide with those under complete information, and no policy intervention is needed. If, by contrast, financial markets are not capable of differentiating constrained and unconstrained firms, both types of incumbents might face relatively similar expansion costs. In this case, our paper shows that government intervention might be welfare improving under certain conditions, even when the regulator is uninformed about the incumbent's cost structure. In particular, we demonstrate that a policy reducing the financial costs associated with expansion induces a change in the equilibrium outcome from a pooling to a separating equilibrium.[3] We also predict that, despite the potential benefits from lowering financial costs —inducing similar entry patterns as under complete information— such policy can be easily *overdone*, which occurs when expansion costs are reduced beyond certain levels. In particular, under extremely low expansion costs, both types of incumbent expand their facilities, changing the equilibrium prediction, from a pooling equilibrium where no type of incumbent expands to one where both types expand. Entry patterns in both pooling equilibria however coincide, thus hindering the ability of the subsidy policy to promote efficient entry.

Our results can help us evaluate the potential effects of recent federal and state policies directed at reducing firms' expansion costs. After the economic crisis, several firms have started to experience larger customer traffic and sales.[4] In this context, policies reducing expansion costs to both constrained and unconstrained firms can help businesses expand. Our paper, however, identifies a potential risk of these policies often overlooked by the existing literature: if subsidies are too generous, they might entail entry in contexts where entrants would not be attracted under complete information. Indeed, such policies can deter entry in markets which would still support the entry of new competitors, or attract entry in markets which cannot sustain further entry. Our results hence suggest that, while small subsidies can facilitate the emergence of efficient entry patterns, very generous subsidies can be completely ineffective in affecting the entry behavior of potential competitors.

Our equilibrium results hold under relatively general conditions. First, the incumbent's capacity constraint can arise from her high efficiency level or high market demand, indicating that our conclusions can be applied to settings where firms are privately informed about their capacity constraint, regardless of

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[3] Our results depend on the severity of the capacity constraint. In particular, when capacity constraints are not severe, no type of incumbent significantly benefits from breaking her capacity constraint. In this case, a policy reducing financial costs would only switch the particular pooling equilibrium being played, namely, from one where no type of incumbent expands to one where both firms expand. Hence, under weak capacity constraints our paper suggests that a policy reducing both firms' financial costs is futile, since it does not modify the entry patterns that arise in the pooling equilibria of the game.

[4] Bloomberg, for instance, reported a 5.5% increase in sales among U.S. retailers (see Bloomberg.com on December 28th, 2010), while the Wall Street Journal recorded a 4.2% sales increase among chain-stores (December 21st, 2010), relative to the same period in 2009.

the source of such constraint. Second, the paper's equilibrium predictions are not qualitatively affected if the incumbent's capacity constraint is very severe. To illustrate the application of our results, we provide a parametric example with linear demand throughout the paper.

Related literature. This paper contributes to both the literature pertaining to capacity constraints under complete information contexts, and that on signaling in entry-deterrence games. On one hand, Dixit's seminal work (1979, 1980) —analyzing the incentives for an incumbent to deter entry by expanding her capacity in a two-period game— has been expanded in other studies where both incumbent and entrant are perfectly informed. Specifically, Ware (1984) examines entry deterrence in a three-stage game, and Formby and Smith (1984) and Mason and Nowell (1992) study the incumbent's incentives to allow entry and then collude with the entrant. The role of capacity constraints as an entry deterrence device has, however, been analyzed using complete information settings, wherein every firm is able to perfectly observe other firms' cost structure. This assumption may not be sensible in certain industries that have been monopolized for long periods of time, where entrants have access to very limited information about the incumbent's cost structure. Our paper hence contributes to the literature on capacity constraints by relaxing this assumption and allowing for incomplete information.[5]

On the other hand, this paper builds upon the literature on entry deterrence in signaling games, such as Milgrom and Roberts (1982), where the entrant is uncertain about the incumbent's unit costs.[6] In their setting, only the separating equilibrium can survive standard equilibrium refinements. In our model, by contrast, incomplete information about the incumbent's capacity constraint does not necessarily imply that the constrained incumbent experiences greater benefits from investing in additional capacity than the unconstrained incumbent.[7] As the Single-Crossing Property does not necessarily hold in our model, both separating and pooling equilibria can be sustained in equilibrium and survive equilibrium refinements.

Another related article is Matthews and Mirman (1983), where the incumbent sets prices that can communicate information about market profitability to potential entrants as in our model. However, the

[5] Arvan (1986) considers an incomplete information version of Dixit's (1980) model but focuses on type-dependent strategy profiles, unlike our paper that examines both type-dependent and type-independent strategy profiles. In addition, our model allows for capacity constraints to originate from both the incumbent's efficiency level and market demand.

[6] In an extension to Milgrom and Roberts' (1982), Harrington (1986) allows for the possibility that the entrant is uncertain about his own costs after entry. Interestingly, this article shows that when the costs of the entrant and the incumbent are sufficiently positively correlated then Milgrom and Roberts' (1982) results are reversed. That is, the incumbent's production is below the simple monopoly output in order to strategically deter entry. Our model is different from Harrington (1986) because in our setting both firms know each other's costs, but the entrant is uninformed about the incumbent's capacity constraint.

[7] Essentially, the constrained incumbent experiences a larger increase in profits from expanding her facility than her unconstrained counterpart if entry is deterred, but may experience a smaller increase if entry follows. As we show in the paper, this result holds even when the constrained incumbent can finance her expansion at a lower cost than the unconstrained incumbent.

actual price in the market (which is also the message observed by the potential entrant) receives a random shock, given that the incumbent sets prices before demand is actually realized. By contrast, we assume that market demand or capacity constraints are perfectly observed by the incumbent across periods.[8] In a recent article, Ridley (2008) analyzes an environment where an informed firm's entry provides a noisy signal about market demand to additional entrants. We consider, however, that the expansion decision of the informed firm may have a favorable effect on her technology since it allows the constrained firm to increase its production, whereas Ridley (2008) assumes that the firms' costs are unaffected. This implies that the informative separating equilibrium can arise in our model only if the incumbent's expansion produces strong technological benefits in addition to serving as a signal to potential entrants. Finally, Espinola-Arredondo *et al.* (2011) considers a similar information structure, where the incumbent is informed about market demand and chooses whether to invest in cost-reducing technologies, e.g., research and development. Unlike our paper, their model assumes that the incumbent is not limited by a capacity constraint in either period, and that the investment helps the incumbent lower her marginal cost in all units.

The paper is organized as follows. Section 2 presents the model. Section 3 describes our equilibrium predictions. Section 4 examines an extension to our model, showing that the equilibrium results are qualitatively unaffected. Section 5 elaborates on the policy implications of our results and the last section concludes.

2. Model

Consider a market with an incumbent monopolist (she) and a potential entrant (he) with inverse demand function $p(Q)$ which satisfies $p'(Q) < 0$ and $p''(Q) \leq 0$. The monopolist is perfectly informed about her (constant) marginal production costs being low, c_L, or high, c_H, where $p(0)>c_H>c_L\geq 0$. In order to introduce the effect of the capacity constraint, we consider that the low-cost incumbent's profit-maximizing output would exceed her available production capacity, $\overline{q}$, whereas that of the high-cost incumbent does not. The monopolist decides whether to expand her facility (which allows the low-cost incumbent to produce her profit-maximizing output in future periods) or not to expand. The time structure of this incomplete information game is described as follows:

1. Nature determines the incumbent's marginal costs: high c_H with probability p, or low c_L with probability 1-p. The incumbent privately observes her cost structure, but the entrant does not.
2. After observing her cost structure, the incumbent decides whether to expand her facility.

[8] Bagwell and Ramey (1990) and Albaek and Overgaard (1994) examine a similar entry deterrence in a model where the potential entrant can perfectly observe both the incumbent's pre-entry pricing strategy and its advertising expenditures.

3. After observing the incumbent's expansion (or no expansion) decision, the entrant updates his beliefs about the incumbent's costs. Let $\mu(H \mid Exp)$ and $\mu(H \mid NoExp)$ denote the entrant's posterior beliefs about a high-cost incumbent after observing expansion or no expansion, respectively.[9]
4. Given these beliefs, the entrant decides whether to enter the incumbent's market, in which case firms compete in quantities or to remain in a perfectly competitive market with associated zero economic profits.[10]

We assume that expansion is costly and specify by K_H and K_L to be the incumbent's expansion costs when her marginal production costs are high and low, respectively. For generality, we do not restrict expansion costs K_H and K_L.[11][12] In order to make the entry decision interesting, we consider that the entrant's marginal costs are high. As a consequence, he has incentives to enter (stay out) when the incumbent's costs are high (low, respectively). Finally, in the case of entry, the entrant observes the incumbent's type and chooses his optimal output. In addition, we assume that the entrant must incur a fixed entry cost $F>0$ while his entry costs are zero if he remains in the perfectly competitive industry.

First period. For a production level $q_{i,K}^{S}$ let subscript "i,K" indicate firm $i=\{inc,ent\}$ (incumbent or entrant) when the incumbent's marginal costs are $K=\{H,L\}$, whereas superscript S denotes the particular market structure in which the firm operates $S=\{M,D\}$, either monopoly or duopoly. For instance, $q_{inc,H}^{M}$ denotes the monopoly profit-maximizing output for the high-cost incumbent. In particular, note that $q_{inc,H}^{M} < \overline{q}$, since this incumbent does not face a capacity constraint, yielding monopoly profits of $\pi_{inc,H}^{M,NE}$, where superscript NE denotes that the incumbent did not expand her facility.[13] By contrast, the low-cost incumbent's profit-maximizing output, $q_{inc,L}^{M}$, satisfies $q_{inc,L}^{M} > \overline{q}$ given that she faces a capacity

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[9] For compactness, we do not include posterior beliefs about the incumbent's type being low, i.e., $\mu(L \mid Exp)$ and $\mu(L \mid NoExp)$, since they are already captured by the case in which $\mu(H \mid Exp) = 0$ and $\mu(H \mid NoExp) = 0$, respectively.

[10] This information structure resembles that in standard entry-deterrence games, such as Milgrom and Roberts (1982), whereby the incumbent's costs are unobserved by the potential entrant, but the entrant's can be anticipated by the incumbent given her experience in the industry.

[11] Expansion costs might be weakly lower for the most efficient incumbent, i.e., $K_H \geq K_L$. Intuitively, this might occur when the incumbent uses a share of previous period profits to finance her expansion decision. We elaborate on this specific case in our discussion of the equilibrium results (Section 5).

[12] Note that if, rather than fixed entry costs, the incumbent faced variable capacity costs, our results would be qualitatively unaffected; see our discussion of the Single-Crossing property in this section.

[13] Note that we use superscript *NE* in first period profits since the incumbent has not expanded her facility yet. In our description of output and profit decisions during the second period, however, this superscript can either be *E* or *NE* to denote that the incumbent expands (does not expand, respectively).

constraint. Because this type of incumbent is affected by her capacity constraint $\overline{q}$, we use $\pi_{inc,L}^{M,NE}\left(\overline{q}\right)$ to represent her monopoly profits when she does not expand, where $\pi_{inc,L}^{M,NE}\left(\overline{q}\right) < \pi_{inc,L}^{M,E}$. Intuitively, this allows the capacity constraint to take several forms: from an extreme context where output cannot be further increased beyond $\overline{q}$, to milder settings where the incumbent's marginal cost experiences an increase for all units surpassing capacity level $\overline{q}$; as in Dixit (1980). Under both circumstances, nonetheless, the constrained (low cost) incumbent enjoys an increase in her profits when she relaxes ("breaks") her capacity constraint by expanding her facility, raising profits from $\pi_{inc,L}^{M,NE}\left(\overline{q}\right)$ to $\pi_{inc,L}^{M,E}$.

Example. Consider a linear inverse demand curve p(Q)=1-Q, and constant marginal costs c_H=1/5 for the high-cost incumbent (and the entrant) and c_L=0 (only for the low-cost incumbent). In addition, assume a capacity constraint $\overline{q}$=1/6 that the low-cost incumbent cannot exceed. Then, for the high-cost incumbent we have a monopoly output of $q_{inc,H}^{M}=\frac{1-c_H}{2}=\frac{1}{10}<\frac{1}{6}=\overline{q}$, yielding profits of $\pi_{inc,H}^{M,NE}=\dfrac{4}{25}$ when the incumbent does not expand, and similarly $\pi_{inc,H}^{M,E}=\dfrac{4}{25}$ when she expands, since she is unaffected by a capacity constraint. By contrast, the low-cost incumbent can only produce $\overline{q}=\frac{1}{6}$ given that its profit-maximizing output under monopoly, $q_{inc,L}^{M}=\frac{1}{2}$, exceeds $\overline{q}$. As a consequence, her profits become $\pi_{inc,L}^{M,NE}(\overline{q})=\dfrac{5}{36}$ if this type of incumbent does not expand, but increase to $\pi_{inc,L}^{M,E}=\dfrac{1}{4}$ if she expands.

Second period, No entry. In the second period, if there is no entry and the incumbent does not expand her facility, then the high(low)-cost incumbent's profits are $\pi_{inc,H}^{M,NE}$ ($\pi_{inc,L}^{M,NE}\left(\overline{q}\right)$, respectively), coinciding with those in the first period. By contrast, if the high-cost incumbent expands, her profits become $\pi_{inc,H}^{M,E}-K_H$, where $\pi_{inc,H}^{M,E}=\pi_{inc,H}^{M,NE}$, since this incumbent did not originally face a capacity constraint.[14] The low-cost incumbent, however, faces a capacity constraint $\overline{q}$ and her expansion decision helps produce her profit-maximizing output $q_{inc,L}^{M}>\overline{q}$ (since she breaks her capacity constraint) entailing profits of $\pi_{inc,L}^{M,E}-K_L$, where $\pi_{inc,L}^{M,E}>\pi_{inc,L}^{M,NE}(\overline{q})$.

[14] This implies that the unconstrained monopolist does not modify her sales after expanding her facility and, as suggested below, her benefits from expansion arise only if entry is deterred.

Second period, Entry. If entry occurs, firms compete as Cournot duopolists. Let us first analyze the case in which the incumbent does not expand her facility. If the incumbent's costs are high, duopoly profits are $\pi_{inc,H}^{D,NE}$ and $\pi_{ent,H}^{D,NE} - F > 0$ for incumbent and entrant, respectively. Intuitively, the entrant obtains a positive profit from entering, since he competes with a high-cost incumbent. If, by contrast, the incumbent's costs are low, the incumbent produces a duopoly profit-maximizing output of $q_{inc,L}^{D,NE}$ in the case that she did not expand her facility. In this section, we consider that this incumbent also faces a capacity constraint under duopoly.[15] Intuitively, the capacity constraint she faces is relatively strong, yielding profits of $\pi_{inc,L}^{D,NE}(\overline{q})$ which are a function of the capacity constraint $\overline{q}$. In this case, the entrant produces $q_{ent,L}^{D,NE}$ with associated profits of $\pi_{ent,L}^{D,NE} - F$.

Let us now examine the case where the incumbent expands her facility. The high-cost incumbent's duopoly profits are $\pi_{inc,H}^{D,E} - K_H$, while the entrant's duopoly profits are $\pi_{ent,H}^{D,E} - F > 0$. Intuitively, note that $\pi_{i,H}^{D,E} = \pi_{i,H}^{D,NE}$ for any firm *i*={*inc*,*ent*} since the production capacity of the high-cost incumbent is unaffected by her expansion decision. Finally, if the incumbent's costs are low, her expansion decision helps break her capacity constraint, producing $q_{inc,L}^{D,E} > \overline{q}$, and yielding profits $\pi_{inc,L}^{D,E} - K_L$ and $\pi_{ent,L}^{D,E} - F$ for the incumbent and entrant, respectively. As suggested above, the entrant has incentives to enter the market only when competing with the high-cost incumbent. This implies that entry costs, *F*, satisfy $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$ when the incumbent does not expand and $\pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,E}$ when the incumbent expands.[16]

Example. Following our previous parametric example, in the case of entry the high-cost incumbent produces a duopoly output of $\frac{1-c_H}{3} = \frac{1}{15}$, both after expansion and no expansion, i.e., $q_{inc,H}^{D,NE} = q_{inc,H}^{D,E} = \frac{1}{15}$, yielding duopoly profits of $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E} = \dfrac{16}{225}$. For the low-cost incumbent, note that: (1) if

[15] At the end of section 3 we relax this assumption and show that our equilibrium results are not qualitatively affected. In particular, we allow for the capacity constraint to be binding (not binding) under monopoly (duopoly, respectively). Thus, the capacity constraint will not be as severe as in our current analysis, where it affects the incumbent both under monopoly and duopoly. Note that this assumption can alternatively be interpreted in terms of the efficiency of the low-cost incumbent. Specifically, for a given capacity constraint, a decrease in her marginal cost c_L implies that the incumbent finds the capacity constraint limiting under both market structures.

[16] Nonetheless, the above two conditions can be summarized as $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$. In particular, the entrant's profits satisfy $\pi_{ent,H}^{D,E} = \pi_{ent,H}^{D,NE}$ since the high-cost incumbent is not affected by the capacity constraint —and therefore the expansion decision does not modify her production capacity in the second period— but $\pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$ given that the entrant's duopoly profits decrease when the low-cost incumbent eliminates her capacity constraint.

she expands, her duopoly output when competing against a high-cost entrant is $q_{inc,L}^{D,E} = \frac{6}{15}$, yielding profits of $\pi_{inc,L}^{D,E} = \frac{4}{25}$; whereas (2) if she does not expand, she is limited by her capacity constraint since $q_{inc,L}^{D,E} = \frac{6}{15} > \frac{1}{6} = \overline{q}$, entailing profits of $\pi_{inc,L}^{D,NE} = \frac{1}{12}$.

2.1. Expansion benefits

The expansion decision produces, on one hand, a direct benefit from enlarging her facility ("breaking" the capacity constraint), which allows the incumbent to produce using her efficient cost structure. Importantly, note that only the low-cost incumbent enjoys this direct benefit, since her production is limited by the capacity constraint, whereas the high-cost incumbent does not. On the other hand, expansion brings a potential loss in profits if such expansion attracts entry. (Alternatively, expansion provides an indirect benefit if it deters entry). Let $BCC_L = \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\overline{q}) > 0$ denote the benefits from breaking the capacity constraint for the low-cost incumbent (direct benefit), and let $PLE_K^E = \pi_{inc,K}^{M,E} - \pi_{inc,K}^{D,E} > 0$ represent the profits loss that the *K*-type incumbent suffers from entry, where $K=\{H,L\}$. If despite not expanding her facility, entry follows, the low-cost incumbent experiences a profit loss of $PLE_L^{NE} = \pi_{inc,L}^{M,NE}(\overline{q}) - \pi_{inc,L}^{D,NE}(\overline{q}) > 0$.[17]

Hence, if expansion deters entry, the low-cost incumbent benefits from BCC_L, while the high-cost incumbent obtains no benefits or losses. If, by contrast, expansion does not deter entry, the low-cost incumbent's BCC_L benefit is reduced by the profit loss of sharing the market with the entrant, i.e., $BCC_L - PLE_L^E < BCC_L$, whereas the high-cost incumbent only bears the profit loss due to entry, i.e., $-PLE_H^E$ <0. Note that expansion benefits when entry ensues, $BCC_L - PLE_L^E$, are only positive if $BCC_L - PLE_L^E = \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q}) > 0$. Intuitively, this condition holds when the low-cost incumbent is significantly limited by her capacity constraint, and thus her monopoly profits before expanding her facility, $\pi_{inc,L}^{M,NE}(\overline{q})$, are lower than her duopoly profits after the expansion, $\pi_{inc,L}^{D,E}$. This implies that her benefit from breaking the capacity constraint offsets the profit loss associated to entry, i.e.,

[17] Note that the profit loss from entry for the high-cost incumbent when she expands, $PLE_H^E = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}$, coincides with that when he does not, $PLE_H^{NE} = \pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}$, since $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$ under monopoly and $\pi_{inc,H}^{D,E} = \pi_{inc,H}^{D,NE}$ under duopoly.

$BCC_L > PLE_L^E$. Finally, if despite not expanding entry occurs, the incumbent only experiences a profit loss due to entry of $-PLE_K^{NE}$ <0 for all $K=\{H,L\}$. For instance, in the above parametric example, a relatively small facility, $\overline{q} = \frac{1}{6}$, yields a benefit from breaking the capacity constraint, $BCC_L = \frac{1}{4} - \frac{5}{36} = \frac{1}{9}$, which exceeds the profit loss if entry ensues, $PLE_L^E = \frac{1}{4} - \frac{4}{25} = \frac{9}{100}$.[18]

Single-Crossing Property. The single-crossing property is not necessarily satisfied under all conditions. In particular, if expansion deters entry, the low-cost incumbent obtains larger benefits from expanding her facility, $BCC_L > 0$, than the high-cost does, i.e., the latter obtains zero profits from breaking the capacity constraint. When expansion attracts entry, however, the benefits for the low-cost incumbent, $BCC_L - PLE_L^E$, are larger than for the high-cost type, $-PLE_H^E < 0$, if and only if $\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q}) > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E}$. More precisely, since $\pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E} < 0$, the previous condition is guaranteed to hold as long as $\pi_{inc,L}^{D,E} > \pi_{inc,L}^{M,NE}(\overline{q})$.[19] From our above discussion, this occurs when $BCC_L - PLE_L^E > 0$ which intuitively implies that the low-cost incumbent's benefit from breaking her capacity constraint outweighs her profit loss from attracting entry.[20]

3. Equilibrium Analysis

Before analyzing the set of Perfect Bayesian Equilibria (PBE) of this signaling game, let us introduce some additional notation. In particular, let p^{NE} denote the probability that makes an entrant indifferent between the expected profits from Cournot competition after no expansion,

[18] Note that $PLE_L^{NE} = 5/36 - 1/12 = 1/18$ for the low-cost incumbent, and that $PLE_H^E = PLE_H^{NE} = 4/25 - 16/225 = 4/48$ for the high-cost incumbent.

[19] Note that $\pi_{inc,L}^{D,E} > \pi_{inc,L}^{M,NE}(\overline{q})$ is a sufficient condition whereas $\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q}) > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E}$ is a necessary condition for the single-crossing property to hold. Hence, if $\pi_{inc,L}^{D,E} > \pi_{inc,L}^{M,NE}(\overline{q})$ is not satisfied, condition $\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q}) > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E}$ can still hold if, for instance, the profit loss due to entry for the high-cost incumbent, PLE_H^E, is relatively large.

[20] A similar argument would also be valid if the incumbent's efficiency, rather than being drawn from only from two levels, high and low, was drawn from a continuum of expansion investments. In general, when expansion is followed by entry, the low-cost incumbent does not necessarily obtain a larger benefit from marginally increasing her investment in comparison to a high-cost incumbent.

$p\left(\pi_{ent,H}^{D,NE}-F\right)+(1-p)\left(\pi_{ent,L}^{D,NE}-F\right)$ and the profits from operating in the alternative market,[21] where superscript *NE* represents that the incumbent *did not expand* her facility. Similarly, let p^E denote the probability that makes the entrant indifferent between the expected profits from Cournot competition after expansion, $p\left(\pi_{ent,H}^{D,E}-F\right)+(1-p)\left(\pi_{ent,L}^{D,E}-F\right)$ and the profits from operating in the alternative market, where superscript *E* represents that the incumbent *expanded* her facility. The following lemma shows that these probability cutoffs satisfy $p^E > p^{NE}$.

Lemma 1. *The potential entrant enters under a larger set of priors when the incumbent does not expand her facility than when she does, i.e.,* $p^E > p^{NE}$.

Intuitively, since the entrant's ex-ante profits are decreasing in the probability that the incumbent's costs are high, *p*, the range of priors for which he is attracted to enter is larger when the incumbent does not expand, $p > p^{NE}$, than if she does, for all $p > p^E$, since $p^E > p^{NE}$.[22] Our description of equilibrium outcomes is separated into three regimes according to the priors: low, $p < p^{NE}$, intermediate, $p^{NE} \leq p < p^E$, and high priors, $p \geq p^E$. The following proposition identifies the set of PBE under the first regime.

Proposition 1. *When priors are relatively low,* $p < p^{NE}$*, the following strategy profiles can be supported as PBEs of the expansion signaling game:*

1. *A separating equilibrium where the incumbent chooses* ($NoExp_H$, Exp_L)*, and the entrant selects* ($Enter_{NoExp}$, $NoEnter_{Exp}$) *if and only if expansion costs satisfy* $K_H > PLE_H^E$ *and* $K_L < BCC_L + PLE_L^{NE}$ *and the entrant's beliefs are* $\mu(H \mid NoExp)=1$ *and* $\mu(H \mid Exp)=0$*;*
2. *A pooling equilibrium with expansion,* (Exp_H, Exp_L)*, and the entrant selects* ($Enter_{NoExp}$, $NoEnter_{Exp}$) *if and only if expansion costs satisfy* $K_L < BCC_L + PLE_L^{NE}$ *and* $K_H < PLE_H^E$ *and the entrant's beliefs are* $\mu(H \mid Exp) = p < p^E$ *and* $\mu(H \mid NoExp) \geq p^{NE}$*;*
3. *A pooling PBE where both types of incumbent do not expand their facility* ($NoExp_H$, $NoExp_L$) *followed by no entry, where either:*

[21] Recall that the entrant obtains zero profits on the alternative perfectly competitive market where information is readily available to any potential entrants.

[22] The expressions for p^{NE} and p^E are obtained by solving for *p* in these indifference conditions. They are both included in the proof of Lemma 1 in the appendix. We show that $p^E, p^{NE} \in [0,1]$, and that these expressions satisfy $p^E > p^{NE}$ under all parameter values.

a) *the entrant's strategy is* $\left(NoEnter_{NoExp}, Enter_{Exp}\right)$ *given that her off-the-equilibrium beliefs are* $\mu(H \mid Exp) = \mu \geq p^E$ *, for expansion costs satisfying* $K_L > BCC_L - PLE_L^E$ *and* $K_H > 0$*; or*

b) *the entrant's strategy is* $\left(NoEnter_{NoExp}, NoEnter_{Exp}\right)$ *given that her off-the-equilibrium beliefs are* $\mu(H \mid Exp) = \mu < p^E$ *, for expansion costs satisfying* $K_L > BCC_L$ *and* $K_H > 0$.

The following figure depicts the set of expansion costs (K_H, K_L) under which each of the above PBEs can be sustained. First, when the low-cost expansion costs are relatively low, $K_L < BCC_L + PLE_L^{NE}$, but those of the high-cost incumbent are relatively high, i.e., $K_H > PLE_H^E$, the former expands her facility while the latter does not. Under these expansion costs, hence, information about the incumbent's cost structure is perfectly transmitted to the entrant, deterring her from entering after observing an expansion. Despite the information transmission, however, note that the incumbent's expansion decision can be supported under different parameter conditions than under complete information, since the incumbent is willing to incur more expensive expansion costs in order to convey her type to the entrant, thus deterring entry. Specifically, if the entrant were informed about the incumbent's costs being low, he would be deterred, and hence the low-cost incumbent would expand if $K_L < BCC_L$, i.e., if expansion costs are lower than the only benefit from expansion under complete information embodied in BCC_L .[23] Therefore, the entrant's lack of information about the incumbent's type induces the low-cost incumbent to expand her facility under a larger set of expansion costs, $K_L < BCC_L + PLE_L^{NE}$, than in the complete information setting, $K_L < BCC_L$, since $BCC_L + PLE_L^{NE} > BCC_L$.

[23] Under complete information, the high-cost incumbent does not expand since entry ensues and such an expansion does not bring any direct benefit. Hence, in such an information context the low (high)-cost incumbent expands (does not expand) if $K_L < BCC_L$ and for any $K_H > 0$.

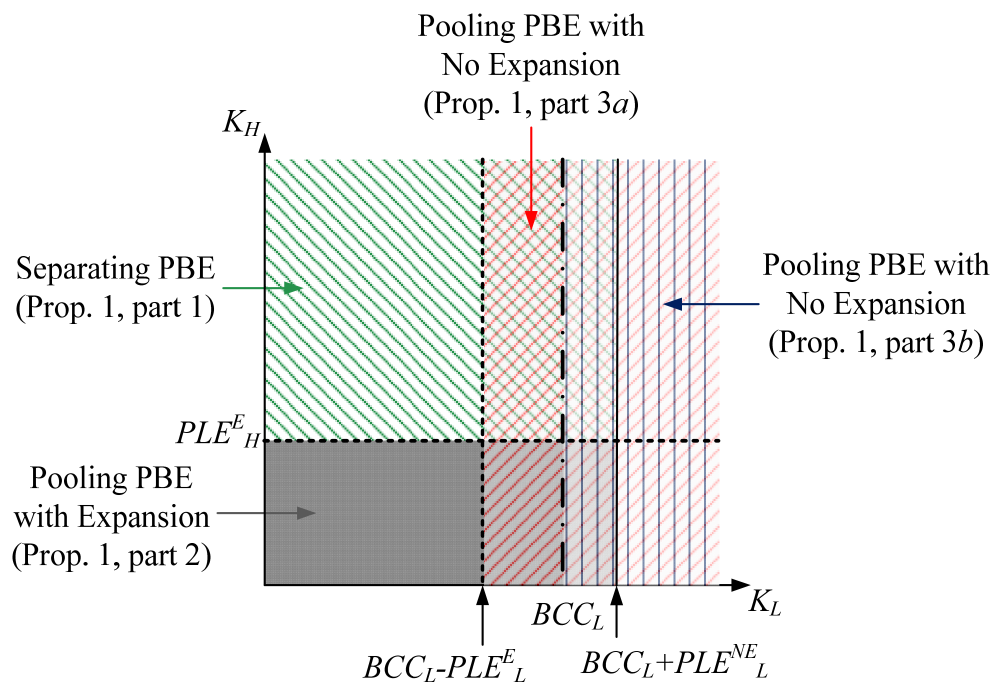

Figure 1: Equilibrium predictions under low priors.

The figure also depicts a pooling equilibrium (described in part 2 of the Proposition 1) whereby both types of incumbent expand their facility given that expansion costs are relatively low, i.e., $K_L < BCC_L + PLE_L^{NE}$ and $K_H < PLE_H^E$. By expanding, incumbents successfully deter entry, which occurs because prior probability p is sufficiently low. Hence, the high-cost incumbent expands if the profit loss she avoids by deterring entry is larger than her expansion costs, i.e., $K_H < PLE_H^E$. On the other hand, the low-cost incumbent expands if her expansion cost is lower than the foregone benefits from expansion, which arise not only from the profit loss she avoids by deterring entry but also from the benefits she obtains by breaking the capacity constraint, i.e., $K_L < BCC_L + PLE_L^{NE}$.

Finally, the pooling equilibrium described in part 3 of Proposition 1 examines the case where no type of incumbent expands given the high expansion costs and that the entrant is deterred after observing no expansion. This outcome is supported in both pooling equilibria described in parts 3a and 3b. These equilibria differ, however, in their off-the-equilibrium predictions. In particular, the equilibrium in part 3a considers that, after observing a deviation towards expansion, the entrant believes that the incumbent's costs must be high. These off-the-equilibrium beliefs are, however, not very sensible and do not survive standard equilibrium refinements.[24] If, by contrast, the entrant believes that a deviation towards expansion

[24] Appendix 1 shows that the deviation towards expansion is more likely to originate from the low- than from the high-cost incumbent. Therefore, the pooling equilibrium in part 3a violates the Cho and Kreps' (1987) Intuitive Criterion under most parameter conditions.

originates from a low-cost incumbent (as described in part 3b), then he does not enter, and both types of incumbent have incentives to expand their facilities.

Comparative statics. Let us next analyze how an increase in the efficiency of the low-cost incumbent affects our equilibrium results. In particular, a reduction in c_L reflects a more efficient low-cost incumbent,.[25] which becomes more limited by her capacity constraint and, therefore, obtains a larger benefit from expanding her facility, i.e., BCC_L raises. Note that the pooling equilibria described in Proposition 1 (part 3) —where the low-cost incumbent does not expand— are sustained under more restrictive parameter conditions as the low-cost incumbent becomes more efficient, i.e., the region corresponding to this equilibrium in figure 1 shrinks. Indeed, a larger efficiency increases her benefits from breaking the capacity constraint, BCC_L, inducing her to expand under larger expansion costs. By contrast, an increase in the efficiency level of the low-cost incumbent expands the set of parameter values under which we can support the separating equilibrium and the pooling equilibrium where both types of incumbent expand their facility.

Let us next examine equilibrium outcomes when priors are intermediate, i.e., $p^{NE} \leq p < p^E$.

Proposition 2. *When priors satisfy $p^{NE} \leq p < p^E$, the strategy profiles described in parts 1 and 2 of Proposition 1 can still be can be supported as PBEs of the expansion signaling game. In addition, a pooling equilibrium with no expansion can be sustained, ($NoExp_H, NoExp_L$), followed by entry (since priors satisfy $\mu(H \mid NoExp) = p \geq p^{NE}$) if either of the following two cases arises:*

a) *The entrant's strategy is $\left(Enter_{NoExp}, NoEnter_{Exp}\right)$ given that her off-the-equilibrium beliefs are $\mu(H \mid Exp) = \mu < p^E$ and expansion costs satisfy $K_L > BCC_L + PLE_L^{NE}$ and $K_H > PLE_H^E$; or*
b) *The entrant's strategy is $\left(Enter_{NoExp}, Enter_{Exp}\right)$ given that her off-the-equilibrium beliefs are $\mu(H \mid Exp) = \mu \geq p^E$ and expansion costs satisfy $K_L > BCC_L + PLE_L^{NE} - PLE_L^E$ and $K_H > 0$.*

The following figure depicts equilibrium outcomes when the prior probability of facing a high-cost incumbent is intermediate.[26]

[25] This implies that $BCC_L < PLE_L^{NE}$ when the incumbent is not very efficient (c_L is relatively high), but $BCC_L > PLE_L^{NE}$ when her efficiency increases (c_L is relatively low).

[26] Note that the figure depicts the case where $PLE_H^E > PLE_L^{NE}$. An analogous figure can be constructed otherwise.

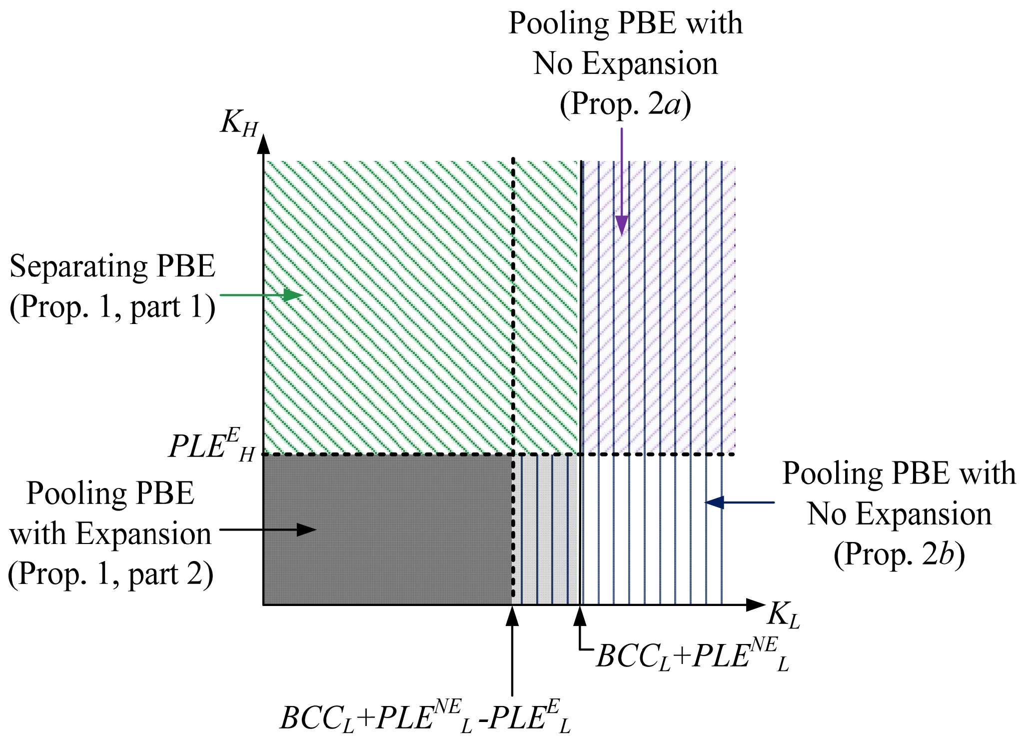

Figure 2: Equilibrium predictions under intermediate priors.

Relative to the set of equilibria when priors are relatively low depicted in figure 1, two equilibrium outcomes can still be supported under intermediate priors: the (fully informative) separating equilibrium and the pooling equilibrium where both types of incumbent expand their facility. Nonetheless, two pooling equilibria with no expansion emerge; as described in Proposition 2a and 2b.[27] First, Proposition 2a specifies a pooling equilibrium similar to that in Proposition 1, part 3a. Unlike that equilibrium, however, the entrant is now attracted to the market after observing no expansion (in equilibrium) since priors are relatively high, both in the equilibria described in Proposition 2a and 2b. First, if entry follows after no expansion but does not otherwise (as described in Proposition 2a), the high-cost incumbent does not expand (as prescribed) if the benefit she would obtain from deterring entry —by deviating towards expansion— is lower than her expansion costs, i.e., $K_H > PLE_H^E$. Similarly, the low-cost incumbent does not expand if the benefit she would obtain from expanding (not only arising from avoiding entry but also from breaking the capacity constraint) is lower than her expansion costs, i.e., $K_L >$ $BCC_L + PLE_L^E$. If, by contrast, the entrant believes that a deviation towards expansion must originate from a high-cost incumbent, then entry follows regardless of the incumbent's action (as described in

[27] In addition, Appendix 2 shows that, under relatively general conditions, a semiseparating equilibrium can be sustained, whereby one or both types of incumbent randomize their expansion decision.

Proposition 2b). These off-the-equilibrium beliefs, however, are not very sensible since the low-cost incumbent is more likely to deviate towards expansion than the high-cost incumbent, and indeed violate standard equilibrium refinements.[28] Let us finally examine our equilibrium predictions under relatively high priors, i.e., $p \geq p^E$.

Proposition 3. *When priors satisfy $p \geq p^E$, only the separating strategy profile described in Proposition 1 (part 1), and the pooling strategy profiles with no expansion specified in Proposition 2a and 2b can be supported as PBEs of the expansion signaling game.*

Therefore, the pooling equilibrium where both types of incumbent expand cannot be sustained when priors are relatively high. Intuitively, the expansion decision by both types of incumbent keeps the entrant "in the dark" about the incumbent's cost structure and entry is deterred when priors satisfy $p<p^E$. When priors are relatively high $p \geq p^E$, however, this strategy would attract entry, leading the incumbent not to use it. All other equilibrium outcomes can still be sustained in this context.

Not severe capacity constraints. In our previous analysis, we consider that the low-cost incumbent is severely limited by her capacity constraint. In particular, when she does not expand her facility, she is constrained both as a monopolist and as a duopolist, yielding profits of $\pi_{inc,L}^{M,NE}(\overline{q})$ and $\pi_{inc,L}^{D,NE}(\overline{q})$, respectively. Therefore, the profit loss from entry was defined as $PLE_L^{NE} \equiv \pi_{inc,L}^{M,NE}(\overline{q}) - \pi_{inc,L}^{D,NE}(\overline{q})$. If, however, the capacity constraint affects the incumbent only as a monopolist, then such a profit loss can be expressed as $\pi_{inc,K}^{M,NE}(\overline{q}) - \pi_{inc,K}^{D,NE}$. Since profits are lower when the capacity constraint is binding, $\pi_{inc,K}^{D,NE}(\overline{q}) < \pi_{inc,K}^{D,NE}$, the profit loss is more substantial when the incumbent is severely limited by the capacity constraint than otherwise.[29] As a consequence, a reduction in PLE_L^{NE} shrinks the set of parameter values supporting the separating equilibrium and the pooling equilibrium with expansion (described in parts 1 and 2 of Proposition 1). Intuitively, the low-cost incumbent obtains a smaller benefit from deterring entry, and therefore expands only under cheaper expansion costs.

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[28] Appendix 1 shows that the pooling equilibrium described in Proposition 2*b* violates the Cho and Kreps' (1987) Intuitive Criterion for all expansion costs lower than the benefits that the low-cost incumbent obtains from breaking her capacity constraint and protecting the market, i.e., $K_L < BCC_L + PLE_L^E$

[29] Note that from our above discussion, only the profit loss from entry is reduced. In particular, PLE_L^{NE} decreases, but cutoffs BCC_L, PLE_H^{NE} and PLE_K^E are unaffected.

Remark. For simplicity, we focus on the set of pure-strategy PBEs. Appendix 2 elaborates on the properties of equilibria in which either one type of incumbent (or both) randomize their expansion decision, i.e., semiseparating equilibria. Intuitively, this appendix shows semiseparating equilibria can be sustained under parameter conditions for which pure-strategy PBE already exist. These semiseparating equilibria, nonetheless, predict that the low-cost incumbent does not necessarily expand her facility with a higher probability than the high-cost incumbent, thus limiting the informative role of the incumbent's expansion decision. Importantly, these equilibria can be sustained when both types of incumbent face relatively low expansion costs. Therefore, a policy that produces a symmetric reduction in the expansion costs of both types of incumbent —which occurs, for instance, when government agencies cannot observe the incumbent's costs— can potentially promote this type of semiseparating equilibria, whereby information is essentially concealed from the entrant.

4. Extension – Market demand as a capacity constraint

In this section we show that similar equilibrium outcomes can be sustained when the incumbent's constraint arises from a significant market demand, rather than from her efficiency level (low production costs) considered above. Consider a similar signaling model as in the previous section, where the incumbent's production costs are common knowledge but market demand is only observed by the incumbent. The entrant, however, only knows the prior probability that demand is high or low. Similarly as in our previous model, the incumbent decides whether to expand her facility, and the entrant, observing the incumbent's decision, chooses to enter the market. In this case, the incumbent experiences a capacity constraint when demand is high, but does not when demand is low.[30] The entrant prefers to enter the market when demand is high but stay out otherwise. Hence, let $BCC_H \equiv \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\overline{q}) > 0$ denote the benefit that the high-demand incumbent obtains from breaking her capacity constraint. Similarly as in our above setting, let $PLE_K^E = \pi_{inc,K}^{M,E} - \pi_{inc,K}^{D,E} > 0$ represent the profit loss from entry that the incumbent suffers after expanding her facility, while $PLE_K^{NE} = \pi_{inc,K}^{M,NE}(\overline{q}) - \pi_{inc,K}^{D,NE}(\overline{q}) > 0$ denote the profit loss after no expansion, for any demand level $K = \{H, L\}$. The following result shows that the strategy profiles specified in propositions 1-3 can also be supported in this information context by switching the type of incumbent who experiences a capacity constraint: from the low-cost incumbent in the previous setting to the high-demand incumbent in this environment. Hence, regions of expansion costs (K_H, K_L) sustaining every equilibrium are switched, from the low-cost (high-cost) incumbent to the high-demand (low-

[30] Similarly as in our previous model, each capacity constraint $\overline{q}$ can be interpreted as a maximum production level that the high-demand incumbent cannot exceed, but also as an increase in the incumbent's marginal costs of production when her output exceeds $\overline{q}$, as in Dixit (1980).

demand) incumbent, respectively. For instance, the separating equilibrium where only the constrained high-demand incumbent expands can be supported if $K_H < BCC_H + PLE_H^{NE}$ and $K_L > PLE_L^E$.

Proposition 4. *In the expansion signaling game where the potential entrant does not observe market demand, the separating and pooling strategy profiles described in Propositions 1-3 can be sustained as PBEs, where the regions of expansion costs supporting each equilibrium are switched across the two types of incumbent, from the low-cost (high-cost) incumbent to the high-demand (low-demand) incumbent, respectively.*

Thus, our results in the previous section can be extended to different information contexts where the incumbent suffering a capacity constraint conveys her type to the entrant in the separating equilibrium of the game regardless of the source of such a constraint. A similar intuition is applicable to the pooling equilibria, whereby the incumbent's actions conceal whether or not she faces a capacity constraint.

5. Discussion and policy implications

Let us next evaluate the welfare properties of the above equilibria. In the separating equilibrium, entry occurs in high-demand markets, but it is deterred in low-demand markets. This implies that the entry pattern described in the separating equilibrium of the game coincides with that arising under complete information. If, instead, the entrant entered into low-demand markets, his overall profits (net of entry costs) would be negative, inducing the entrant to fail in the long run and exit the industry. By contrast, entry in pooling equilibria does not ensue in similar conditions as under complete information, since it might occur (not occur) when entry is not (is, respectively) profitable given the entrant's lack of accurate information. For instance, entry follows in the pooling equilibrium even when demand is low if priors are sufficiently high, whereas entry does not occur despite demand being actually high when priors are sufficiently low. A similar argument can be extended to the context where the entrant is uninformed about the incumbent's efficiency level.

Our discussion suggests that public agencies should promote separating equilibria in order to support more desirable entry profiles. We can, hence, examine under which conditions this type of equilibrium occurs and what policies can facilitate it. The separating equilibrium can be sustained if the expansion costs that the constrained incumbent faces are significantly lower than those of the unconstrained incumbent. Such difference in expansion costs can arise if, for instance, constrained incumbents —either very efficient firms or incumbents operating in high-demand markets— accumulate profits before their expansion decision. Hence, a constrained incumbent could self-finance a larger portion of her expansion than the unconstrained incumbent, thus not having to access capital markets to the same extent as the unconstrained firm. In this case, no government intervention would be necessary since entry patterns in the separating equilibrium are similar to those under complete information.

If, however, financial institutions are unable to differentiate among both types of incumbent, firms face similar expansion costs. In such a case, our results suggest socially improving policies. Specifically, if expansion costs are symmetric and relatively high, a policy reducing expansion costs can induce a change in incumbents' expansion decision, from a pooling equilibrium where no type of incumbent expands (producing undesirable entry patterns) to a separating equilibrium where only the constrained incumbent expands (generating desirable entry). The following figure illustrates the effect of this policy, as a reduction in the symmetric expansion costs (those along the diagonal $K_H=K_L$) from point A to B.[31] Importantly, note that a policy radically reducing firms' expansion costs might produce undesirable outcomes; as depicted by point C in the figure where expansion costs are so significantly reduced at C that both types of incumbent choose to expand their facility, thus hindering the ability of expansion decisions to serve as an informative signal for potential entrants.

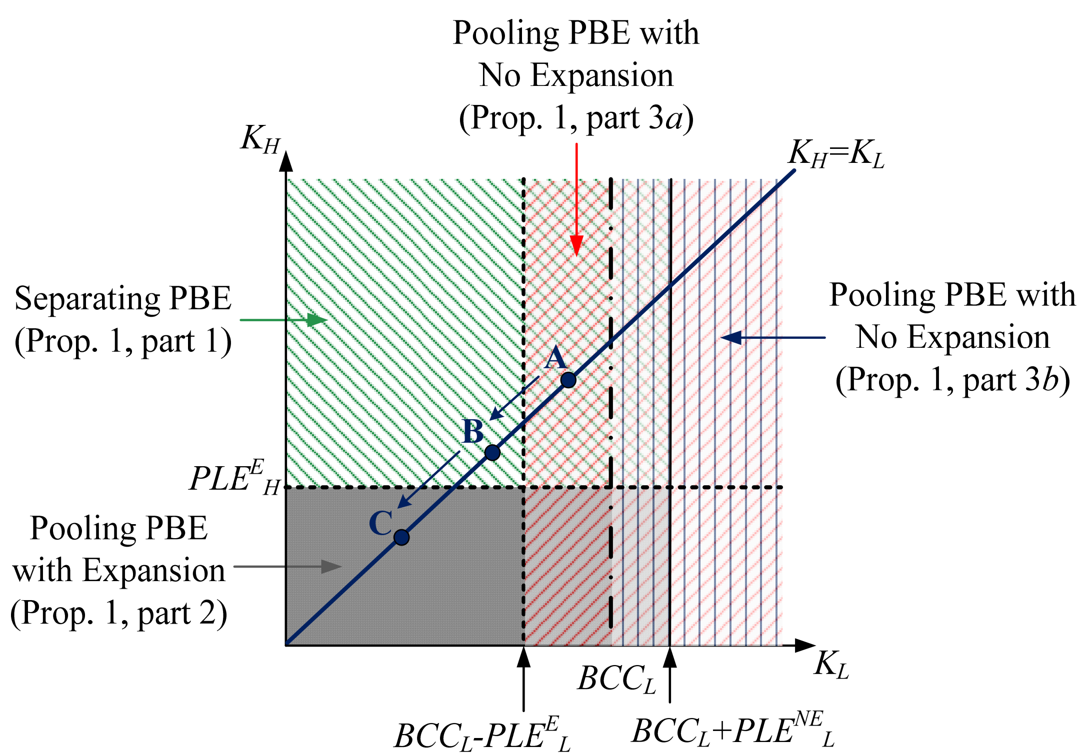

Figure 3: Effects of a policy reducing expansion costs.

[31] For simplicity, the figure considers $p<p^{NE}$ as in figure 1. Analogous figures can be applied to different prior probabilities.

Alternative policies which could promote the emergence of the separating equilibrium are those increasing the benefits that firms experience when breaking their capacity constraint, e.g., subsidizing their production costs.[32] In our model, such a subsidy would increase the constrained incumbent's post-entry profits, thereby increasing BCC_L, and ultimately enlarging the set of parameter values supporting the separating equilibrium. A similar argument can be applied for utility discount programs, such as the "Power for Jobs" program in New York State, which also reduces marginal production costs.[33]

Finally, note that the reduction of symmetric expansion costs promotes the separating equilibrium only if $BCC_L - PLE_L^E > PLE_H^E$. Intuitively, this occurs when the incumbent's benefit from breaking her capacity constraint is significant, e.g., the capacity constraint is severe. Otherwise, a policy that reduces expansion costs for both types of incumbents only switches the pooling equilibrium being played, from one where no type of incumbent expands to that in which both types expand.[34] In line with our above discussion, in these contexts a policy reducing marginal production costs might be more appropriate than a subsidy in expansion costs.

6. Conclusions

This paper examines entry deterrence and signaling in a context where the incumbent experiences a capacity constraint. We demonstrate that separating and pooling equilibria can be sustained and that most of them survive standard equilibrium refinements. Our results suggest that severe capacity constraints expand the set of parameter values that support the fully-informative separating equilibrium. Otherwise, this set shrinks, leading to an expansion of the set sustaining pooling equilibria. Furthermore, the separating equilibrium can be supported under large parameter values if financial institutions are able to discriminate constrained and unconstrained incumbents and, as a consequence, the financial costs they charge to these firms are substantially different. In this case, information about the incumbent's cost structure (or market demand) is perfectly transmitted to the entrant, and entry patterns are desirable, recommending no government intervention. If, by contrast, both types of incumbent face similar expansion costs, we identify a policy that can help move the industry towards the separating equilibrium (with similar entry patterns to those under complete information), namely, a reduction in expansion costs.

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[32] One example of this type of policies is, for instance, the subsidy programs of several European countries (such as Germany and Spain) that reduce the production cost of firms installing solar cell panels; as documented in The Economist, on December 9th 2010.

[33] However, note that in order to guarantee that the entrant stays out of the market where the efficient incumbent operates the above policy might have to be accompanied by an increase in the administrative costs of entry, *F*.

[34] Graphically, this implies that cutoff $BCC_L - PLE_L^E$ would cross cutoff PLE_H^E above the main diagonal $K_H=K_L$ in figure 3.

Nonetheless, our results also show that this policy should not be overemphasized. Otherwise, such subsidy policy would be costly, and yet leave undesirable entry patterns unaffected.

Our model offers several extensions for further research. First, the paper considers that the incumbent can only choose one specific investment level in order to expand her facility, which is sufficiently large to eliminate her capacity constraint. In richer settings, however, the incumbent might choose among a continuum of investment levels, each of them yielding a different capacity.[35] Second, we consider that potential entrants only observe the incumbent's expansion decision, but are not able to observe the incumbent's output. In some industries, nonetheless, the entrant might observe both incumbent's actions. Entry patterns can hence be different to those in signaling games where the entrant either observes the incumbent's output alone —as in Milgrom and Roberts (1982)— or her expansion decision alone —as in this paper. Specifically, this is due to the fact that the incumbent's expansion decision breaks her capacity constraint in the second period but not in the first. Hence, a constrained incumbent in this setting would not be able to increase her first-period production level beyond $\overline{q}$ in order to convey her efficiency level to the potential entrant, hampering the role of output as an informative signal. Hence, the introduction of an additional signal, rather than improving information transmission to the entrant, could potentially limit the dissemination of information.

[35] Both separating and pooling equilibria might still emerge in this context, since when expansion is followed by entry, the constrained incumbent does not necessarily obtain a larger benefit from marginally increasing her investment than the unconstrained incumbent does, as suggested in our discussion of the single-crossing property.

APPENDIX

Proof of Lemma 1:

We need to show that $p^{NE} < p^{E}$. Indeed, $p^{NE} \equiv \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} < \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^{E}$, and solving for the entry cost, *F*, we obtain

$$F < \frac{\pi_{ent,H}^{D,E}\pi_{ent,L}^{D,NE} - \pi_{ent,L}^{D,E}\pi_{ent,H}^{D,NE}}{\left(\pi_{ent,L}^{D,NE} - \pi_{ent,L}^{D,E}\right) - \left(\pi_{ent,H}^{D,NE} - \pi_{ent,H}^{D,E}\right)}$$

and since $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E}$ we can reduce the above expression to $F < \pi_{ent,H}^{D,E}$, which holds by definition.

Proof of Propositions 1, 2 and 3:

Pooling equilibrium with no expansion. Let us investigate if the strategy profile $\{NoExp_H, NoExp_L\}$ can be supported as a pooling PBE of this signaling game. First, the entrant's beliefs are $\mu(H \mid NExp) = p$ after observing no expansion (in equilibrium) and $\mu(H \mid Exp) = \mu \in [0,1]$ after observing expansion (off-the-equilibrium). Given these beliefs after observing *no expansion* (in equilibrium) the entrant enters if and only if

$$p\left(\pi_{ent,H}^{D,NE} - F\right) + (1-p)\left(\pi_{ent,L}^{D,NE} - F\right) \geq 0,$$

where the right-hand side represents the entrant's profits from staying in the perfectly competitive market (with zero profits). Solving for *p*, we obtain that the entrant enters if $p \geq \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$. Note that this cutoff is positive and smaller than one, $1 > p^{NE} > 0$, since entry costs, *F*, satisfy $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$ by definition. Hence, after observing no expansion (in equilibrium) the entrant enters the market if $p \geq p^{NE}$ and stays out otherwise. Similarly, after observing expansion (off-the-equilibrium), the entrant enters if and only if

$$\mu\left(\pi_{ent,H}^{D,E} - F\right) + (1-\mu)\left(\pi_{ent,L}^{D,E} - F\right) \geq 0$$

Solving for μ, we find that the entrant enters if $\mu \geq \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^{E}$. Note that this cutoff is positive and smaller than one, $1 > p^{NE} > 0$, since $\pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,E}$ is satisfied by definition. Indeed, $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$ given that the entrant's profits are not affected by the (unconstrained) high-cost incumbent decision to expand, $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E}$, and the entrant's profits are higher when the low-cost incumbent does not expand than when she does, $\pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$. Hence, after observing expansion (off-the-equilibrium) the entrant enters if $\mu \geq p^{E}$ and stays out otherwise. Given the entrant's strategies let us now analyze the incumbent:

- If $p < p^{NE}$ and $\mu \geq p^{E}$ then the entrant does not enter after observing no expansion (in equilibrium) but enters otherwise. Hence, the low-cost incumbent prefers not to expand (as prescribed) if and only if $\pi_{inc,L}^{M,NE}(\overline{q}) > \pi_{inc,L}^{D,E} - K_L$, or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q})$, where $BCC_L - PLE_L^E \equiv \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q})$. Similarly, the high-cost incumbent does not expand if $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{D,E} - K_H$, or $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE} < 0$ is satisfied, which holds for any expansion cost K_H>0. Thus, the strategy profile in which both types of incumbent do not expand their facility can be supported as a pooling PBE in the signaling game if $K_L > BCC_L - PLE_L^E$; as described in Proposition 1, Part 3a.

- If $p < p^{NE}$ and $\mu < p^{E}$ then the entrant does not enter after observing any action from the incumbent. Therefore, the low-cost monopolist prefers not to expand (as prescribed) if and only if $\pi_{inc,L}^{M,NE}(\overline{q}) > \pi_{inc,L}^{M,E} - K_L$, or $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\overline{q}) \equiv BCC_L$. Similarly, the high-cost incumbent prefers not to expand since $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$ or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} < 0$ which is satisfied for any K_H>0. Thus, this strategy profile can be sustained as a pooling PBE in the signaling game if expansion costs satisfy $K_L > BCC_L$; as described in Proposition 1, Part 3b.
- If $p \geq p^{NE}$ and $\mu < p^{E}$ then the entrant enters after observing no expansion (in equilibrium) but does not enter otherwise. Hence, the low-cost incumbent does not expand (attracting entry) if and only if $\pi_{inc,L}^{D,NE}(\overline{q}) > \pi_{inc,L}^{M,E} - K_L$, or $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\overline{q}) \equiv BCC_L + PLE_L^{NE}$. Similarly, the high-cost incumbent does not expand if and only if $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{M,E} - K_H$, or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E} = PLE_H^{E}$, since $\pi_{inc,H}^{D,E} = \pi_{inc,H}^{D,NE}$ given that the high-cost incumbent is unaffected by the capacity constraint. Thus, this strategy profile can be supported as a pooling PBE in the signaling game under expansion costs K_L> $BCC_L + PLE_L^{NE}$ and K_H> PLE_H^{E}; as described in Proposition 2a, and Proposition 3.
- If $p \geq p^{NE}$ and $\mu \geq p^{E}$ then the entrant enters after observing any action from the incumbent. Therefore, the low-cost incumbent does not expand (as prescribed) if and only if $\pi_{inc,L}^{D,NE}(\overline{q}) > \pi_{inc,L}^{D,E} - K_L$, or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\overline{q}) \equiv BCC_L + PLE_L^{NE} - PLE_L^{E}$. Similarly, the high-cost incumbent does not expand since $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{D,E} - K_H$, or $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE} = 0$, which holds for any K_H>0. Thus, this strategy profile can be supported as a pooling PBE for expansion costs K_L> $BCC_L + PLE_L^{NE} - PLE_L^{E}$ and K_H>0; as described in Proposition 2b, and Proposition 3.

Separating equilibrium. Let us now consider the separating strategy profile where only the low-cost incumbent expands, i.e., $\{NotExpand_H, Expand_L\}$. First, entrant's updated beliefs become $\mu(H \mid NExp) = 1$ and $\mu(H \mid Exp) = 0$. Given these beliefs, the entrant enters after observing no expansion since $\pi_{ent,H}^{D,NE} - F > 0$, or $\pi_{ent,H}^{D,NE} > F$, which satisfies our initial assumptions. On the other hand, after observing expansion the entrant stays out since $\pi_{ent,L}^{D,E} - F < 0$, or $\pi_{ent,L}^{D,E} < F$, which also holds by definition. Therefore, given the entrant's responses, the high-cost incumbent does not expand (as prescribed) if and only if $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{M,E} - K_H$, or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}$. Since $\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E} \equiv PLE_H^{E}$ given that $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$, we can then conclude that the high-cost incumbent does not expand if $K_H > PLE_H^{E}$. By contrast, the low-cost incumbent expands (as prescribed) if and only if $\pi_{inc,L}^{D,NE}(\overline{q}) < \pi_{inc,L}^{M,E} - K_L$ or $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\overline{q}) \equiv BCC_L + PLE_L^{NE}$. Thus, this strategy profile can be sustained as a separating PBE for expansion costs $K_H > PLE_H^{E}$ and $K_L < BCC_L + PLE_L^{NE}$; as described in Proposition 1 (Part 1) and Proposition 3.

For completeness, let us check that the opposite separating strategy profile $\{Exp_H, NoExp_L\}$ cannot be supported as a PBE of the signaling game. In this case, the entrant's updated beliefs become $\mu(H \mid NExp) = 0$ and $\mu(H \mid Exp) = 1$. Given these beliefs, the entrant enters after observing expansion since $\pi_{ent,H}^{D,E} - F > 0$ or $\pi_{ent,H}^{D,E} > F$, which holds by definition. However, the entrant does not enter after observing no expansion given that $\pi_{ent,L}^{D,NE} - F < 0$, or $\pi_{ent,L}^{D,NE} < F$, which is satisfied by definition. Given the entrant's responses, the low-cost incumbent does not expand (as prescribed) if and only if $\pi_{inc,L}^{M,NE}(\overline{q}) > \pi_{inc,L}^{D,E} - K_L$, or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q})$. On the other hand, the high-cost incumbent expands (as prescribed) if $\pi_{inc,H}^{M,NE} < \pi_{inc,H}^{D,E} - K_H$, or $K_H < \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE} = \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E} < 0$ (since $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$), which cannot hold for any $K_H > 0$. Thus, this strategy profile cannot be supported as a separating PBE of the signaling game.

Pooling equilibrium with expansion. Let us investigate if the pooling strategy profile in which both types of incumbent expand their facility, i.e., $\{Exp_H, Exp_L\}$, can be supported as a PBE of the signaling game. First, the entrant's beliefs are $\mu(H \mid Exp) = p$ after observing expansion (in equilibrium) and $\mu(H \mid NExp) = \gamma \in [0,1]$ after observing no expansion (off-the-equilibrium). Given these beliefs, after observing expansion, the entrant enters if

$$p\left(\pi_{ent,H}^{D,E} - F\right) + (1-p)\left(\pi_{ent,L}^{D,E} - F\right) \geq 0$$

Solving for *p*, we obtain $p \geq \dfrac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E$, where $0 \leq p^E \leq 1$ from our above discussion. Hence, entry ensues after observing expansion if $p > p^E$, but does not otherwise. If, instead, the entrant observes *no* expansion, she enters if

$$\gamma\left(\pi_{ent,H}^{D,NE} - F\right) + (1-\gamma)\left(\pi_{ent,L}^{D,NE} - F\right) \geq 0,$$

Solving for *γ*, we find $\gamma \geq \dfrac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$, where $0 \leq p^{NE} \leq 1$ holds from our above discussion. Therefore, the entrant enters after observing no expansion if $\gamma \geq p^{NE}$, but does not otherwise. Given the entrant's strategies let us now examine the incumbent:

- If $p \geq p^E$ and $\gamma \geq p^{NE}$ then the entrant enters both after observing expansion and no expansion. Therefore, the low-cost incumbent expands (as prescribed) if and only if $\pi_{inc,L}^{D,NE}(\overline{q}) < \pi_{inc,L}^{D,E} - K_L$, or $K_L < \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\overline{q})$. However, the high-cost incumbent does not expand since $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{D,E} - K_H$, or $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE} = 0$, which holds for any expansion costs K_H >0. Thus, the strategy profile in which both types of incumbent expand cannot be supported as a pooling PBE of the signaling game.
- If $p \geq p^E$ but $\gamma < p^{NE}$ then the entrant enters after observing expansion (in equilibrium), but does not enter otherwise. Hence, the low-cost incumbent expands (as prescribed) if and only if $\pi_{inc,L}^{M,NE}(\overline{q}) < \pi_{inc,L}^{D,E} - K_L$, or $K_L < \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\overline{q}) \equiv BCC_L - PLE_L^E$. By contrast, the high-cost incumbent does not expand since $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{D,E} - K_H$, or $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE} < 0$, which holds for all expansion costs K_H >0. Therefore, this strategy profile cannot be sustained as a pooling PBE of the signaling game.
- If $p < p^E$ but $\gamma \geq p^{NE}$ then the entrant does not enter after observing expansion (in equilibrium), but enters otherwise. Hence, the low-cost incumbent expands (as prescribed) if and only if $\pi_{inc,L}^{D,NE}(\overline{q}) < \pi_{inc,L}^{M,E} - K_L$, or $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\overline{q})$. Since

  $$BCC_L + PLE_L^{NE} = \left(\pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\overline{q})\right) + \left(\pi_{inc,L}^{M,NE}(\overline{q}) - \pi_{inc,L}^{D,NE}(\overline{q})\right) = \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\overline{q}),$$

  the low-cost incumbent expands if $K_L < BCC_L + PLE_L^{NE}$. Similarly, the high-cost incumbent expands (as prescribed) if and only if $\pi_{inc,H}^{D,NE} < \pi_{inc,H}^{M,E} - K_H$, or $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} \equiv PLE_H^{NE}$, since $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E}$ and $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$ for the unconstrained high-cost incumbent under monopoly and duopoly, respectively. Thus, this strategy profile can be supported as a pooling PBE under expansion costs $K_L < BCC_L + PLE_L^{NE}$ and $K_H < PLE_H^E$; as described in Proposition 1, Part 2.

- If $p < p^E$ and $\gamma < p^{NE}$ then the entrant does not enter after observing any action from the incumbent. Therefore, the low-cost incumbent expands (as prescribed) if and only if $\pi_{inc,L}^{M,NE}(\overline{q}) < \pi_{inc,L}^{M,E} - K_L$, or $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\overline{q}) \equiv BCC_L$. By contrast, the high-cost incumbent does not expand since $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$, or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} = 0$, which holds for any expansion cost K_H>0. Thus, this strategy profile cannot be sustained as a pooling PBE.

Proof of Proposition 4:

Pooling equilibrium with no expansion. Let us investigate if the pooling strategy profile $\{NoExp_H, NoExp_L\}$ can be supported as a pooling PBE of this signaling game. First, the entrant's beliefs are $\mu(H \mid NExp) = p$ after observing no expansion (in equilibrium) and $\mu(H \mid Exp) = \mu \in [0,1]$ after observing expansion (off-the-equilibrium). Given these beliefs after observing *no expansion* (in equilibrium) the entrant enters if and only if

$$p\left(\pi_{ent,H}^{D,NE} - F\right) + (1-p)\left(\pi_{ent,L}^{D,NE} - F\right) \geq 0,$$

where the right-hand side represents the entrant's profits from staying in the perfectly competitive market (with zero profits). Solving for *p*, we obtain that the entrant enters if $p \geq \dfrac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$. Note that this cutoff is positive and smaller than one, $1 > p^{NE} > 0$, since entry costs, *F*, satisfy $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$ by definition. Hence, after observing no expansion (in equilibrium) the entrant enters the market if $p \geq p^{NE}$ and stays out otherwise. Similarly, after observing expansion (off-the-equilibrium), the entrant enters if and only if

$$\mu\left(\pi_{ent,H}^{D,E} - F\right) + (1-\mu)\left(\pi_{ent,L}^{D,E} - F\right) \geq 0$$

Solving for μ, we find that the entrant enters if $\mu \geq \dfrac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E$. Note that this cutoff is positive and smaller than one, $1 > p^{NE} > 0$, since $\pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,E}$ is satisfied by definition. Indeed, $\pi_{ent,H}^{D,NE} > \pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,NE} = \pi_{ent,L}^{D,E}$ given that the entrant's profits are not affected by the (unconstrained) low-demand incumbent decision to expand, $\pi_{ent,L}^{D,NE} = \pi_{ent,L}^{D,E}$, and the entrant's profits are higher when the high-demand incumbent does not expand than when she does, $\pi_{ent,H}^{D,NE} > \pi_{ent,H}^{D,E}$. Hence, after observing expansion (off-the-equilibrium) the entrant enters if $\mu \geq p^E$ and stays out otherwise. Finally, note that $p^{NE} < p^E$ as it shown in the proof of Proposition 1.

Given the entrant's strategies let us now analyze the incumbent:

- If $p < p^{NE}$ and $\mu \geq p^E$ then the entrant does not enter after observing no expansion (in equilibrium) but enters otherwise. Hence, the high-demand incumbent prefers not to expand (as prescribed) if and only if $\pi_{inc,H}^{M,NE}(\overline{q}) > \pi_{inc,H}^{D,E} - K_H$, or $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\overline{q})$, where $BCC_H - PLE_H^E \equiv \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\overline{q})$. Similarly, the low-demand incumbent does not expand if $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{D,E} - K_L$, or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE} < 0$ is satisfied, which holds for any expansion cost K_L>0. Thus, the strategy profile in which both types of incumbent do not expand their facility can be supported as a pooling PBE if $K_H > BCC_H - PLE_H^E$.

- If $p < p^{NE}$ and $\mu < p^{E}$ then the entrant does not enter after observing any action from the incumbent. Therefore, the high-demand monopolist prefers not to expand (as prescribed) if and only if $\pi_{inc,H}^{M,NE}(\overline{q}) > \pi_{inc,H}^{M,E} - K_H$, or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\overline{q}) \equiv BCC_H$. Similarly, the low-demand incumbent prefers not to expand since $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{M,E} - K_L$ or $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE} < 0$ which is satisfied for any K_L>0. Thus, this strategy profile can be sustained as a pooling PBE in the signaling game if expansion costs satisfy $K_H > BCC_H$.
- If $p \geq p^{NE}$ and $\mu < p^{E}$ then the entrant enters after observing no expansion (in equilibrium) but does not enter otherwise. Hence, the high-demand incumbent does not expand (attracting entry) if and only if $\pi_{inc,H}^{D,NE}(\overline{q}) > \pi_{inc,H}^{M,E} - K_H$, or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\overline{q}) \equiv BCC_H + PLE_H^{NE}$. Similarly, the low-demand incumbent does not expand if and only if $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{M,E} - K_L$, or $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE} = PLE_L^{E}$. Thus, this strategy profile can be supported as a pooling PBE in the signaling game under expansion costs K_H> $BCC_H + PLE_H^{NE}$ and K_L> PLE_L^{E}.
- If $p \geq p^{NE}$ and $\mu \geq p^{E}$ then the entrant enters after observing any action from the incumbent. Therefore, the high-demand incumbent does not expand (as prescribed) if and only if $\pi_{inc,H}^{D,NE}(\overline{q}) > \pi_{inc,H}^{D,E} - K_H$, or $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE}(\overline{q}) \equiv BCC_H + PLE_H^{NE} - PLE_H^{E}$. Similarly, the low-demand incumbent does not expand since $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{D,E} - K_L$, or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE} = 0$, which holds for any K_L >0. Thus, this strategy profile can be supported as a pooling PBE for expansion costs K_H> $BCC_H + PLE_H^{NE} - PLE_H^{E}$ and K_L>0.

Separating equilibrium. Let us now consider the separating strategy profile where only the high-demand incumbent expands, i.e., $\{Expand_H, NotExpand_L\}$. First, entrant's updated beliefs become $\mu(H \mid NExp) = 0$ and $\mu(H \mid Exp) = 1$. Given these beliefs, the entrant enters after observing expansion since $\pi_{ent,H}^{D,E} - F > 0$, or $\pi_{ent,H}^{D,E} > F$. On the other hand, after observing no expansion the entrant stays out since $\pi_{ent,L}^{D,NE} - F < 0$, or $\pi_{ent,L}^{D,NE} < F$, which also holds by definition. Therefore, given the entrant's responses, the low-demand incumbent does not expand (as prescribed) if and only if $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{M,E} - K_L$, or $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}$. Since $\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE} = \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,E} \equiv PLE_L^{E}$ given that $\pi_{inc,L}^{D,NE} = \pi_{inc,L}^{D,E}$, we can then conclude that the high-demand incumbent does not expand if $K_L > PLE_L^{E}$. By contrast, the high-demand incumbent expands (as prescribed) if and only if $\pi_{inc,H}^{D,NE}(\overline{q}) < \pi_{inc,H}^{M,E} - K_H$ or $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\overline{q}) \equiv BCC_H + PLE_H^{NE}$. Thus, this strategy profile can be sustained as a separating PBE for expansion costs $K_L > PLE_L^{E}$ and $K_H < BCC_H + PLE_H^{NE}$.

For completeness, let us check that the opposite separating strategy profile $\{NoExp_H, Exp_L\}$ cannot be supported as a PBE of the signaling game. In this case, the entrant's updated beliefs become $\mu(H \mid NExp) = 1$ and $\mu(H \mid Exp) = 0$. Given these beliefs, the entrant does not enter after observing expansion since $\pi_{ent,L}^{D,E} - F < 0$ or $\pi_{ent,L}^{D,E} < F$, which holds by definition. However, the entrant enters after observing no expansion given that $\pi_{ent,H}^{D,NE} - F > 0$, or $\pi_{ent,H}^{D,NE} > F$, which is satisfied by definition. Given the entrant's responses, the high-demand incumbent does not expand (as prescribed) if and only if $\pi_{inc,H}^{M,NE}(\overline{q}) > \pi_{inc,H}^{D,E} - K_H$, or $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\overline{q})$. On the other hand, the low-demand incumbent expands (as prescribed) if $\pi_{inc,L}^{M,NE} < \pi_{inc,L}^{D,E} - K_L$, or $K_L < \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE} = \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,E} < 0$ (since $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$), which cannot hold for any $K_L > 0$. Thus, this strategy profile cannot be supported as a separating PBE of the signaling game.

Pooling equilibrium with expansion. Let us investigate if the pooling strategy profile in which both types of incumbent expand their facility, i.e., $\{Exp_H, Exp_L\}$, can be supported as a PBE of the signaling game. First, the entrant's beliefs are $\mu(H \mid Exp) = p$ after observing expansion (in equilibrium) and $\mu(H \mid NExp) = \gamma \in [0,1]$ after observing no expansion (off-the-equilibrium). Given these beliefs, after observing expansion, the entrant enters if

$$p\left(\pi_{ent,H}^{D,E} - F\right) + (1-p)\left(\pi_{ent,L}^{D,E} - F\right) \geq 0$$

Solving for *p*, we obtain $p \geq \dfrac{F - \pi_{ent,H}^{D,E}}{\pi_{ent,L}^{D,E} - \pi_{ent,H}^{D,E}} \equiv p^E$, where $0 \leq p^E \leq 1$ from our above discussion. Hence, entry ensues after observing expansion if $p \geq p^E$, but does not otherwise. If, instead, the entrant observes *no* expansion, she enters if

$$\gamma\left(\pi_{ent,H}^{D,NE} - F\right) + (1-\gamma)\left(\pi_{ent,L}^{D,NE} - F\right) \geq 0,$$

Solving for *γ*, we find $\gamma \geq \dfrac{F - \pi_{ent,H}^{D,NE}}{\pi_{ent,L}^{D,NE} - \pi_{ent,H}^{D,NE}} \equiv p^{NE}$, where $0 \leq p^{NE} \leq 1$ holds from our above discussion. Therefore, the entrant enters after observing no expansion if $\gamma \geq p^{NE}$, but does not otherwise. Given the entrant's strategies let us now examine the incumbent:

- If $p \geq p^E$ and $\gamma \geq p^{NE}$ then the entrant enters both after observing expansion and no expansion. Therefore, the high-demand incumbent expands (as prescribed) if and only if $\pi_{inc,H}^{D,NE}(\overline{q}) < \pi_{inc,H}^{D,E} - K_H$, or $K_H < \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE}(\overline{q})$. However, the low-demand incumbent does not expand since $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{D,E} - K_L$, or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE} = 0$, which holds for any expansion costs K_L >0. Thus, the strategy profile in which both types of incumbent expand cannot be supported as a pooling PBE of the signaling game.
- If $p \geq p^E$ but $\gamma < p^{NE}$ then the entrant enters after observing expansion (in equilibrium), but does not enter otherwise. Hence, the high-demand incumbent expands (as prescribed) if and only if $\pi_{inc,H}^{M,NE}(\overline{q}) < \pi_{inc,H}^{D,E} - K_H$, or $K_H < \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\overline{q}) \equiv BCC_H - PLE_H^E$. By contrast, the low-demand incumbent does not expand since $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{D,E} - K_L$, or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE} < 0$, which holds for all expansion costs K_L >0. Therefore, this strategy profile cannot be sustained as a pooling PBE of the signaling game.
- If $p < p^E$ but $\gamma \geq p^{NE}$ then the entrant does not enter after observing expansion (in equilibrium), but enters otherwise. Hence, the high-demand incumbent expands (as prescribed) if and only if $\pi_{inc,H}^{D,NE}(\overline{q}) < \pi_{inc,H}^{M,E} - K_H$, or $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\overline{q})$. Since
$$BCC_H + PLE_H^{NE} = \left(\pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\overline{q})\right) + \left(\pi_{inc,H}^{M,NE}(\overline{q}) - \pi_{inc,H}^{D,NE}(\overline{q})\right) = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\overline{q}),$$
the high-demand incumbent expands if $K_H < BCC_H + PLE_H^{NE}$. Similarly, the low-demand incumbent expands (as prescribed) if and only if $\pi_{inc,L}^{D,NE} < \pi_{inc,L}^{M,E} - K_L$, or $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE} \equiv PLE_L^{NE}$, since $\pi_{inc,L}^{M,NE} = \pi_{inc,L}^{M,E}$ and $\pi_{inc,L}^{D,NE} = \pi_{inc,L}^{D,E}$ for the unconstrained low-demand incumbent under monopoly and duopoly, respectively. Thus, this strategy profile can be supported as a pooling PBE under expansion costs $K_H < BCC_H + PLE_H^{NE}$ and $K_L < PLE_L^E$.

If $p < p^E$ and $\gamma < p^{NE}$ then the entrant does not enter after observing any action from the incumbent. Therefore, the high-demand incumbent expands (as prescribed) if and only if $\pi_{inc,H}^{M,NE}(\overline{q}) < \pi_{inc,H}^{M,E} - K_H$, or

$K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\overline{q}) \equiv BCC_H$. By contrast, the low-demand incumbent does not expand since $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{M,E} - K_L$, or $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE} = 0$, which holds for any expansion cost K_L >0. Thus, this strategy profile cannot be sustained as a pooling PBE.

Appendix 1 - *Equilibrium refinement*

Proposition A. *All equilibria identified in Propositions 1 and 2 survive the Cho and Kreps' (1987) Intuitive Criterion, except for:*

1. *the pooling equilibrium of no expansion followed by* $\left[NoEnter_{NoExp}, Enter_{Exp}\right]$ *as described in Proposition 1(part 3a), if expansion costs satisfy* $BCC_L > K_L > BCC_L - PLE_L^E$ *and* $K_H > 0$*;*
2. *the pooling equilibrium of no expansion followed by* $\left[NoEnter_{NoExp}, NoEnter_{Exp}\right]$ *as described in Proposition 1(part 3b), if expansion costs satisfy* $BCC_L + PLE_L^{NE} > K_L > BCC_L$ *and* $PLE_H^{NE} > K_H > 0$*;*
3. *the pooling equilibrium of no expansion followed by* $\left[Enter_{NoExp}, Enter_{Exp}\right]$ *as described in Proposition 2b, if expansion costs satisfy* $BCC_L + PLE_L^E > K_L > BCC_L + PLE_L^{NE} - PLE_L^E$ *and* $PLE_H^{NE} > K_H > 0$*; and if expansion costs satisfy* $BCC_L + PLE_L^E > K_L > BCC_L + PLE_L^{NE} - PLE_L^E$ *and* $K_H > PLE_H^{NE}$*.*

Proof.

Pooling equilibrium (Proposition 2b)**:** If the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is $\pi_{inc,H}^{M,E} - K_H$, which strictly exceeds her equilibrium payoff of $\pi_{inc,H}^{D,NE}$ if and only if $\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} > K_H$, and since $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$ this condition implies $PLE_H^{NE} \equiv \pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE} > K_H$. Hence, considering the equilibrium condition for the high-cost incumbent ($K_H > 0$), she deviates towards expansion if and only if $PLE_H^{NE} > K_H > 0$. Similarly, if the low-cost incumbent deviates towards expansion, the highest payoff she can obtain is $\pi_{inc,L}^{M,E} - K_L$, which strictly exceeds her equilibrium payoff of $\pi_{inc,L}^{D,NE}$ if and only if $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\overline{q}) \equiv BCC_L + PLE_L^E$. Considering the equilibrium condition for the low-cost incumbent (K_L>BCC_L), she deviates towards expansion if and only if $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$. Hence, the following cases can arise:

- If $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$ and $0 < K_H < PLE_H^{NE}$ for the low-cost and high-cost incumbent, respectively, then both types of incumbent deviate towards expansion. Then the entrant's off-the-equilibrium beliefs are updated to $\mu(H \mid Exp) = p$, where $p^{NE} \le p < p^E$, leading the entrant to stay out after observing this expansion. But then both types of incumbent have incentives to deviate towards expansion, and the pooling equilibrium without expansion violates the Intuitive Criterion for all $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$ and $0 < K_H < PLE_H^{NE}$ and intermediate priors $p^{NE} \le p < p^E$. If, instead, priors are relatively high, $p \ge p^E$, the entrant enters after observing the deviation towards expansion. Hence, the high-cost incumbent does not deviate since its profit from deviating towards expansion, $\pi_{inc,H}^{D,E} - K_H$, is lower than her equilibrium profit from not expanding, $\pi_{inc,H}^{D,NE}$, since $\pi_{inc,H}^{D,E} = \pi_{inc,H}^{D,NE}$. Similarly, the low-cost incumbent does not deviate towards expansion either if her profits from doing so, $\pi_{inc,L}^{D,E} - K_L$, is lower than her equilibrium profit, $\pi_{inc,L}^{D,NE}\left(\overline{q}\right)$ or $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}\left(\overline{q}\right) \equiv BCC_L + PLE_L^{NE} - PLE_L^E$, which holds in this pooling equilibrium. Therefore, no type of incumbent deviates and the pooling equilibrium without expansion of Proposition 2b survives the

Intuitive Criterion for all $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$ and $0 < K_H < PLE_H^{NE}$ and priors are relatively high, i.e., $p \geq p^E$.

- If $K_H > PLE_H^{NE}$ but $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$, then the high-cost incumbent does not deviate towards expansion, but the low-costs incumbent does deviate. The entrant's off-the-equilibrium beliefs (after observing a expansion) become $\mu(H \mid Exp) = 0$, whereas his beliefs after observing no expansion are $\mu(H \mid NoExp) = p \in [0,1]$. Since $p>p^{NE}$ holds in this equilibrium, the entrant does not enter after observing expansion, but enters otherwise, i.e., $\left[NoEnter_{\exp}, Enter_{no\exp}\right]$. Given this response by the entrant after updating his off-the-equilibrium beliefs, the high-cost incumbent does not deviate towards expansion if $\pi_{inc,H}^{D,NE}(\overline{q}) > \pi_{inc,H}^{M,E} - K_H$ or $K_H > PLE_H^{NE}$, which is satisfied in the case we consider. However, the low-cost incumbent deviates towards expansion since $\pi_{inc,L}^{D,NE}(\overline{q}) < \pi_{inc,L}^{M,E} - K_L$, or
$$0 < K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\overline{q}) \equiv BCC_L + PLE_L^E .$$
Considering, in addition, this incumbent's equilibrium conditions ($BCC_L + PLE_L^{NE} - PLE_L^E < K_L$), the above condition becomes $BCC_L < K_L < BCC_L + PLE_L^E$, which indeed holds in the case we consider. As a consequence, the low-cost incumbent deviates towards expansion. Hence, the pooling equilibrium ($NoExp_H$, $NoExp_L$) with ($Enter_{NoExp}$, $Enter_{Exp}$), as described in Proposition 2b, *violates* the Intuitive Criterion when expansion costs satisfy $K_H > PLE_H^{NE}$ and $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$.
- If $K_L > BCC_L + PLE_L^{NE}$ and $K_H > PLE_H^{NE}$, then no incumbent deviates towards expansion. This implies that the entrant does not update his beliefs and therefore he responds by using the prescribed strategy ($Enter_{EXP}$, $Enter_{NEXP}$). Hence, the pooling ($NoExp_H$;$NoExp_L$) with ($Enter_{EXP}$; $Enter_{NEXP}$) survives the Intuitive Criterion.
- If $K_L > BCC_L + PLE_L^{NE}$ but $0 < K_H < PLE_H^{NE}$, then the low-cost incumbent does not deviate towards expansion, but the high-cost incumbent does deviate. The entrant's off-the-equilibrium beliefs become $\mu(H \mid Exp) = 1$, whereas his beliefs after observing no expansion (in equilibrium) are $\mu(H \mid NoExp) = p \in [0,1]$. Since $p>p^{NE}$ holds in this equilibrium, the entrant enters after observing expansion, and also enters after observing no expansion, i.e., $\left[Enter_{Exp}, Enter_{NoExp}\right]$. Hence, the entrant's strategy coincides with that in equilibrium, and therefore both types of incumbent's equilibrium strategy are unaffected. Thus, this pooling equilibrium survives the Intuitive Criterion under expansion costs $K_L > BCC_L + PLE_L^{NE}$ and $0 < K_H < PLE_H^{NE}$.

***Pooling equilibrium (Proposition 1, Part 3a)*:** If the low-cost incumbent deviates towards expansion the highest payoff she can obtain is $\pi_{inc,L}^{M,E} - K_L$, which strictly exceeds her equilibrium payoff of $\pi_{inc,L}^{M,NE}$ if and only if $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE} \equiv BCC_L$. Combining this condition with the parameter values under which this equilibrium is supported ($K_L > BCC_L - PLE_L^E$), we obtain that the low-cost incumbent deviates towards expansion if $BCC_L > K_L > BCC_L - PLE_L^E$. Similarly, if the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is $\pi_{inc,H}^{M,E} - K_H$, which does not exceed her equilibrium payoff of $\pi_{inc,H}^{M,NE}$ since $\pi_{inc,H}^{M,E} - K_H > \pi_{inc,H}^{M,NE}$ implies $0 = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} > K_H$. As consequence, the high-cost incumbent does not deviate under any parameter values. The following two cases can hence arise:

- If $BCC_L > K_L > BCC_L - PLE_L^E$ only the low-cost incumbent has incentives to deviate towards expansion, which helps the entrant restrict his off-the-equilibrium beliefs to $\mu(H \mid Exp) = 0$. These beliefs induce no entry after observing expansion (and no entry after expansion either since $\mu(H \mid NoExp) = p < p^{NE}$ in this equilibrium), i.e., $\left[NoEnter_{NoExp}, NoEnter_{Exp}\right]$. Given this strategy for the entrant, the low-cost incumbent deviates towards expansion since $\pi_{inc,L}^{M,E} - K_L > \pi_{inc,L}^{M,NE}(\overline{q})$ given that $K_L < BCC_L$ holds in this case. By contrast, the high-cost incumbent does not deviate towards expansion since $\pi_{inc,H}^{M,E} - K_H < \pi_{inc,H}^{M,NE}$ for all K_H>0. Therefore, only the low-cost incumbent deviates towards expansion, and the pooling PBE where ($NoExp_H$, $NoExp_L$) with ($NEnter_{NoExp}$, $Enter_{Exp}$), as described in Proposition 1, Part 3a, *violates* the Intuitive Criterion if expansion costs satisfy $BCC_L > K_L > BCC_L - PLE_L^E$ and K_H>0.

- If, instead, $K_L > BCC_L > BCC_L - PLE_L^E$, then no type of incumbent has incentives to deviate towards no expansion. Hence, the entrant's beliefs are unaffected, his strategy still coincides with that in the pooling equilibrium, i.e., $\left[NEnter_{NoExp}, Enter_{Exp}\right]$, and this pooling equilibrium survives the Intuitive Criterion.

***Pooling equilibrium (Proposition 2a)*:** If the low-cost incumbent deviates towards expansion the highest payoff she can obtain is $\pi_{inc,L}^{M,E} - K_L$, which strictly exceeds her equilibrium payoff of $\pi_{inc,L}^{D,NE}(\overline{q})$ if and only if $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\overline{q}) \equiv BCC_L + PLE_L^{NE}$. This inequality, however, contradicts the parameter condition for the low-cost incumbent supporting this pooling PBE. As a consequence, she does not deviate towards expansion. Similarly, if the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is $\pi_{inc,H}^{M,E} - K_H$, which strictly exceeds her equilibrium payoff of $\pi_{inc,H}^{D,NE}$ if and only if $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} \equiv PLE_H^E$. This inequality also contradicts the parameter condition for the high-cost incumbent supporting this pooling PBE. Therefore, she does not deviate towards expansion either. Hence, no type of incumbent has incentives to deviate towards expansion, and the pooling equilibrium ($NoExp_H$, $NoExp_L$) with (E_{NEXP}; NE_{EXP}) survives the Intuitive Criterion.

***Pooling equilibrium (Proposition 1b)*:** If the low-cost incumbent deviates towards expansion the highest payoff she can obtain is $\pi_{inc,L}^{M,E} - K_L$, which strictly exceeds her equilibrium payoff of $\pi_{inc,L}^{M,NE}$ if and only if $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\overline{q}) \equiv BCC_L$. This inequality, however, contradicts the parameter condition for the high-cost incumbent supporting this pooling PBE (K_L>BCC_L). As a consequence, she does not deviate towards expansion. Similarly, if the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is $\pi_{inc,H}^{M,E} - K_H$, which does not exceed her equilibrium payoff of $\pi_{inc,H}^{M,NE}(\overline{q})$ for any K_H>0. Therefore, the high-cost incumbent does not deviate towards expansion either, and the pooling equilibrium ($NoExp_H$, $NoExp_L$) with ($NoEntry_{NEXP}$; $NoEntry_{EXP}$) survives the Intuitive Criterion.

***Pooling equilibrium (Proposition 1, Part 3b)*:** If the high-cost incumbent deviates towards no expansion, the highest payoff she can obtain is $\pi_{inc,H}^{M,NE}$, which strictly exceeds her equilibrium payoff of $\pi_{inc,H}^{M,E} - K_H$ for any K_H>0, i.e., $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$, or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} = 0$. Considering the equilibrium condition for the high-cost incumbent (K_H< PLE_H^{NE}), this implies that she deviates towards no expansion if and only if $0 < K_H < PLE_H^{NE}$. Regarding the low-cost incumbent, if she deviates towards no expansion the highest payoff she can obtain is $\pi_{inc,L}^{M,NE}(\overline{q})$, which strictly exceeds her equilibrium payoff of $\pi_{inc,L}^{M,E} - K_L$ if and only if $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\overline{q}) \equiv BCC_L$. Considering the equilibrium condition for the low-cost incumbent ($K_L < BCC_L + PLE_L^{NE}$), she deviates towards no expansion if and only if $BCC_L < K_L < BCC_L + PLE_L^{NE}$. Hence, the following cases can arise:

- If $BCC_L < K_L < BCC_L + PLE_L^{NE}$ and $0 < K_H < PLE_H^{NE}$, then both incumbents deviate towards no expansion, and the entrant's off-the-equilibrium beliefs become $\mu\left(H \mid NoExp\right) = p$, where $p < p^{NE}$ (the pooling equilibrium of Proposition 1, part 3b holds only for relatively low priors). The entrant hence stays out after observing a deviation towards no expansion. Therefore, both types of incumbents have incentives to deviate towards no expansion if $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$, or $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} = 0$ for the high-cost incumbent and $\pi_{inc,L}^{M,NE}\left(\overline{q}\right) > \pi_{inc,L}^{M,E} - K_L$, or $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}\left(\overline{q}\right) \equiv BCC_L$ for the low-cost incumbent. Since both conditions on the expansion costs of the low and high-cost incumbent are satisfied, both types of incumbent deviate towards no expansion, and the pooling equilibrium with expansion of Proposition 1 (part 3b) violates the Intuitive Criterion for all: $BCC_L < K_L < BCC_L + PLE_L^{NE}$ and $0 < K_H < PLE_H^{NE}$.

- If, instead, $K_L < BCC_L < BCC_L + PLE_L^{NE}$ but $0 < K_H < PLE_H^{NE}$, then the low-cost incumbent does not deviate towards no expansion, while the high-cost incumbent does deviate. The entrant's off-the-equilibrium beliefs become $\mu(H \mid NExp) = 1$, whereas his beliefs after observing expansion are $\mu(H \mid Exp) = p \in \left[0,1\right]$. Since in this equilibrium priors satisfy $p < p^E$, the entrant stays out after observing expansion but enters after observing no expansion, i.e., ($NoEnter_{Exp}$,$Enter_{NExp}$). The optimal response by the entrant after updating his beliefs, however, coincides with his response in this pooling equilibrium. As a consequence, the incumbent's expansion decision is unaffected, and we can conclude that this pooling equilibrium survives the Intuitive Criterion if expansion costs satisfy $K_L < BCC_L < BCC_L + PLE_L^{NE}$ and $0 < K_H < PLE_H^{NE}$.

Appendix 2 – *Semiseparating equilibria*

Proposition B. *The following strategy profiles can be supported as semi-separating PBEs of the game:*

1. *A strategy profile where the incumbent expands her facility when her costs are low,* p_L=1*, but expands with probability* $p_H \in \left(0,1\right)$ *when her costs are high, where* $p_H = \frac{p^E}{1-p^E}\frac{1-p}{p}$. *In this equilibrium, after observing no expansion the entrant enters,* s=1*, and after observing expansion, the entrant enters with probability* $r = 1 - \frac{K_H}{PLE_H^E}$, *given beliefs* $\mu(H \mid NoExp) = 1$ *and* $\mu(H \mid Exp) = p^E$. *This equilibrium can be only supported if priors are relatively high,* $p>p^E$*, and expansion costs satisfy* $0 < K_H < \min\left\{PLE_H^E, K_H{}^1\right\}$, *where* $K^1{}_H \equiv -\frac{B}{PLE_L^E} + \frac{PLE_H^E}{PLE_L^E}K_L$ *and* $B \equiv PLE_H^E\left[BCC_L + \left(PLE_L^{NE} - PLE_L^E\right)\right]$.
2. *A strategy profile where the incumbent expands her facility when her costs are high,* p_H=1*, but expands with probability* $p_L \in \left(0,1\right)$ *when her costs are low, where* $p_L = \frac{p}{1-p}\frac{1-p^E}{p^E}$. *In this equilibrium, after observing no expansion the entrant stays out, s=0, and after observing expansion, the entrant enters with probability* $r = 1 - \frac{K_H}{PLE_H^E}$, *given beliefs* $\mu(H \mid NoExp) = 0$ *and* $\mu(H \mid Exp) = p^E$. *This equilibrium can be only supported if priors are relatively high,* $p>p^E$*, and expansion costs satisfy* $K_L < PLE_L^E$.
3. *A strategy profile where the high-cost incumbent expands with probability* $p_H = \frac{p^E\left(p - p^{NE}\right)}{\left(p^E - p^{NE}\right)p}$*,and the low-cost incumbent expands with probability* $p_L = \frac{\left(1-p^E\right)\left(p-p^{NE}\right)}{\left(p^E - p^{NE}\right)(1-p)}$, *where* $p_H, p_L \in (0,1)$ *After observing expansion (no expansion) the entrant enters with probability r (s, respectively), where*

$$r=\left[K_L-BCC_L-\frac{K_H\times PLE_L^{NE}}{PLE_H^E}\right]\times\frac{1}{PLE_L^{NE}-PLE_L^E} \quad \text{and}$$

$$s=\left[K_L-BCC_L-\frac{K_H\times PLE_L^E}{PLE_H^E}\right]\times\frac{1}{PLE_L^{NE}-PLE_L^E}$$

given beliefs $\mu(H\mid NoExp)=p^{NE}$ *and* $\mu(H\mid Exp)=p^E$*. This equilibrium can be only supported if priors are intermediate,* $p^{NE}\geq p>p^E$*, and: (1) expansion costs satisfy* $0<K_H<\frac{C}{PLE_L^{NE}}$ *and* $0<K_L<\frac{PLE_L^E}{PLE_H^E}K_H+\frac{B}{PLE_H^E}$ *if* $PLE_L^{NE}>PLE_L^E$*; (2) expansion costs satisfy* $K_H>0$ *for all* $K_L\leq BCC_L$ *and* $K_H>\frac{C}{PLE_L^E}$ *otherwise if* $PLE_L^{NE}\leq PLE_L^E$*, where* $B\equiv PLE_H^E\left[BCC_L+\left(PLE_L^{NE}-PLE_L^E\right)\right]$ *and* $C\equiv PLE_H^E\left(K_L-BCC_L\right)$.

4. *A strategy profile where the incumbent does not expand her facility when her costs are high,* $p_H=0$*, but expands with probability* $p_L\in(0,1)$ *when her costs are low, where* $p_L=\frac{p^{NE}-p}{p^{NE}(1-p)}$ *In this equilibrium, after observing expansion the entrant stays out, r=0, and after observing no expansion the entrant enters with probability* $s=\frac{BCC_L}{PLE_L^E}+\frac{K_L}{PLE_L^E}$*, given beliefs* $\mu(H\mid Exp)=0$ *and* $\mu(H\mid NoExp)=p^{NE}$*. This equilibrium can be only supported if priors are relatively low,* $p<p^{NE}$*, and expansion costs satisfy* $PLE_L^E-BCC_L>K_L>\frac{PLE_L^E}{PLE_H^E}K_H-BCC_L$.
5. *A strategy profile where the incumbent does not expand her facility when her costs are low,* $p_L=0$*, but expands with probability* $p_H\in(0,1)$ *when her costs are low, where* $p_H=\frac{p-p^{NE}}{p\left(1-p^{NE}\right)}$ *In this equilibrium, after observing expansion the entrant enters, r=0, and after observing no expansion the entrant enters with probability* $s=\frac{K_H}{PLE_H^E}$*, given beliefs* $\mu(H\mid Exp)=1$ *and* $\mu(H\mid NoExp)=p^{NE}$*. This equilibrium can be only supported for priors ,* $p>p^{NE}$*, and expansion costs satisfying*

$$PLE_H^E>K_H>\frac{PLE_H^E}{PLE_L^E}K_L+\frac{BCC_L\times PLE_H^E}{PLE_L^E}$$

Proof.

p_L=1 and $p_H\in(0,1)$. In this equilibrium, the entrant's beliefs after observing no expansion become $\mu(H\mid NoExp)=1$, which leads him to enter since $\pi_{ent,H}^{D,NE}>F$. In the case that the entrant observes expansion, he mixes if his beliefs $\mu(H\mid Exp)$ satisfy

$$\mu(H\mid Exp)\times\left(\pi_{ent,H}^{D,E}-F\right)+(1-\mu(H\mid Exp))\left(\pi_{ent,L}^{D,E}-F\right)=0$$

and solving for $\mu(H\mid Exp)$, we obtain $\mu(H\mid Exp)=\frac{F-\pi_{ent,H}^{D,E}}{\pi_{ent,L}^{D,E}-\pi_{ent,H}^{D,E}}\equiv p^E$. We can now use the entrant's posterior beliefs $\mu(H\mid Exp)=p^E$ in order to find the probability, p_H, with which the high-cost incumbent randomizes, by using Bayes' rule as follows

$$\mu(H \mid Exp) = p^E = \frac{p \times p_H}{\left(p \times p_H\right) + (1-p)}$$

Solving for p_H we obtain $p_H = \frac{p^E}{(1-p^E)} \times \frac{(1-p)}{p}$, where $p_H \in (0,1)$ for all $p^E < p$. In addition, note that p_H is decreasing in p since $\frac{\partial p_H}{\partial p} = \frac{p^E}{\left(p^E - 1\right) \times p^2} < 0$, starting from $\lim_{p \to p^E} p_H = 1$ and converging to $\lim_{p \to 1} p_H = 0$.

Regarding the incumbent, when her costs are high, she mixes as prescribed, $p_H \in (0,1)$, if and only if

$$r \times \left(\pi^{D,E}_{inc,H} - K_H\right) + (1-r) \times \left(\pi^{M,E}_{inc,H} - K_H\right) = \pi^{D,NE}_{inc,H},$$

where r is the probability with which the entrant enters after observing expansion. Solving for r, we obtain

$$r = \frac{\pi^{M,E}_{inc,H} - \pi^{D,NE}_{inc,H} - K_H}{\pi^{M,E}_{inc,H} - \pi^{D,E}_{inc,H}}$$

and since $PLE^E_H = \pi^{M,E}_{inc,H} - \pi^{D,E}_{inc,H}$ and $\pi^{D,E}_{inc,H} = \pi^{D,NE}_{inc,H}$, we can rewrite this probability as $r = 1 - \frac{K_H}{PLE^E_H}$, where $r \in (0,1)$ only if expansion costs satisfy $K_H < PLE^E_H$. On the other hand, the low-cost incumbent expands as prescribed (p_L=1) if and only if

$$r \times \left(\pi^{D,E}_{inc,L} - K_L\right) + (1-r) \times \left(\pi^{M,E}_{inc,L} - K_L\right) > \pi^{D,NE}_{inc,L}.$$

which implies

$$r > \frac{\pi^{M,E}_{inc,L} - \pi^{D,NE}_{inc,L} - K_L}{\pi^{M,E}_{inc,L} - \pi^{D,E}_{inc,L}} = \frac{BCC_L + PLE^{NE}_L - K_L}{PLE^E_L} \equiv \hat{r},$$

Hence, probability r must satisfy $r > \hat{r}$ which implies $K_H < -\frac{B}{PLE^E_L} + \frac{PLE^E_H}{PLE^E_L} K_L \equiv K^1{}_H$ where $B \equiv PLE^E_H \left[BCC_L + \left(PLE^{NE}_L - PLE^E_L\right)\right]$. Hence, this semiseparating strategy profile can be supported as an equilibrium if expansion costs satisfy $0 < K_H < \min\left\{PLE^E_H, K_H{}^1\right\}$ and priors are relatively high, i.e. $p > p^E$.

p_H=1 and p_L∈(0,1). In this equilibrium, the entrant's posterior beliefs after observing no expansion are $\mu(H \mid NoExp) = 0$, which leads him to stay out since $\pi^{D,NE}_{ent,L} < F$. In the case that the entrant observes expansion, he mixes if his beliefs $\mu(H \mid Exp)$ satisfy

$$\mu(H \mid Exp) \times \left(\pi^{D,E}_{ent,H} - F\right) + (1 - \mu(H \mid Exp))\left(\pi^{D,E}_{ent,L} - F\right) = 0$$

and solving for $\mu(H \mid Exp)$, we obtain $\mu(H \mid Exp) = p^E$. Using Bayes' rule,

$$\mu(H \mid Exp) = p^E = \frac{p}{p + (1-p)p_L}$$

and solving for p_L we obtain $p_L = \frac{p}{1-p} \times \frac{1-p^E}{p^E}$, where $p_L \in (0,1)$ for all $p < p^E$. In addition, p_L is increasing in p since $\frac{\partial p_L}{\partial p} = \frac{1-p^E}{p^E (1-p)^2} > 0$, starts at $\lim_{p \to 0} p_L = 0$ and converges to $\lim_{p \to p^E} p_L = 1$.

Regarding the incumbent, when her costs are high, she expands as prescribed (p_H=1) if and only if

$$r \times \left(\pi^{D,E}_{inc,H} - K_H\right) + (1-r) \times \left(\pi^{M,E}_{inc,H} - K_H\right) > \pi^{M,NE}_{inc,H},$$

solving for r, and using the property that $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E}$, we find

$$r > \frac{-K_H}{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}} = -\frac{K_H}{PLE_H^E} \equiv \tilde{r}$$

where cutoff $\tilde{r} < 0$ for all parameter values. On the other hand, the low-cost incumbent randomizes as prescribed, $p_L \in (0,1)$, if and only if

$$r \times \left(\pi_{inc,L}^{D,E} - K_L\right) + (1-r) \times \left(\pi_{inc,L}^{M,E} - K_L\right) = \pi_{inc,L}^{D,E}.$$

which implies

$$r = \frac{\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,E} - K_L}{\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,E}} = 1 - \frac{K_L}{PLE_L^E},$$

where $r \in (0,1)$ if $K_L < PLE_L^E$. Finally, since cutoff $\tilde{r} < 0$, $r > \tilde{r}$ under all parameter values. Hence, this semiseparating strategy profile can be supported as an equilibrium if expansion costs satisfy $K_L < PLE_L^E$ for low and intermediate priors, i.e., $p < p^E$.

***p_L, p_H*∈(0,1).** In this equilibrium, after observing an expansion, the entrant is indifferent between entering and not entering the incumbent's market if and only if his posterior beliefs $\mu(H \mid Exp)$ satisfy

$$\mu(H \mid Exp) \times \left(\pi_{ent,H}^{D,E} - F\right) + (1 - \mu(H \mid Exp))\left(\pi_{ent,L}^{D,E} - F\right) = 0$$

and solving for $\mu(H \mid Exp)$, we obtain $\mu(H \mid Exp) = \frac{F - \pi_{ent,H}^{D,E}}{\pi_{ent,L}^{D,E} - \pi_{ent,H}^{D,E}} \equiv p^E$. We can then use the entrant's posterior beliefs $\mu(H \mid Exp) = p^E$ in order to find probability, p_H, with which the incumbent randomizes when her costs are high, by using Bayes' rule, as follows

$$\mu(H \mid Exp) = p^E = \frac{p \times p_H}{(p \times p_H) + ((1-p) \times p_L)}$$

Solving for p_H we obtain $p_H(p_L) = \frac{p^E \times p_L \times (1-p)}{p \times (1-p^E)}$. Similarly, after observing that the incumbent does not expand, the entrant is indifferent between entering and not entering the incumbent's market if and only if his posterior beliefs $\mu(H \mid NoExp)$ satisfy

$$\mu(H \mid NoExp) \times \left(\pi_{ent,H}^{D,NE} - F\right) + (1 - \mu(H \mid NoExp))\left(\pi_{ent,L}^{D,NE} - F\right) = 0$$

and solving for $\mu(H \mid NoExp)$ we obtain $\mu(H \mid NoExp) = \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$. We can hence use the entrant's posterior beliefs $\mu(H \mid NoExp) = p^{NE}$ in order to find probability, p_L, with which the incumbent randomizes when her costs are low, by using Bayes' rule, as follows

$$\mu(H \mid NoExp) = p^{NE} = \frac{p \times (1-p_H)}{\left(p \times (1-p_H)\right) + \left((1-p) \times (1-p_L)\right)}$$

Solving for p_L we obtain $p_L(p_H) = \frac{p^{NE} + (p_H - 1)p - p_H \times p^{NE} \times p}{p^{NE} \times (1-p)}$. Solving for p_H and p_L simultaneously, we obtain $p_L = \frac{(1-p^E) \times (p - p^{NE})}{(p^E - p^{NE}) \times (1-p)}$ and $p_H = \frac{p^E \times (p - p^{NE})}{(p^E - p^{NE}) \times p}$. First, note that $p_L \geq 0$ if and only if $p \geq p^{NE}$, given that $p^{E} > p^{NE}$ and $p, p^{E}, p^{NE} \in (0,1)$ under all parameter values. In addition, $p_L < 1$ for all $p < p^{E}$. Therefore, $p_L \in (0,1)$ if and only if priors are intermediate, i.e., $p^{NE} \leq p < p^{E}$. Second, note that $p_H \geq 0$ if and only if $p \geq p^{NE}$. Furthermore, $p_H < 1$ for all $p < p^{E}$. Therefore, $p_H \in (0,1)$ only if priors are intermediate, i.e., $p^{NE} \leq p < p^{E}$. We can therefore conclude that under intermediate priors both types of incumbent randomize their expansion decisions, $p_H, p_L \in (0,1)$, where note that $p_H \geq p_L$ for all $p < p^{E}$, which holds in this regime of intermediate priors. We next show that these probabilities are both increasing in p, since

$$\frac{\partial p_L}{\partial p} = \frac{\left(1-p^{E}\right)\left(1-p^{NE}\right)}{\left(1-p\right)^{2}\left(p^{E}-p^{NE}\right)} > 0 \quad \text{and} \quad \frac{\partial p_H}{\partial p} = \frac{p^{NE} p^{E}}{p^{2}\left(p^{E}-p^{NE}\right)} > 0.$$

Finally, note that at the lower bound of $p \in \left[p^{NE}, p^{E}\right)$, the incumbent's randomization becomes $\lim_{p \to p^{NE}} p_L = \lim_{p \to p^{NE}} p_H = 0$ whereas at the upper bound, we obtain $\lim_{p \to p^{E}} p_L = \lim_{p \to p^{E}} p_H = 1$.

Let us now examine the incumbent's strategy in this equilibrium. If the high-cost incumbent expands with probability $p_H \in (0,1)$, as prescribed, it must be that the entrant makes her indifferent between expanding and not expanding her facility,

$$r \times \left(\pi_{inc,H}^{D,E} - K_H\right) + \left(1-r\right) \times \left(\pi_{inc,H}^{M,E} - K_H\right) = s \times \pi_{inc,H}^{D,NE} + \left(1-s\right) \times \pi_{inc,H}^{M,NE},$$

where r and s are the probability with which the entrant enters after observing expansion and no expansion, respectively. Solving for r, we obtain

$$r(s) = \frac{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} - K_H}{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}} + s\left(\frac{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}}{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}}\right)$$

and since $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$ and $\pi_{inc,H}^{D,E} = \pi_{inc,H}^{D,NE}$, the above expression simplifies to $r(s) = s - \frac{K_H}{PLE_H^{E}}$.

Similarly, when the incumbent's costs are low, the entrant makes the incumbent indifferent between expanding and not expanding her facility,

$$r \times \left(\pi_{inc,L}^{D,E} - K_L\right) + \left(1-r\right) \times \left(\pi_{inc,L}^{M,E} - K_L\right) = s \times \pi_{inc,L}^{D,NE} + \left(1-s\right) \times \pi_{inc,L}^{M,NE}.$$

Solving for s, we obtain

$$s(r) = \frac{\pi_{inc,L}^{M,NE} + K_L - \pi_{inc,L}^{M,E}}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}} - r\left(\frac{\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,E}}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}}\right),$$

which simplifies to $s(r) = \frac{K_L - BCC_L}{PLE_L^{NE}} + r\left(\frac{PLE_L^{E}}{PLE_L^{NE}}\right)$. Solving for probabilities s and r simultaneously, we obtain

$$s = \left[K_L - BCC_L - \frac{K_H \times PLE_L^{E}}{PLE_H^{E}}\right] \times \frac{1}{PLE_L^{NE} - PLE_L^{E}}$$

$$r = \left[K_L - BCC_L - \frac{K_H \times PLE_L^{NE}}{PLE_H^{E}}\right] \times \frac{1}{PLE_L^{NE} - PLE_L^{E}}$$

First, note, that probability s is positive if and only if $PLE_L^{NE} > PLE_L^{E}$ and $\frac{PLE_H^{E} \times \left(K_L - BCC_L\right)}{PLE_L^{E}} > K_H$ hold.

Secondly, s<1 if and only if

$$K_H > \frac{PLE_H^{E}}{PLE_L^{E}} K_L - \frac{PLE_H^{E} \times \left(BCC_L + \left(PLE_L^{NE} - PLE_L^{E}\right)\right)}{PLE_L^{E}} \tag{1}$$

Similarly, note that probability r is positive if and only if $PLE_L^{NE} > PLE_L^E$ and $\frac{PLE_H^E \times (K_L - BCC_L)}{PLE_L^{NE}} > K_H$ hold. Finally, note that r<1 if and only if

$$K_H > \frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{PLE_H^E \left(BCC_L + \left(PLE_L^{NE} - PLE_L^E \right) \right)}{PLE_L^{NE}} \qquad (2)$$

Let us first analyze the case in which condition $PLE_L^{NE} > PLE_L^E$ holds. In this case, probabilities r,s∈(0,1) if expansion costs satisfy

$$\frac{C}{PLE_L^E} > K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E} \quad \text{and} \quad \frac{C}{PLE_L^{NE}} > K_H > \frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{B}{PLE_L^{NE}}$$

where $B \equiv PLE_H^E \left[BCC_L + \left(PLE_L^{NE} - PLE_L^E \right) \right]$ and $C \equiv PLE_H^E \left(K_L - BCC_L \right)$.

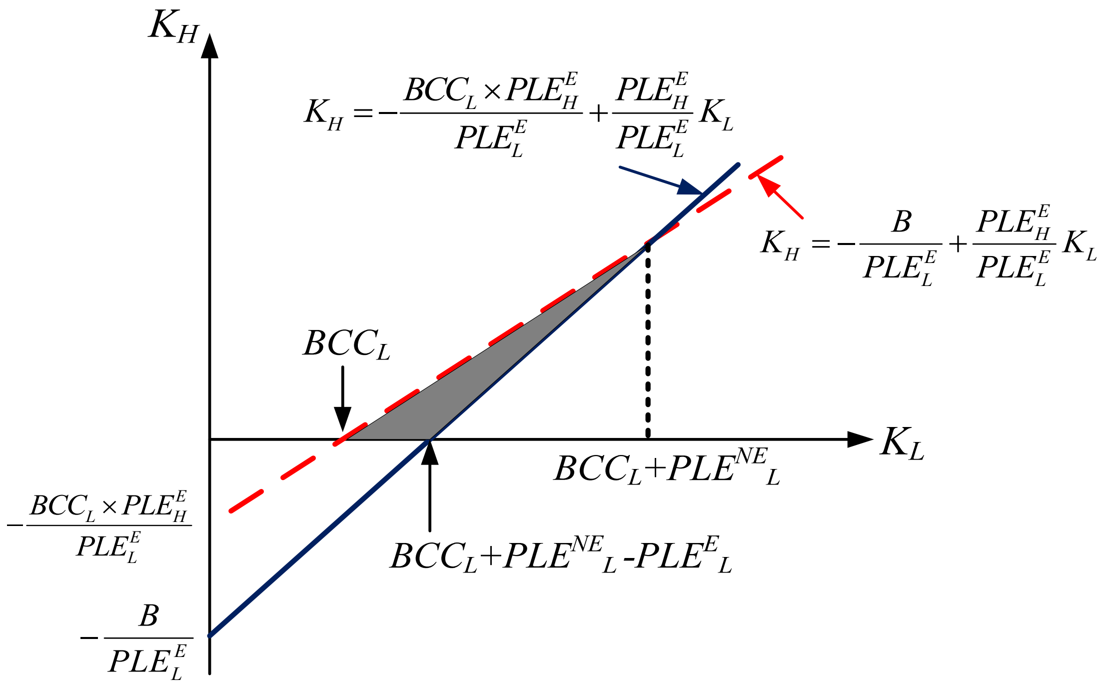
First, note that $\frac{C}{PLE_L^E} > \frac{C}{PLE_L^{NE}}$ since $PLE_L^{NE} > PLE_L^E$ in the case we consider. Hence $\frac{C}{PLE_L^{NE}} > K_H$ is more restrictive than $\frac{C}{PLE_L^E} > K_H$. In order to rank expressions $\frac{PLE_H^E}{PLE_L^E} \times K_L - \frac{B}{PLE_L^E}$ and $\frac{PLE_H^E}{PLE_L^{NE}} \times K_L - \frac{B}{PLE_L^{NE}}$, note that vertical intercepts satisfy $0 > \frac{B}{PLE_L^{NE}} > \frac{B}{PLE_L^E}$ and the equation in condition (2) is flatter than that in (1) since $\frac{PLE_H^E}{PLE_L^E} > \frac{PLE_H^E}{PLE_L^{NE}}$. Hence, condition (1) is more restrictive than (2). Thus, we only need to use

$$K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E}$$

Regarding condition $\frac{C}{PLE_L^{NE}} > K_H$, note that it can be expressed as

$$\frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{BCC_L \times PLE_H^E}{PLE_L^{NE}} > K_H$$

which is flatter than condition (1) since $\frac{PLE_H^E}{PLE_L^{NE}} < \frac{PLE_H^E}{PLE_L^E}$, and the vertical intercept is also smaller (absolute value) than that of condition (1) since $BCC_L \times PLE_H^E < B$ and $PLE_L^{NE} < PLE_L^E$, as depicted in the following figure.



Hence, when condition $PLE_L^{NE} > PLE_L^E$ holds, this semiseparating equilibrium with $p_H, p_L \in (0,1)$ can be supported for intermediate priors $p^{NE} \geq p > p^E$ and expansion costs satisfying

$$0 < K_H < \frac{C}{PLE_L^{NE}} \text{ and } 0 < K_L < \frac{PLE_L^E}{PLE_H^E} K_H + \frac{B}{PLE_H^E}.$$

If, by contrast, $PLE_L^{NE} < PLE_L^E$ holds, probabilities $r,s \in (0,1)$ if expansion costs satisfy

$$K_H > \frac{C}{PLE_L^E} \text{ and } K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E}$$

and

$$K_H > \frac{C}{PLE_L^{NE}} \text{ and } K_H > \frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{B}{PLE_L^{NE}}$$

From our previous discussion, $\frac{C}{PLE_L^E} > \frac{C}{PLE_L^{NE}}$. In addition, $K_H > \frac{C}{PLE_L^E}$ is more restrictive than $K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E}$ since both expressions have the same slope but the former originates at a higher vertical intercept than the latter since $BCC_L \times PLE_H^E < B$. Therefore, when $PLE_L^{NE} < PLE_L^E$ holds this semiseparating equilibrium with $p_H, p_L \in (0,1)$ can be supported for intermediate priors $p^{NE} \geq p > p^E$ and expansion costs satisfying

$$K_H > 0 \text{ for all } K_L \leq BCC_L \text{, and } K_H > \frac{C}{PLE_L^E} \text{ otherwise.}$$

p_H =0 and $p_L \in (0,1)$. We now check other semiseparating strategy profiles where either type of incumbent does not expand her facility. Let us first analyze the case where p_H =0 and $p_L \in (0,1)$. In this case, the entrant's posterior beliefs become $\mu(H \mid Exp) = 0$ after observing an expansion, which leads him to stay out since $\pi_{ent,L}^{D,E} < F$. In the case that the entrant observes no expansion from the incumbent, the entrant mixes if and only if his beliefs $\mu(H \mid Exp)$ satisfy

$$\mu(H \mid NoExp) \times \left(\pi_{ent,H}^{D,NE} - F\right) + (1 - \mu(H \mid NoExp))\left(\pi_{ent,L}^{D,NE} - F\right) = 0$$

and solving for $\mu(H \mid NoExp)$, we obtain $\mu(H \mid NoExp) = p^{NE}$. We can hence use Bayes' rule to find

$$\mu(H \mid NoExp) = p^{NE} = \frac{p}{p + (1-p)(1-p_L)}$$

Solving for p_L we obtain $p_L = \frac{p^{NE} - p}{p^{NE}(1-p)}$, where $p_L \in (0,1)$ for low priors, i.e., $p < p^{NE}$. In addition, p_L is decreasing in p since $\frac{\partial p_L}{\partial p} = \frac{p^{NE} - 1}{p^{NE}(1-p)^2} < 0$ starting at $\lim_{p \to p^{NE}} p_L = 0$ and converging to $\lim_{p \to 0} p_L = 1$.

Regarding the high-cost incumbent, she does not expand as prescribed, p_H=0, if

$$\pi_{inc,H}^{M,E} - K_H < s \times \pi_{inc,H}^{D,NE} + (1-s) \times \pi_{inc,H}^{M,NE}.$$

Solving for s, we obtain

$$s > \frac{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{M,E}}{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}} + \frac{K_H}{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}}$$

and since $PLE_H^{NE} = \pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}$, $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E}$ and $PLE_H^{NE} = PLE_H^E$, then this expression reduced to $s > \frac{K_H}{PLE_H^E} \equiv \hat{s}$. On the other hand, the low-cost incumbent mixes as prescribed, $p_L \in (0,1)$, if and only if

$$\pi_{inc,L}^{M,E} - K_L = s \times \pi_{inc,L}^{D,NE} + (1-s) \times \pi_{inc,L}^{M,NE}.$$

Solving for s, we obtain

$$s = \frac{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,E}}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}} + \frac{K_L}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}},$$

and since $BCC_L = \pi_{inc,L}^{M,NE} - \pi_{inc,L}^{M,E}$ and $PLE_L^{NE} = \pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}$, this expression becomes

$$s = \frac{BCC_L}{PLE_L^E} + \frac{K_L}{PLE_L^E} \equiv \tilde{s}$$

where probability cutoff $\tilde{s}$ satisfies $\tilde{s} \in (0,1)$ if $K_L < PLE_L^E - BCC_L$. Hence, we need $\tilde{s} > \hat{s}$, or $K_L > \frac{PLE_L^E}{PLE_H^E} K_H - BCC_L$. Hence, this semiseparating equilibrium can be sustained for relatively low priors, $p < p^{NE}$, and expansion costs satisfying $PLE_L^E - BCC_L > K_L > \frac{PLE_L^E}{PLE_H^E} K_H - BCC_L$.

p_L =0 and $p_H \in (0,1)$. Let us finally check the strategy profile where only the high-cost incumbent randomizes and the low-cost incumbent does not expand. In this case, the entrant's posterior beliefs after observing expansion are $\mu(H \mid Exp) = 1$, which leads him to enter since $\pi_{ent,H}^{D,E} > F$. In the case that the entrant observes no expansion from the incumbent, the entrant mixes if his beliefs $\mu(H \mid NoExp)$ are such that

$$\mu(H \mid NoExp) \times \left(\pi_{ent,H}^{D,NE} - F\right) + (1 - \mu(H \mid NoExp))\left(\pi_{ent,L}^{D,NE} - F\right) = 0$$

and solving for $\mu(H \mid NoExp)$, we obtain $\mu(H \mid NoExp) = p^{NE}$. Using Bayes' rule we have

$$\mu(H \mid NoExp) = p^{NE} = \frac{p\left(1-p_H\right)}{p\left(1-p_H\right)+(1-p)}$$

and solving for p_H we obtain $p_H = \dfrac{p-p^{NE}}{p\left(1-p^{NE}\right)}$, where $p_H \in (0,1)$ for all $p > p^{NE}$. In addition, p_H is increasing in p since $\dfrac{\partial p_H}{\partial p} = \dfrac{p^{NE}}{\left(1-p^{NE}\right)\times p^2} > 0$, starting at $\lim\limits_{p \to p^{NE}} p_H = 0$ and converging to $\lim\limits_{p \to 1} p_H = 1$.

Regarding the low-cost incumbent, she does not expand as prescribed (p_L=0) if and only if

$$\pi_{inc,L}^{M,E} - K_L < s \times \pi_{inc,L}^{D,NE} + \left(1-s\right) \times \pi_{inc,L}^{M,NE},$$

and solving for s, we find

$$s > \frac{BCC_L}{PLE_L^E} + \frac{K_L}{PLE_L^E} \equiv \tilde{s}$$

On the other hand, the high-cost incumbent mixes as prescribed, $p_H \in (0,1)$, if and only if

$$\pi_{inc,H}^{M,E} - K_H = s \times \pi_{inc,H}^{D,NE} + \left(1-s\right)\pi_{inc,H}^{M,NE}.$$

Solving for s, we obtain $s = \dfrac{K_H}{PLE_H^E} \equiv \hat{s}$, where $\hat{s} \in (0,1)$ for all $K_H < PLE_H^E$. Hence, we need that $\hat{s} > \tilde{s}$, or $K_H > \dfrac{PLE_H^E}{PLE_L^E} K_L + \dfrac{BCC_L \times PLE_H^E}{PLE_L^E}$. Hence, this semiseparating strategy profile can be sustained for priors $p > p^{NE}$ and expansion costs satisfying

$$PLE_H^E > K_H > \frac{PLE_H^E}{PLE_L^E} K_L + \frac{BCC_L \times PLE_H^E}{PLE_L^E}.$$

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