3 Static imperfect competition

Firms have various strategic options at their disposal. In the most basic models of strategic interaction, firms choose a single strategic variable, quantity or price, once. In many markets, firms are seen as price-setters. It thus appears natural to start with the analysis of price-setting firms in an environment with a few firms; we do so in Section 3.1. In other markets, however, it seems more reasonable to assume that firms choose quantities rather than prices; we analyse quantity competition in Section 3.2. We then proceed to compare price and quantity competition. First, in Section 3.3, we show that quantity competition can sometimes be mimicked by a two-stage model in which firms choose their capacity of production and next, set their price; we also directly compare price and quantity competition in a unified model of product differentiation. Second, in Section 3.4, we bring the comparison of price and quantity competition to a more general level by introducing the concepts of 'strategic complements' and 'strategic substitutes'. Finally, in Section 3.5, we discuss the empirical investigation of industries with market power.

3.1 Price competition

We analyse here several models of price competition. We start with the standard Bertrand (1883) model where products are homogeneous. Then, we extend the model in two directions: first, we assume that firms have private information about their marginal costs of production; second, we consider differentiated products.

3.1.1 The standard Bertrand model

In the simplest version of the price-competition or Bertrand model, there are two firms with homogeneous products and identical constant marginal costs c. Both firms set price simultaneously to maximize profit. The firm with the lower price attracts all the market demand $Q(p)$, where p is the relevant price. Suppose that, at equal prices, the market splits somehow, say at α_1 and $\alpha_2 = 1 - \alpha_1$. Then firm i faces demand

$$Q_i(p_i) = \begin{cases} Q(p_i) & \text{if } p_i < p_j, \\ \alpha_i Q(p_i) & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases}$$

In this simple Bertrand model, there is a unique pure-strategy equilibrium in which both firms set price equal to marginal costs, $p = c$. To prove this result, it suffices to show that for all other price combinations, there is at least one firm that has an incentive to deviate. If $p_i > p_j > c$, firm i can increase its profits by setting $p_i' \in (c, p_j)$; if $p_i = p_j > c$, each firm can increase its

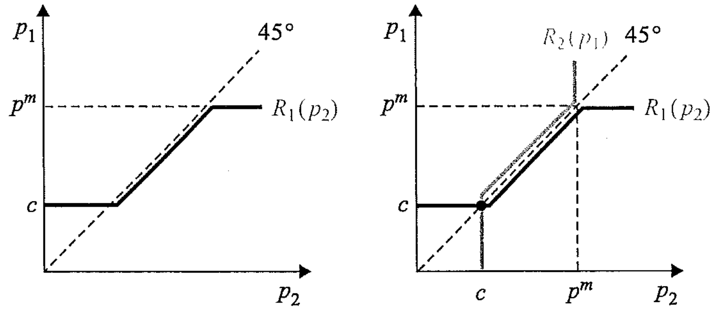

Figure 3.1 Reaction functions and equilibrium in the Bertrand duopoly (with homogeneous product and identical and constant marginal costs)

profits by slightly undercutting the rival price; if $p_i > p_j = c$, firm j can increase its profits by increasing its price above c and below p_i.

Another useful way to show this result is to derive the firms' reaction functions. As the previous reasoning clearly shows, firm i's best response is to set a price p_i just below the price p_j of its rival so as to attract all consumers and maximize its profits. Naturally, this rule suffers two exceptions. First, for values of p_j less than the marginal cost c, firm i chooses $p_i = c$ as undercutting firm j would entail losses. Second, define p^m as the price that maximizes $(p - c)\,Q(p)$; that is, p^m is the monopoly price.[b] Then, for values of p_j larger than p^m, firm i chooses $p_i = p^m$ as a price just below p_j would not maximize profits. The left panel of Figure 3.1 depicts firm 1's reaction function, denoted $R_1(p_2)$, and the right panel superposes the two reaction functions under the assumption that the strategy space of each firm is discrete. For instance, if the price and marginal costs have to be calculated in cents, there are three pure-strategy equilibria, namely $p_1 = p_2 = c$, $p_1 = p_2 = c + 1$ and $p_1 = p_2 = c + 2$. In the limit, as the step between two subsequent prices turns to zero, one obtains that $p_1 = p_2 = c$ is the unique Nash equilibrium of the game. We note for future reference that the *reaction functions of Bertrand duopolists are (weakly) upward-sloping*: when one firm raises its price, the best response of the other firm is to raise its price too.[c]

The previous result is often called a paradox, since we consider a market with just two firms and still the perfectly competitive outcome prevails. It shows that under some circumstances, the competitive pressure under duopoly can be sufficiently strong to give rise to marginal-cost pricing.

Lesson 3.1 **In the homogeneous product Bertrand duopoly with identical and constant marginal costs, the equilibrium is such that firms set prices equal to marginal costs and thus do not enjoy any market power.**

[b] We assume that $Q(p)$ is such that the monopoly profit function has a unique maximizer.

[c] With a continuous strategy space the reaction function is not well-defined (open set problem). The reaction function approximates the situation with a large but finite number of strategies.

The intuition behind this result is clear: firms cannot support prices above marginal cost because small price cuts can lead to large increases in quantity demanded and profits.[d]

Let us now introduce cost asymmetries in the Bertrand model. Firms might indeed have access to different technologies, resulting in different marginal costs of production. As we will describe in Part VII, this is typically what happens when a firm is granted a patent for the exclusive use of an improved technology. In particular, suppose that there are n firms in the market, which are ordered according to their marginal costs $c_i < c_{i+1}$, $i = 1, \dots, n-1$. Firms set price simultaneously. If two firms set the same price, the more efficient firm gets all the demand. Then, any price $p \in [c_1, c_2]$ is an equilibrium. Note, however, that if there is a small probability that the less efficient firm gets a small market share, firm 2 would not set a price $p < c_2$. Therefore, we select the equilibrium in which $p = c_2$ as the most reasonable equilibrium.[e] In the selected equilibrium, firm 1 enjoys positive profit, contrary to the symmetric pure-price competition model. Firm 1 collects the efficiency gains compared to the next best alternative technology (i.e., the difference between c_2 and c_1).[f]

Note that in the equilibrium of the Bertrand game with symmetric costs, both firms use weakly dominated strategies: setting a higher price cannot make the firm worse off but, in a case where the competitor deviates from its Nash equilibrium strategy, such a deviation can be worthwhile. If a firm faces a competitor of unknown costs, it appears that the firm no longer has an incentive to set price equal to marginal costs. We turn to this setting next.

3.1.2 Price competition with uncertain costs

Suppose that each firm has private information about its marginal costs. In particular, the marginal costs of each firm are drawn independently from some distribution and firms learn the realization of their own cost parameter but not those of their competitors. When setting their prices, firms face a trade-off between margins and the likelihood of winning the competition for the best deal they offer to consumers. In this setting, we will show that firms typically set price above marginal costs, so that firms earn positive profits in equilibrium. This price–cost margin is decreasing in the marginal costs. This setting appears to be a natural next step in the analysis of price competition models. It delivers the result that even in the context of price competition with homogeneous goods, all firms in a market set price above marginal costs. The analysis is slightly involved and may therefore be skipped at a first reading.

We analyse this result in a simple setting with linear market demand and a uniform distribution of marginal costs.[2] Suppose that n firms face the market demand function $Q(p) = 1 - p$ and a marginal cost that is independently drawn from the uniform distribution on $[0, 1]$. Denote by $\widehat{p}_{-i}$ the lowest price charged by the competitors of firm i: $\widehat{p}_{-i} \equiv \min\{p_1, p_{i-1}, p_{i+1}, p_n\}$. Then firm i faces demand $q_i = 1 - p_i$ if $p_i < \widehat{p}_{-i}$, $q_i = (1 - p_i)/m$ if $p_i = \widehat{p}_{-i}$ where m is the number

[d] This result largely carries over to Bertrand oligopolies, with the qualification that there also exist Nash equilibria where at least two firms set the price equal to marginal cost (while the other firms may set the price above marginal cost).

[e] Formally speaking, all equilibria with $p < c_2$ are not trembling-hand perfect.

[f] It is implicitly assumed here that the difference $c_2 - c_1$ is not large enough for the optimal price that would be set by a monopoly producing at cost c_1 to fall below c_2; otherwise, firm 1 would set this monopoly price and not a price equal to c_2. In Part VII, we will say that the 'innovation' (represented by the cost reduction from c_2 to c_1) is supposed to be 'minor' or 'non-drastic'.

[2] The analysis goes back to Hansen (1988) in an auction-theoretic setting. For a general analysis of the Bertrand game, see Spulber (1995). For a derivation of expressions in the uniform-linear case, see for example Lofaro (2002).

of firms that set the lowest price p_i, and $q_i = 0$ if $p_i > \hat{p}_{-i}$. We solve for the symmetric Bayesian Nash equilibrium of this game, which is given by a function $p^*(\cdot)$ that maps marginal costs into prices. This function has the property that $p^*(1) = 1$ because the highest-cost firm will always set price equal to marginal cost. For this reason, we do not gain additional insights by considering a demand function of the type $a - p$ with $a > 1$. We look for a price function that is strictly increasing on [0, 1] so that each cost type sets a different price.[g] Since a firm receives the same cost parameter as some of their competitors with probability zero (and profits are bounded), we can restrict attention to situations in which firms have different cost parameters. Firm i's expected profit is

$$(p_i - c_i)(1 - p_i)\text{Prob}(p_i < \hat{p}_{-i}),$$

because a firm of type c_i sells the product with probability $\text{Prob}(p_i < \hat{p}_{-i})$. Due to the independence assumption, this probability is simply the product of the probabilities of setting the lowest price in all pairwise comparisons with the firm's competitors: $\text{Prob}(p_i < \hat{p}_{-i}) = \text{Prob}(p_i < p^*(c_1)) \times \cdots \times \text{Prob}(p_i < p^*(c_{i-1})) \times \text{Prob}(p_i < p^*(c_{i+1})) \times \cdots \times \text{Prob}(p_i < p^*(c_n))$. For a firm of type c_i, we have $p_i = p^*(c_i)$ in equilibrium. Therefore, $p^{*-1}(p_j)$ denotes the marginal cost of a competitor who sets p_j and follows the equilibrium strategy. Since p^* is strictly increasing, we can write $\text{Prob}(p_i < p^*(c_j)) = \text{Prob}(p^{*-1}(p_i) < c_j)$ which, since c_j is uniformly distributed on [0, 1], is equal to $1 - p^{*-1}(p_i)$. Consequently, $\text{Prob}(p_i < \hat{p}_{-i}) = [1 - p^{*-1}(p_i)]^{n-1}$ and each firm solves the following maximization problem:

$$\max_{p_i} (p_i - c_i)(1 - p_i)[1 - p^{*-1}(p_i)]^{n-1}.$$

The first-order condition of profit maximization is

$$\begin{aligned}&(1 + c_i - 2p_i)[1 - p^{*-1}(p_i)]^{n-1}\\ &\quad - (p_i - c_i)(1 - p_i)(n - 1)[1 - p^{*-1}(p_i)]^{n-2}\frac{\partial p^{*-1}(p_i)}{\partial p_i} = 0.\end{aligned}$$

In a symmetric equilibrium, $p^*(c_i) = p_i$ and thus $c_i = p^{*-1}(p_i)$. We also note that the derivative of the inverse price-setting function is the inverse of the derivative. Hence, we can rewrite the above equation as

$$\begin{aligned}&p^{*\prime}(c_i)(1 + c_i - 2p^*(c_i))[1 - c_i]^{n-1}\\ &\quad - (p^*(c_i) - c_i)(1 - p^*(c_i))(n - 1)[1 - c_i]^{n-2} = 0.\end{aligned}$$

Dividing by $[1 - c_i]^{n-2}$ and rearranging gives the differential equation

$$p^{*\prime}(c_i) = \frac{(n-1)(p^*(c_i) - c_i)(1 - p^*(c_i))}{(1 - c_i)(1 + c_i - 2p^*(c_i))}.$$

Suppose the solution to this equation is of the form $p^*(c_i) = a + bc_i$. Taking into account that $p^{*\prime}(c_i) = b$ and substituting, we obtain an equation in a, b, n and c_i. The only admissible solution is $a = 1/(n + 1)$ and $b = n/(n + 1)$. The price schedule is then given by

$$p^*(c_i) = \frac{1 + nc_i}{n + 1}. \tag{3.1}$$

[g] The argument why this is the case is presented formally in the context of search models in Chapter 7.

To verify that this is indeed the unique equilibrium, we note that demand is

$$\text{Prob}(p_i < \widehat{p}_{-i}) = \left[\frac{(n+1)(1-p_i)}{n}\right]^{n-1}$$

for prices p_i with $1/(n+1) < p_i < 1$. On this set of prices, firm i's maximization problem is therefore

$$\max_{p_i} (p_i - c_i)(1-p_i)^n \left(\frac{n+1}{n}\right)^{n-1}.$$

The unique solution to this problem is indeed given by Equation (3.1).

The equilibrium has a number of properties. First, all firms (except the firm with $c_i = 1$) set prices above marginal cost. The price–cost margin is increasing with the efficiency of the firm. Note also that the resulting equilibrium price $p^*(\widehat{c})$ that consumers pay, where $\widehat{c} = \min\{c_1, \ldots, c_n\}$, is decreasing with the number of firms even if the lowest cost in the market does not change. In other words, an increase in the number of firms leads to a more competitive outcome because it increases the competitive pressure in the market. In addition, it leads to a lower expected cost of the most efficient firm. This amplifies the effect of an increase in the number of firms on equilibrium price. Correspondingly, total industry output increases with the number of firms (although it is always one firm that is active). In the limit, as n turns to infinity, price converges to marginal costs. In this respect, the model delivers qualitatively similar results as the Cournot model (see Section 3.2). We summarize our main insight by the following lesson.

Lesson 3.2 **In the price competition model with homogeneous products and private information about marginal costs, firms set price above marginal costs and make strictly positive expected profits in equilibrium. More firms in the industry lead to lower price–cost margins, higher output and lower profits. As the number of firms converges to infinity, the competitive limit is reached.**

In contrast to a Cournot model (and price competition models with consumer search that will be analysed in Chapter 7), even though all active firms set price above marginal costs, only the most efficient firm makes strictly positive revenues. In other words, firms post prices above marginal costs but all except one firm do not generate any revenues at any point in time. While this appears to often be violated in real markets because of other market frictions (see, in particular, Part III), we believe that the model has some intuitive appeal: firms listing their products, for example on Amazon Marketplace, at a high price can only hope to make profits if more efficient firms happen to be absent. The present model provides a rationale as to why these high-cost firms are around in spite of this: more efficient supplies may have dried up so that only high-price alternatives remain available. If such markets operate over time and firms are repeatedly subject to cost shocks, the results of the model are compatible with price changes over time and a changing identity of the most efficient firm.

3.1.3 Price competition with differentiated products

While pure price competition takes away any margins that are not due to lower costs, competitors may avoid intense competition if they offer products that are not perfect substitutes. If products are only imperfect substitutes for one another, the market features product differentiation. In this case demand may respond smoothly to price changes. As we already saw in the previous chapter, Merck and other former patent holders do not set price equal to marginal costs and typically charge a premium over the price of generics. If reading this book gives you a headache and you decide to buy some Aspirin®, you may want to keep in mind that this is more costly than buying a pill that has the same chemical composition but is sold under a different name. Here, product differentiation is present even though the physical composition of the products is the same. Many firms outside the pharmaceutical industries have also long recognized the need to differentiate their products from competitors in order to increase their market power.[h] Our next case illustrates this behaviour.

Case 3.1 Bananas and oranges

Fruits such as bananas and oranges may look like a bad example to illustrate the idea of product differentiation. However, for more than 100 years, the Sunkist brand has been with us. Long ago, California citrus growers decided to sell their products in a distinct way and created the Sunkist trademark. This allowed them to advertise their products without being confused with competitors. Other firms have followed this example. For example, bananas with the Chiquita brand are sold at a premium. Sunkist and Chiquita managed to convince consumers that their brand offers certain features, such as taste and freshness, that competitors cannot guarantee.[3]

We now turn to a formal analysis of product differentiation and to this end, we use again the Hotelling line that we introduced in the dominant firm model of Chapter 2; this is the so-called Hotelling model. Suppose that two products (denoted 1 and 2) are located at the extreme locations of the $[0, 1]$ interval. Firms have constant and identical marginal costs c of production and maximize profits $\pi_i = (p_i - c)Q_i(p_i, p_j)$. Consumers are uniformly distributed on the unit interval and incur a disutility from travelling to the location of the product, which is linear in distance. A consumer's indirect utility is written as $r - \tau|l_i - x| - p_i$ if the consumer buys one unit of product i. Additional units of this product do not increase a consumer's utility. Furthermore, a consumer is interested in exactly one of the products. Consumer x's purchasing decision solves $\max_{i=1,2}\{r - \tau|l_i - x| - p_i\}$. For prices such that both firms are active, there is exactly one indifferent consumer $\widehat{x}$ who is defined by

$$r - \tau\widehat{x} - p_1 = r - \tau(1 - \widehat{x}) - p_2 \text{ or, equivalently,}$$

$$\widehat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2\tau}.$$

[h] We investigate the incentives to differentiate products in Chapter 5.

[3] The Sunkist story is nicely told in Arens (2004).

Hence, the demand of firm 1 consists of all consumers to the left of $\widehat{x}$ and the demand of firm 2 consists of all consumers to the right of $\widehat{x}$. For a mass 1 of consumers, demand functions are

$$Q_i(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2\tau}.$$

Profit functions then become

$$\pi_i = (p_i - c)\left(\frac{1}{2} + \frac{p_j - p_i}{2\tau}\right).$$

The first-order condition of profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2\tau}\left(p_j - 2p_i + c + \tau\right) = 0.$$

Solving the previous equation for p_i, we derive firm i's reaction function:

$$p_i = \tfrac{1}{2}(p_j + c + \tau),$$

which is upward-sloping as in our previous model of Bertrand competition with homogeneous products.

At the intersection of the two reaction functions, we find the equilibrium prices: $p_i = p_j = c + \tau$. This demonstrates that due to product differentiation each firm faces a demand function that is not perfectly price-elastic. The more products are differentiated, that is the higher τ, the higher the price–cost margin of the firms in equilibrium.

Lesson 3.3 **If products are more differentiated, firms enjoy more market power.**

The above analysis was performed under the implicit assumption that all consumers prefer to purchase the product; that is, we did not consider the consumers' participation decision. Let us introduce the possibility of abstaining from buying a product in the market, in which case the consumer's utility of the outside option is set equal to zero. Then, for τ sufficiently large, the participation constraint of some consumers is violated. It follows that for high values of τ, firms enjoy (local) monopoly power: each firm sets the monopoly price and ignores the presence of the other firm.

Duopolies with price competition not only appear in textbooks but do from time to time pop up in the real world, as the following case demonstrates.

Case 3.2 Airbus vs. Boeing and the market for wide-bodied aircraft[4]

The market for large commercial jets is currently dominated by two firms: Boeing, of the USA and Airbus, of Europe. It can therefore be described as a duopoly, and it is likely to remain a duopoly for years to come, in spite of its high profitability (the market is forecast to be worth $2.6 trillion over the next two decades). Potential entrants do exist. China and Russia need to replace the old Tupolevs and other Russian-built aircraft that fly in both countries, but they do not want to rely on Boeing or Airbus without attempting to develop their own

[4] Based on 'China and Russia take on the might of Boeing and Airbus', *The Times* (20 March 2007).

industries first. However, the two countries face enormous entry barriers: (i) new types of aircraft cost up to \$10 billion to develop; (ii) it took decades for Boeing and Airbus to establish their safety and reliability records, while Russian and Chinese manufacturing suffer from a reputation of poor quality control.

Although this market provides a good example of a duopoly, it is less clear whether it is adequately described by price competition. The pure price-competition models we have analysed so far fail to capture one important feature of this market, namely that capacity constraints may lead to delays. For instance, the Airbus A380 suffered a series of delays and was finally launched two years behind its original schedule. Boeing also had to delay the launch of its new 'Dreamliner' aircraft (B787) in 2007–8. This suggests that capacity constraints may play a role. We consider this issue formally in Section 3.3.

We extend the analysis to a setting of localized competition with n firms. Suppose firms are equidistantly located on a circle with circumference 1 and consumers are uniformly distributed on this circle. This is the so-called *Salop model*.[5] The consumers' decision-making corresponds to that in the Hotelling model: consumers buy at most one unit of the product and they buy it from the firm offering them the lowest 'generalized price', that is, the price augmented by the transportation cost. We assume a unit transportation cost of τ. Hence, consumer x's purchasing decision solves $\max_{k=i,i+1}\{r - \tau|l_k - x| - p_i\}$, where firms $k = i, i+1$ are the firms between which consumer x is located and where firm k's location is $l_k = k/n$. The consumer $\widehat{x}_{i,i+1}$ who is indifferent between firms i and $i+1$ is defined by

$$r - \tau\left(\widehat{x}_{i,i+1} - \tfrac{i}{n}\right) - p_i = r - \tau\left(\tfrac{i+1}{n} - \widehat{x}_{i,i+1}\right) - p_{i+1} \text{ or, equivalently,}$$

$$\widehat{x}_{i,i+1} = \frac{2i+1}{2n} + \frac{p_{i+1} - p_i}{2\tau}.$$

By analogy, we can identify the consumer who is indifferent between firm i and its left neighbour, firm $i-1$, as

$$\widehat{x}_{i-1,i} = \frac{2i-1}{2n} + \frac{p_i - p_{i-1}}{2\tau}.$$

Firm i attracts all consumers located between $\widehat{x}_{i-1,i}$ and $\widehat{x}_{i,i+1}$. Because firms are located symmetrically, we focus on a symmetric equilibrium in which all firms charge the same price p. Hence, setting $p_{i-1} = p_{i+1} = p$ in the above expressions, we compute the demand for firm i as

$$\begin{aligned} Q_i(p_i, p) &= \left(\frac{2i+1}{2n} + \frac{p_{i+1} - p_i}{2\tau}\right) - \left(\frac{2i-1}{2n} + \frac{p_i - p_{i-1}}{2\tau}\right) \\ &= \frac{1}{n} + \frac{p - p_i}{\tau}. \end{aligned}$$

Supposing that all firms have the same constant marginal costs of production c, we can write firm i's maximization programme as

$$\max_{p_i} (p_i - c)\left(\frac{1}{n} + \frac{p - p_i}{\tau}\right).$$

[5] This model has become widely used since Salop (1979).

The first-order condition gives $1/n + (p - 2p_i + c)/\tau = 0$. Setting $p_i = p$ yields

$$p = c + \tau/n,$$

which is analogous to the result we obtained in the Hotelling model. An additional parameter, the number of firms, also affects the equilibrium outcome. A larger number of firms leads to closer substitutes on the circle. This increases the competitive pressure. As the number of firms turns to infinity, prices converge to marginal costs.

3.1.4 Asymmetric competition with differentiated products

In the previous models, firms did not produce the same product but these products were symmetric in the sense that each firm's incentives did not depend on whether it was called firm i or j. However, in some instances products are not only horizontally differentiated, but one product might also be of superior quality or offer additional features.

Suppose that firms operate in the same environment as in the previous Hotelling model but that a consumer's indirect utility is $v_i = r_i - \tau |l_i - x| - p_i$. Above, we had $r_1 = r_2$. Let us now assume that the willingness to pay for product 1 is greater than for product 2 at the ideal location l_i (i.e., $r_1 > r_2$), but that for some consumers, product 2 is more attractive than product 1 (i.e., $r_2 + \tau > r_1$). Here, products are horizontally differentiated but product 1 is of superior quality or offers additional features that are of the same value to all consumers.[i] The indifferent consumer is given by

$$\widehat{x} = \frac{1}{2} + \frac{(r_1 - r_2) - (p_1 - p_2)}{2\tau}. \tag{3.2}$$

Since $Q_1(p_1, p_2) = \widehat{x}$ and $Q_2(p_1, p_2) = 1 - \widehat{x}$, profit functions become

$$\pi_i = (p_i - c)\left(\frac{1}{2} + \frac{(r_i - r_j) - (p_i - p_j)}{2\tau}\right).$$

The first-order condition of profit maximization of firm i (on the range of prices such that demand is strictly positive for both firms) is

$$\frac{1}{2\tau}[p_j - 2p_i + c + \tau + (r_i - r_j)] = 0.$$

Solving the system of two linear equations, we obtain

$$\begin{cases} p_1^* = c + \tau + \frac{1}{3}(r_1 - r_2), \\ p_2^* = c + \tau - \frac{1}{3}(r_1 - r_2). \end{cases}$$

We observe that the high-quality firm, firm 1, sets a higher price; the price difference between firms is $p_1^* - p_2^* = (2/3)(r_1 - r_2)$. Hence in equilibrium, demand for firm 1 is

$$Q_1(p_1^*, p_2^*) = \frac{1}{2} + \frac{r_1 - r_2}{6\tau},$$

and correspondingly for firm 2. Note that demand is strictly positive also for the low-quality firm (under our assumption that $r_2 + \tau > r_1$). However, the high-quality firm has larger demand than the low-quality firm in equilibrium.

[i] The distinction between horizontal and vertical product differentiation will be clarified in Chapter 5.

To maximize welfare (measured as total surplus), prices must be equal to marginal cost. Introducing $p_1 = p_2 = c$ in expression (3.2), we obtain the socially optimal allocation

$$Q_1(c,c) = \frac{1}{2} + \frac{r_1 - r_2}{2\tau} > Q_2(c,c) = \frac{1}{2} + \frac{r_2 - r_1}{2\tau} > 0,$$

which shows that the ranking of demand also holds for the solution that maximizes welfare.

We can now ask whether the number of consumers served by firm 1 is socially sufficient. It is immediate to see that $Q_1(p_1^*, p_2^*) < Q_1(c,c)$: the equilibrium demand of firm 1 is too low from a social point of view. This is due to the fact that firm 1 sets a higher price than firm 2 under strategic price-setting. This is a general feature of imperfect competition. Note that instead of considering a model in which firm i offers a more attractive product, we could analyse a situation in which firm i produces at lower costs. Our insight would be confirmed under this alternative assumption: the low-cost firm sells too few units from a welfare perspective.

Lesson 3.4 **Under imperfect competition, the firm with higher quality or lower marginal costs sells too few units from a welfare perspective.**

If a social planner wanted to correct this inefficiency, he would need to subsidize the high-quality (or low-cost) firm or to tax the low-quality (or high-cost) firm. This appears in contrast to government programmes that protect feeble firms.

3.2 Quantity competition

In this section we analyse situations in which firms set quantities, as first analysed by Cournot (1838). The price clears the market and is thus equal to the inverse demand, $p = P(q)$, where q is total output in the industry. We may wonder where this price p comes from. In real markets, we typically observe some price-setting, which makes it difficult to provide a literal interpretation of Cournot competition. However, the price-setting is sometimes done on behalf of the firms by some auctioneer. If there is a small number of big players that provide most of the industry output, then these firms may commit themselves to bring a certain amount of a product to the market. The market clearing may be performed by an auctioneer (who, for simplicity, is assumed not to charge for market transactions) who finds the highest price at which all offered units are sold. The resulting equilibrium allocation is then equivalent to the outcome in the quantity-setting game.

We start our analysis of the Cournot model with the simple case of an oligopoly facing linear demand for a homogeneous product and producing at constant marginal costs (Subsection 3.2.1). In order to gain further insight, we extend the initial setting by using general demand and cost functions (Subsection 3.2.2).

3.2.1 The linear Cournot model

We consider a homogeneous product market with n firms in which firm i sets q_i. Total output is then $q = q_1 + \cdots + q_n$. The market price is given by the linear inverse demand $P(q) = a - bq$ (with

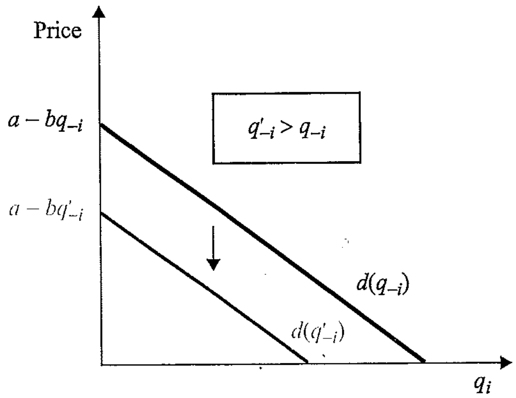

Figure 3.2 Residual demand for a Cournot oligopolist

$a, b > 0$). Let us also suppose that the cost functions are linear: $C_i(q_i) = c_i q_i$ (with $0 \leq c_i < a$ $\forall i = 1, \ldots, n$). We first solve the model in the most general case for any number of potentially heterogeneous firms ($c_i \neq c_j$ for any $i \neq j$). We then use this general analysis in two specific cases: in a duopoly ($n = 2$) and in a symmetric oligopoly ($c_i = c\ \forall i$).

Cournot oligopoly with heterogeneous firms

Let us denote by $q_{-i} \equiv q - q_i$ the sum of the quantities produced by all firms but firm i. The inverse demand can then be rewritten as

$$P(q) = (a - bq_{-i}) - bq_i \equiv d_i(q_i; q_{-i}).$$

As firm i conjectures that the other firms do not modify their choice of quantity no matter what it decides itself to produce (this is known as the *Cournot conjecture*), the function $d_i\,(q_i; q_{-i})$ can be seen as the *residual demand* facing firm i. Clearly, if firm i expects the total quantity of the other firms to increase, it faces a lower residual demand, as illustrated in Figure 3.2.

Accordingly, firm i will produce a lower quantity. We now confirm this intuition analytically. Firm i chooses q_i to maximize its profits $\pi_i = (a - b\,(q_i + q_{-i}))\,q_i - c_i q_i$, which can also be written as $d_i\,(q_i; q_{-i})\,q_i - c_i q_i$, meaning that firm i acts as a monopolist on its residual demand. The first-order condition of profit maximization is expressed as

$$a - c_i - 2bq_i - bq_{-i} = 0 \tag{3.3}$$

or, solving for q_i, as

$$q_i\,(q_{-i}) = \tfrac{1}{2b}(a - c_i - bq_{-i}). \tag{3.4}$$

Expression (3.4) gives firm i's best-response (or reaction) function. One checks that the best-response function slopes downward: faced with a larger quantity produced by the rival firms (i.e., a larger q_{-i}), firm i optimally reacts by lowering its own quantity ($q_i\,(q_{-i})$ decreases). This is illustrated below, for the duopoly case, in Figure 3.3.

At the Cournot equilibrium, Equation (3.4) is satisfied for each of the n firms. In other words, each firm 'best responds' to the choices of the other firms. Summing the equations (3.4) derived for the n firms, we obtain

$$\sum_{i=1}^{n} q_i = \tfrac{1}{2b}\left(na - \sum_{i=1}^{n} c_i - b\sum_{i=1}^{n} q_{-i}\right).$$

By definition, $\sum_i q_i = q$ and it is easily understood that $\sum_i q_{-i} = (n-1)\,q$. Denoting C for $\sum_i c_i$, we can rewrite the previous equation as

$$q = \tfrac{1}{2b}\left(na - C - b\,(n-1)\,q\right) \Leftrightarrow q^* = \frac{na - C}{b\,(n+1)}.$$

Introducing q^* (i.e., the total quantity produced at the Cournot equilibrium) into Equation (3.4), we find the quantity that firm i produces at the Cournot equilibrium (where $C_{-i} \equiv \sum_{j \neq i} c_j$):

$$q_i^* = \tfrac{1}{2b}\left(a - c_i - b\left(\frac{na - C}{b\,(n+1)} - q_i^*\right)\right) \Leftrightarrow q_i^* = \frac{a - (n+1)\,c_i + C}{b\,(n+1)} \Leftrightarrow$$

$$q_i^* = \frac{a - nc_i + C_{-i}}{b\,(n+1)}. \tag{3.5}$$

Evaluated at the equilibrium, the first-order condition (3.3) can be rewritten as $bq_i^* = a - b(q_i^* + q_{-i}^*) - c_i = P(q^*) - c_i$. It follows that firm i's equilibrium profits are computed as[j]

$$\pi_i^* = (P(q^*) - c_i)\,q_i^* = b\left(q_i^*\right)^2 = \frac{(a - nc_i + C_{-i})^2}{b\,(n+1)^2}. \tag{3.6}$$

We observe that π_i^* decreases with c_i and increases with C_{-i}, which allows us to state the following lesson.

Lesson 3.5 **In the linear Cournot model with homogeneous products, a firm's equilibrium profits increase when the firm becomes relatively more efficient than its rivals (i.e., all other things being equal, when its marginal cost decreases or when the marginal cost of any of its rivals increases).**

Implicit in the previous analysis was the assumption that the equilibrium is interior, in the sense that all firms find it optimal to be active at equilibrium. This is so if, for all firms i, we have $q_i^* \geq 0$, which is equivalent to $c_i \leq (1/n)\,(a + C_{-i})$. If we order firms according to their marginal costs ($c_i \leq c_{i+1}, i = 1, \ldots, n-1$), the latter inequality is the most stringent for firm n. Hence, what the condition for an interior equilibrium says is that the less efficient firm cannot be 'too inefficient' relative to the rival firms (i.e., its marginal cost must be small enough).

[j] As we will frequently use the linear Cournot model in the rest of the book, it is worth putting a bookmark at this page or, better, remembering expressions (3.5) and (3.6).

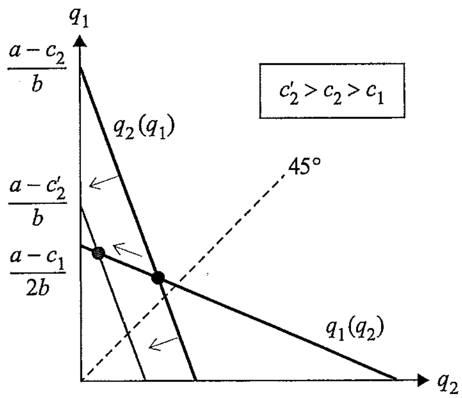

Figure 3.3 Cournot duopoly

Cournot duopoly

In order to illustrate the previous results, both analytically and graphically, we briefly redo the analysis in the duopoly case. Using expression (3.4), we can write the system of reaction functions:

$$\begin{cases} q_1 = \frac{1}{2b}\left(a - c_1 - bq_2\right), \\ q_2 = \frac{1}{2b}\left(a - c_2 - bq_1\right). \end{cases}$$

The solution of this system yields the following Nash equilibrium quantities:

$$q_1^* = \frac{a - 2c_1 + c_2}{3b} \quad \text{and} \quad q_2^* = \frac{a - 2c_2 + c_1}{3b}.$$

We let the reader compute the equilibrium profits and check that they correspond to the general formula given by expression (3.6). If we assume that $c_1 \leq c_2$, the condition for an interior equilibrium is $c_2 \leq (a + c_1)/2$.[k]

Figure 3.3 nicely illustrates our previous results. First, the two firms' reaction functions are shown to be downward-sloping. Second, the assumption that $c_1 < c_2$ implies that at equilibrium, firm 1 produces a larger quantity (and achieves a larger profit) than firm 2. Third, keeping c_1 constant, we observe that firm 2's disadvantage widens when its marginal cost increases from c_2 to $c_2' > c_2$: firm 2's reaction function shifts down and the equilibrium moves up along firm 1's reaction function. Firm 2 remains active (i.e., $q_2^* \geq 0$) as long as the vertical intercept of firm 2's reaction function lies above the vertical intercept of firm 1's reaction function, or $(a - c_2')/b \geq (a - c_1)/(2b)$, which is equivalent to $c_2' \leq (a + c_1)/2$.

Symmetric Cournot oligopoly

Later in this book, we will frequently make the simplifying assumption that firms are ex ante symmetric in the sense that they all have the same cost structure. In the case of constant marginal costs, this amounts to assuming that $c_i = c$ for all i. Then, using expression (3.5), we derive the

[k] The condition can also be interpreted as follows. If firm 1 was in a monopoly position, it would choose a quantity $q_1^m = (a - c_1)/(2b)$ and sell it at $p_1^m = (a + c_1)/2$. Yet, in the presence of firm 2, firm 1 is not able to set this monopoly price if $p_1^m \geq c_2$, which is equivalent to $c_2 \leq (1/2)(a + c_1)$. As already noted above, we will use a similar analysis in Chapter 18 when introducing the distinction between major and minor innovations.

quantity produced by any firm at the Cournot equilibrium as

$$q_i^*(n) = \frac{a-c}{b(n+1)}. \quad (3.7)$$

The total quantity and the market price are equal to

$$q^*(n) = \frac{n(a-c)}{b(n+1)} \quad \text{and} \quad p^*(n) = a - bq^*(n) = \frac{a+nc}{n+1}.$$

It follows that the markup (or Lerner index) at the equilibrium of the (symmetric linear) Cournot model is equal to

$$L(n) = \frac{p^*(n)-c}{p^*(n)} = \frac{a-c}{a+nc}.$$

If we let the number of firms (n) increase, we obtain the following comparative statics results: (i) the individual quantity decreases; (ii) the total quantity increases; (iii) the market price decreases; (iv) the markup decreases. Moreover, if we let the number of firms tend to infinity, we observe that the markup tends to zero, meaning that market power vanishes.

Lesson 3.6 **The (symmetric linear) Cournot model converges to perfect competition as the number of firms increases.**

The intuition for this result is simple: as the number of firms increases, each firm sees its influence on the market price diminish and is therefore more willing to expand its output. As a result, the market price decreases with the number of Cournot competitors. This result can be shown to hold in more general settings than the specific one considered here.[6]

3.2.2 Implications of Cournot competition

Let us now consider a general inverse demand function, $P(q)$, and general cost functions, $C_i(q_i)$. We then write firm i's profits as

$$\pi_i = P(q)q_i - C_i(q_i).$$

Each firm maximizes profits with respect to its own output. The first-order condition of profit maximization then is $P'(q)q_i + P(q) - C_i'(q_i) = 0$. Defining $\alpha_i = q_i/q$ as the market share of firm i and recalling that the inverse price elasticity of demand is $1/\eta = -P'(q)q/P(q)$, we can rewrite the first-order condition of profit maximization as

$$\frac{P(q) - C_i'(q_i)}{P(q)} = \frac{\alpha_i}{\eta}.$$

[6] For instance, Amir and Lambson (2000) study the symmetric case wherein all firms have the same twice continuously differentiable and non-decreasing cost function, and demand is continuously differentiable and downward-sloping. They show that the equilibrium price falls with an increase in the number of Cournot competitors if, for all Q, $p'(Q) < c(q)$ for all q in $[0, Q]$.

This is the basic *Cournot pricing formula.*

Lesson 3.7 **In the Cournot model, the markup of firm i is larger the larger is the market share of firm i and the less elastic is market demand.**

Hence, the Cournot model gives the empirically testable prediction that in a given market, a larger firm should have a larger markup. Assuming that costs are convex, $C''(q_i) \geq 0$, a sufficient condition for a Cournot equilibrium to exist is that $P'(q)q_i$ is decreasing in q_i. This condition is equivalent to

$$\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = P'(q) + q_i P''(q) \leq 0,$$

which is the condition of strategic substitutability, as will be defined below. If the cross derivative is indeed negative, then the best-response functions are downward-sloping.

In the Cournot model with constant marginal costs ($C_i(q_i) = c_i q_i$), first-order conditions of profit maximization can be rewritten as

$$\frac{p - c_i}{p} = \frac{\alpha_i}{\eta}. \tag{3.8}$$

Equilibrium profits are $(p - c_i)\alpha_i Q(p)$ where p is the equilibrium price under Cournot competition. We can write industry-wide profits in two equivalent ways:

$$\sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} (p - c_i)q_i = \sum_{i=1}^{n} (p - c_i)\alpha_i q = \begin{cases} \left(p - \sum_{i=1}^{n} \alpha_i c_i\right) q, \\ \frac{pq}{\eta} \sum_{i=1}^{n} \alpha_i^2, \end{cases}$$

where the second line uses the first-order conditions, $p - c_i = \alpha_i p/\eta$. Equating the two alternative expressions and rearranging terms, we have

$$\frac{p - \sum_{i=1}^{n} \alpha_i c_i}{p} = \frac{\sum_{i=1}^{n} \alpha_i^2}{\eta} = \frac{I_H}{\eta}.$$

We recall from the previous chapter that I_H denotes the Herfindahl index, which measures the degree of concentration of the industry. We observe thus that *the average Lerner index (weighted by market shares) is proportional to the Herfindahl index.* This means that in the linear Cournot model, there is a one-to-one relationship between market power and concentration. To the extent that the linear Cournot model with constant marginal costs is a good description of real markets, this implies that calculating the Herfindahl index and estimating the price elasticity of demand allows for calculation of the average markup (or average Lerner index) in the market.

Lesson 3.8 **In the linear Cournot model with homogeneous products, the Herfindahl index is an appropriate measure of market power since it captures the average markup in equilibrium.**

3.3 Price vs. quantity competition

Collecting the results of the previous two sections, we observe marked differences between price and quantity competition. Indeed, consider a market with linear demand, that is, $Q(p) = a - p$. Suppose that two firms operate in this market and have the same constant marginal costs of production $c_1 = c_2 = c$. In the Bertrand model, we have that under simultaneous price-setting $p_1 < p_2$ implies $\pi_2 = 0$ and $\pi_1 = (p_1 - c)(a - p_1)$, and the reverse for $p_1 > p_2$. At $p_1 = p_2$ with the tie-breaking rule that each firm attracts half of the demand, $\pi_1 = (1/2)(p_1 - c)(a - p_1)$ and $\pi_2 = (1/2)(p_2 - c)(a - p_2)$. The Bertrand equilibrium is then $p_1 = p_2 = c$. Output in this market is equal to output in a perfectly competitive market, $q_1^B = q_2^B = (a - c)/2$, and firms' profits are zero, $\pi_1^B = \pi_2^B = 0$. Now contrast these results with what obtains under Cournot competition. Here, setting $b = 1$ and $n = 2$ in expression (3.7), one easily finds that $q_1^C = q_2^C = (a - c)/3$. Cournot competitors thus produce less than Bertrand competitors. As a result, the markup is positive, $p^C - c = a - (q_1^C + q_2^C) - c = (a - c)/3 > 0$, and firms make positive equilibrium profits, $\pi_1^C = \pi_2^C = (a - c)^2/9 > 0$.

Lesson 3.9 **In the homogeneous product case, price is higher, quantity is lower and profits are higher under quantity competition than under price competition.**

In the rest of this section, we want to refine the previous statement by providing a deeper comparison of price and quantity competition. We will first relax one of the assumptions of the Bertrand model and suppose, more realistically, that firms may face limited capacities of production. In this framework, we will show that quantity competition can be mimicked by a two-stage model in which firms choose their capacity of production and next, set their price. Second, we will directly compare price and quantity competition in a unified model of product differentiation. Finally, we will try to identify industry characteristics to decide whether price or quantity competition is the appropriate modelling choice.

3.3.1 Limited capacity and price competition

The Bertrand model presumes that firms can serve any demand at constant marginal costs. This means that they do not face capacity constraints. For a large part of industrial production, the assumption of constant (or even decreasing) unit costs may be an appropriate assumption. However, this only holds as long as capacity is not fully utilized. Increasing output beyond capacity limits is often prohibitively costly so that in the short run, a firm has to respect these capacity choices. This critique of the Bertrand setting was first made by Edgeworth (1897). A concrete example related to retailing is that retailers have to order supplies well in advance; they then have to respect capacity limits at the price-setting stage.[1] Case 3.3 provides another example.

[1] Think for instance of traditional retailing for clothing: firms have to order at the beginning of the season and are thus constrained later by this limit in capacity. Only in recent years some (mostly vertically integrated) clothing companies have increased their flexibility to the effect that retail outlets can react quickly to demand.

Case 3.3 When capacity choices condition pricing decisions in the DVD-by-mail industry

Recall the case of the DVD-by-mail industry that we described in the introduction.[7] We mentioned the fact that at any point in time, demand for newly released movies is larger than for movies released, say, six months or one year before. To meet this larger demand, firms like Netflix or Blockbuster should hold an extra stock of copies of the latest movies. Yet, these copies are also more expensive. This has led firms to develop appropriate user interface and pricing schemes so as to steer subscribers towards renting back-catalogue movies instead of new releases. This illustrates that the choice of costly capacities precedes and conditions pricing decisions. A similar argument helps us understand why flights are much more expensive around Christmas: more people wish to travel at that period of the year but capacities are fixed (i.e., there is a fixed number of seats in any airplane and a given airport can accommodate a fixed number of airplanes per day). One also understands why the airline industry has invented the practice of 'yield (or revenue) management', which is a form of price discrimination (see Part IV) adapted to situations where capacities are fixed while demand is fluctuating (like, e.g., in air transport, lodging or the car rental business). The underlying principle of yield management can be summarized as setting the right prices so as to sell the right product, to the right client, at the right time. Again, capacity and price decisions are closely intertwined.

As an approximation for such markets, we consider that firms can pre-commit to a capacity of production before they engage in price competition. We will see that under a number of assumptions, this capacity-then-price model leads to the same outcome as the Cournot model of quantity competition. We will also discuss what happens when these assumptions are relaxed.

The model

To establish that result, we study the following two-stage game:[8] in stage 1, firms set capacities $\overline{q}_i$ simultaneously; in stage 2, firms set prices p_i simultaneously. It is assumed that the marginal cost of capacity is c and is incurred in the first stage; then, once capacity is installed, the marginal cost of production in the second stage is zero. In this set-up, we characterize the subgame-perfect equilibrium. This implies that at stage 1, firms are aware that their capacity choice may affect equilibrium prices. Firms not only know their own capacity choice but are also assumed to observe the competitors' capacity choice. For many industrial products, this is an appropriate assumption as factory sizes are known. The assumption may be more problematic in the retailer example, but nevertheless appropriate in some instances. For instance, if you think of a local farmers' market, the vendors can easily observe the capacity constraints of competitors.

When allowing for capacity constraints, it is quite possible that one firm will set its price so low that the quantity demanded at that price exceeds its supply. This implies that some

[7] Borland, J. and Hansen, E. (2004). DVD price wars: How low can they go? CNET News.com (last consulted 9 March 2015).

[8] This is due to the seminal analysis by Kreps and Scheinkman (1983).

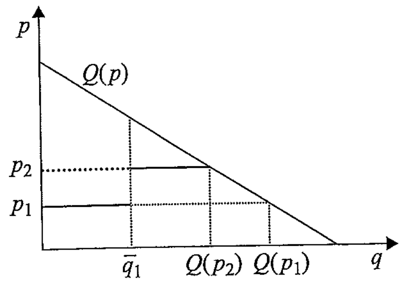

Figure 3.4 Efficient rationing with limited capacities

consumers have to be rationed. Suppose that there is a second firm on the market that offers the product at a higher price. Who will be served at the low price, who will not? At this point, we have to make an assumption on the rationing scheme. We assume that there is *efficient rationing*, that is, consumers with higher willingness to pay are served first. We can provide two justifications for this particular rationing scheme. If there is rationing, products may be allocated according to who is first in the queue. Suppose each consumer demands 0 or 1 unit. Then consumers with a higher willingness to pay will be first in the queue. Alternatively, independently of the way a product in excess demand is allocated, there can be secondary markets that operate without costs; then, consumers with low willingness to pay will resell to consumers with high willingness to pay. Therefore, consumers with a high willingness to pay will never be rationed.[m]

Figure 3.4 illustrates efficient rationing. The first unit is purchased by the consumer with the highest willingness to pay, the second unit by the second highest and so on. Hence, if firm 1 has capacity of $\overline{q}_1$ units, these units are sold to the $\overline{q}_1$ consumers with the highest willingness to pay. If $p_1 < p_2$ is such that quantity $\overline{q}_1$ is insufficient to serve all consumers, so $Q(p_1) > \overline{q}_1$, some consumers are rationed and there is positive residual demand for firm 2 (this cannot be the case in the pure Bertrand model, in which capacity is never binding).

We first want to analyse the price-setting game for given capacities. Before doing so, it is helpful to observe that a firm never sets a very large capacity since capacity is costly. To be precise, a firm will never set a capacity such that its revenues are less than costs independently of the decision of the competitor. Taking the linear demand $Q(p) = a - p$, the maximal revenue of a firm is $\max_q q(a - q) = a^2/4$; the quantity then is $q = a/2$. Costs at stage 1 have to be lower than maximal revenues, $c\overline{q}_i \leq a^2/4$. Hence the profit-maximizing capacity choice must satisfy

$$\overline{q}_i \leq a^2/(4c). \tag{3.9}$$

We now analyse the price-setting stage for capacities which satisfy the above inequality. If firm 1 offers the product at a lower price, firm 2 faces the following residual demand for product 2:

$$\widehat{Q}(p_2) = \begin{cases} Q(p_2) - \overline{q}_1 & \text{if } Q(p_2) - \overline{q}_1 \geq 0, \\ 0 & \text{else.} \end{cases}$$

Hence, for $p_1 < p_2$, profits are

$$\begin{cases} \pi_1 = (p_1 - c)\overline{q}_1, \\ \pi_2 = p_2\widehat{Q}(p_2) - c\overline{q}_2 = p_2[Q(p_2) - \overline{q}_1] - c\overline{q}_2. \end{cases}$$

[m] As consumers with a higher willingness to pay buy at the lowest price, consumer surplus is maximized under this rationing rule, which explains why it is called 'efficient' (see Levitan and Shubik, 1972).

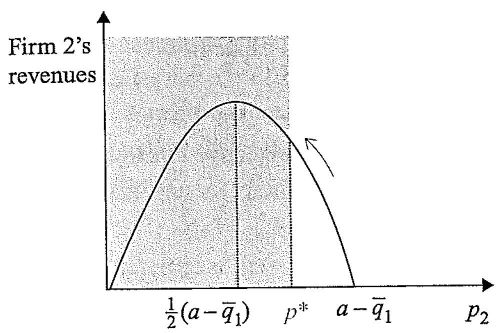

Figure 3.5 Setting $p_2 > p^*$ is not a profitable deviation

We now want to prove that *the equilibrium at the second stage of the game is such that both firms set the market-clearing price*: $p_1 = p_2 = p^* = a - \overline{q}_1 - \overline{q}_2$ (this price does indeed clear the market as it equalizes demand and supply, i.e., total capacity). This result holds provided that the demand parameter a is not too large; in particular, we impose the following condition:

(C1) $c < a < (4/3)c$.

To prove this result, we proceed as follows: supposing that $p_1 = p^*$, we need to show that $p_2 = p^*$ is a best response; that is, firm 2 cannot earn a larger profit by setting either a lower or a higher price than p^*. First, it is easy to see that setting a lower price, $p_2 < p^*$, is not a profitable deviation. Indeed, at $p_1 = p_2 = p^*$, firm 2 sells all its capacity $\overline{q}_2$. By lowering its price, firm 2 would increase the demand for its product but would not be able to serve this additional demand because it is capacity-constrained. Therefore, firm 2 would sell the same number of units as before but at a lower price, which would decrease its profit.

Second, knowing that firm 1 is capacity-constrained, firm 2 could find it profitable to raise its price ($p_2 > p^*$). Indeed, due to firm 1's limited capacity, firm 2 can sell a positive volume at a price above p_1. This case requires some more reasoning than the previous one. Recall that firms have incurred the marginal cost c when installing their capacity in the first stage. Hence, in the second stage, they maximize their revenues (as the marginal production cost is zero). Provided that $p_1 < p_2$, firm 2's revenues are

$$p_2 \widehat{Q}(p_2) = \begin{cases} p_2(a - p_2 - \overline{q}_1) & \text{if } a - p_2 \geq \overline{q}_1, \\ 0 & \text{else.} \end{cases}$$

We have to show that the proposed equilibrium price p^* is located to the right of the maximum of this revenue function (as illustrated in Figure 3.5). If this holds, the proof is complete because increasing the price above p^* decreases profit, meaning that such a deviation is not profitable. The maximum of the revenue function is equal to $\bar{p}_2 = (a - \overline{q}_1)/2$. Then,

$$p^* > \bar{p}_2 \Leftrightarrow a - \overline{q}_1 - \overline{q}_2 > (a - \overline{q}_1)/2 \Leftrightarrow a > \overline{q}_1 + 2\overline{q}_2.$$

Invoking (3.9), we know that $\overline{q}_1 + 2\overline{q}_2 \leq (3/4)(a^2/c)$. Therefore, $a > \overline{q}_1 + 2\overline{q}_2$ is necessarily satisfied if

$$a > (3/4)(a^2/c) \Leftrightarrow a < (4/3)c,$$

which is guaranteed by condition (C1). Hence, it is not profitable to set $p_2 > p^*$, which completes our proof.

We can now insert these stage-2 equilibrium prices in the profit functions and thus obtain reduced profit functions for stage 1, which only depend on capacities:

$$\tilde{\pi}_i(\overline{q}_1, \overline{q}_2) = (a - \overline{q}_1 - \overline{q}_2)\overline{q}_i - c\overline{q}_i.$$

We see that if we reinterpret capacities as quantities, the objective function is the same as in the Cournot model, in which prices are not set by firms but where, for any quantity choice, the price clears the market.

Lesson 3.10 **In the capacity-then-price game with efficient consumer rationing (and with linear demand and constant marginal costs), the chosen capacities are equal to those in a standard Cournot market.**

Discussion

We have to stress that Lesson 3.10 is drawn under a parameter restriction and for a particular rationing rule. What happens if we relax these assumptions? Note first that the key for the previous result to hold is that firm i has no incentive to increase its price above p^* when firm j sets $p_j = p^*$.[n] Under the efficient rationing rule, we have seen that firm i, ignoring its capacity constraint, would like to set $\bar{p}_i = (a - \overline{q}_j)/2$. Firm i will have sufficient capacity to satisfy residual demand at that price if $\bar{p}_i > p^*$. As we showed above, the parameter restriction we imposed excludes this possibility. In particular, this restriction guarantees that firms do not install capacities that are larger than $a/3$ (indeed, for $a < (4/3)c$, the upper bound on profitable capacities, $a^2/(4c)$, is below $a/3$). Note that $a/3$ is precisely the output firms produce at the Cournot equilibrium with no production costs. We can thus generalize our previous result by stating that *under efficient rationing, $p_1 = p_2 = p^*$ is the unique second-stage equilibrium when each firm's capacity is less than or equal to its Cournot best response to the other firm's capacity.* Outside this region of capacity (which is possible when relaxing the parameter restriction, i.e., for $a > (4/3)c$), a pure-strategy equilibrium fails to exist at stage 2: the only equilibria are in mixed strategies in which firms randomize prices over a common interval of prices. However, it can be shown that the first-stage capacity choices continue to correspond to the Cournot quantity equilibrium.[9]

Consider now an alternative rationing rule. Edgeworth (1897) proposed allocating the cheapest units of the product randomly across consumers. Under this *proportional rationing rule*, all consumers have the same probability of being rationed.[o] Under this rule, the highest price charged is always the monopoly price p^m. Indeed, ignoring its capacity constraint, firm i maximizes $\alpha p_i Q(p_i)$, where α is the expected fraction of consumers that firm i serves. If $p^m < p^*$, firm i does not have sufficient capacity to satisfy residual demand at p^m and chooses then to set $p_i = p^*$. Given that $p^m = a/2$, the latter condition is equivalent to $\overline{q}_1 + \overline{q}_2 < a/2$, which is

[n] We know that firm i has no incentive to *lower* its price as this would induce an excess of total demand over total capacity, meaning that firm i could increase its price without losing sales.

[9] See Kreps and Scheinkman (1983).

[o] The proportional rule is clearly not efficient as some consumers may end up buying the good because they were offered the lower price although they would not have bought it at the higher price. From the point of view of the higher-priced firm, this rule yields a relatively high contingent demand, while the efficient rule yields the worst possible one.

more demanding than the corresponding condition we obtained under the efficient rationing rule (actually, capacities sufficiently close to the upper bound given by (3.9) violate this condition as $a > c$ implies that $a^2/(2c) > a/2$). For capacities outside this region, the only equilibria are again in mixed strategies. Here, these mixed strategies are generally difficult to derive. However, it is possible to show that *under proportional rationing, the equilibrium tends to be more competitive than Cournot.*[10]

3.3.2 Differentiated products: Cournot vs. Bertrand

While the purpose of the previous model was to show that quantity competition can be mimicked by a model with price competition at the last stage, the purpose of the present model is to compare the competitiveness between price and quantity competition. As indicated above in Lesson 3.9, the results are obvious in the homogeneous product case: quantity competition leads to higher prices, lower quantities and higher profits than price competition. These results are less obvious once we allow for product differentiation. Consider a simple duopoly model in which firms have constant marginal costs c_1 and c_2, respectively.[11] To obtain linear demand, we assume that there is a large number of identical consumers with a linear–quadratic utility function. In particular, suppose that the utility function takes the form

$$U(q_0, q_1, q_2) = aq_1 + aq_2 - \left(bq_1^2 + 2dq_1q_2 + bq_2^2\right)/2 + q_0,$$

where q_0 is the Hicksian composite commodity with a price normalized to 1.[p] Here we assume that $b > |d|$, which implies that products are differentiated. Consumers maximize their utility $U(q_0, q_1, q_2)$ subject to the budget constraint $y = q_0 + p_1q_1 + p_2q_2$. This gives rise to the following inverse demand functions:

$$\begin{cases} P_1(q_1, q_2) = a - bq_1 - dq_2, \\ P_2(q_1, q_2) = a - dq_1 - bq_2, \end{cases}$$

for strictly positive prices and zero otherwise. Goods are substitutes if $d > 0$, they are independent if $d = 0$ and they are complements if $d < 0$. The ratio d/b can be interpreted as an inverse measure of the degree of product differentiation. It ranges from -1 when products are perfect complements to 1 when they are perfect substitutes; a value of zero means that products are independent. This system of two equations can be inverted (for $-1 < d/b < 1$) to obtain direct demand functions. Let $\widetilde{a} = a/(b + d)$, $\widetilde{b} = b/(b^2 - d^2)$ and $\widetilde{d} = d/(b^2 - d^2)$. Demand functions then take the form

$$\begin{cases} Q_1(p_1, p_2) = \widetilde{a} - \widetilde{b}p_1 + \widetilde{d}p_2, \\ Q_2(p_1, p_2) = \widetilde{a} + \widetilde{d}p_1 - \widetilde{b}p_2, \end{cases}$$

for strictly positive quantities and zero otherwise. With quantity competition, firm i maximizes $(a - bq_i - dq_j - c_i)q_i$ taking q_j as given. With price competition, firm i maximizes $(\widetilde{a} - \widetilde{b}p_i + \widetilde{d}p_j)(p_i - c_i)$ taking p_j as given.

Let us underline an important difference between quantity and price competition, which relates to the form of the best-response functions. To see this, take the case of substitutable

[10] See Davidson and Deneckere (1986) for a characterization of mixed strategies under proportional rationing.

[11] The analysis is based on Singh and Vives (1984).

[p] The Hicksian composite commodity contains all other goods outside the market under consideration. Since we are interested in real prices, we can normalize the price of one unit of this basket to 1.

products (i.e., $d > 0$). Under quantity competition, the best response is downward-sloping in q_j: $q_i(q_j) = (a - dq_j - c_i)/(2b)$. That is, facing an increase in firm j's quantity, firm i reacts by reducing its own quantity. Strategic choices move in opposite directions and, as we define more generally below, quantities can then be said to be *strategic substitutes*. This confirms what we observed above in the Cournot duopoly with perfect substitutes (i.e., for $d = 1$). The opposite holds under price competition. The best response under price competition is upward-sloping in p_j: $p_i(p_j) = (\tilde{a} + \tilde{d}p_j + \tilde{b}c_i)/(2\tilde{b})$. Here, facing an increase in firm j's price, firm i reacts by increasing its own price. Strategic choices move here in the same direction, meaning that prices are *strategic complements*. This is also what we observed in Section 3.1 when considering the standard Bertrand model and the Hotelling model.

Notice that in the presence of complementary products (i.e., for $d < 0$), the previous results are reversed: best-response functions slope upwards under quantity competition and downwards under price competition, meaning that, in this case, quantities are strategic complements whereas prices are strategic substitutes. Admittedly, the terminology might induce some confusion, but it should already be clear by now that the concepts of strategic substitutability and strategic complementarity have to do with the direction of strategic reactions, and not with the demand links between the products. Actually, as we will see below, these concepts go far beyond price and quantity competition as they can be applied to any type of strategic interaction.

We now compare the equilibrium under price and quantity competition. To simplify the exposition, we suppose that marginal costs are symmetric and equal to zero. A few lines of computation establish that equilibrium prices and quantities under quantity competition are $p_i^C = bq_i^C$ and $q_i^C = a/(2b + d)$, while equilibrium prices and quantities under price competition are $p_i^B = \tilde{a}/(2\tilde{b} - \tilde{d})$ and $q_i^B = \tilde{b}p_i^B$. To compare prices and quantities, we can rewrite p_i^B as $p_i^B = a(b - d)/(2b - d)$. Then

$$p_i^C - p_i^B = \frac{ab}{(2b + d)} - \frac{a\,(b - d)}{(2b - d)} = \frac{ab(2b - d) - a(b - d)(2b + d)}{(2b + d)(2b - d)}$$

$$= \frac{ad^2}{4b^2 - d^2} = \frac{a}{4\left(b^2/d^2\right) - 1} > 0.$$

Lesson 3.11 **Price competition always leads to lower prices and larger quantities than quantity competition. Hence, price as the strategic variable gives rise to a more competitive outcome than quantity as the strategic variable.**

To understand this result, look at the slope of the perceived demand function in the two cases. Under price competition, the perceived demand function is $q_i = \tilde{a} - \tilde{b}p_i + \tilde{d}p_j$, with a slope (in absolute value) of $\tilde{b} = b/(b^2 - d^2)$. Under quantity competition, the perceived demand function is $q_i = a - (1/b)p_i - (d/b)q_j$, with a slope (in absolute value) of $1/b$. It is easily checked that $b/(b^2 - d^2) > 1/b$, meaning that a firm perceives a larger elasticity of demand when it takes the price of the rival as fixed rather than its quantity. It follows that firms quote lower prices under price competition than under quantity competition.

We also observe that the price difference depends on the degree of product differentiation d/b. The more differentiated the products, the smaller the difference between prices. As products become independent, $d/b \to 0$, the price difference turns to zero as in both environments both firms tend to behave as monopolists. Since firms produce too little from a social point of view, price competition is socially preferred to quantity competition. However, the profit comparison is less clear-cut. It depends on the sign of d. If products are substitutes, so $d > 0$, quantity competition performs better than price competition from the firms' point of view. If products are complements, so $d < 0$, price competition performs better.

3.3.3 What is the appropriate modelling choice?

One of the important basic insights of oligopoly theory is that the market outcome under imperfect competition depends on the variable, price or quantity, that is chosen for the analysis. From a real-world perspective, the difference between outcomes appears to be cumbersome. Since we want to explain market behaviour, we should have a good idea what is the appropriate model of competition. Note first that this question is pointless in a monopoly setting. As we know from the monopoly analysis, if we fix the environment in which one firm operates (in particular if we fix the action of the competitor), profit maximization with respect to price gives the same result as profit maximization with respect to quantity. Thus, it is immaterial whether firms set price or quantity.

However, in an oligopoly setting, the difference between price and quantity competition materializes in the residual demand a firm faces given the action of the competitor. Suppose $\widetilde{Q}_i(p_i)$ is the residual demand for firm i and $Q(p)$ is the market demand that a monopolist would face. Then, under price competition, price p_j is given. This means that the competitor is willing to serve any demand at price p_j. Then firm i's residual demand is $\widetilde{Q}_i(p_i) = Q(p_i)$ if $p_i < p_j$ and $\widetilde{Q}_i(p_i) = 0$ if $p_i > p_j$ (and demand is equally split for $p_i = p_j$). Hence, firm i's residual demand curve reacts very sensitively to price changes: it is perfectly elastic at $p_i = p_j$. Under quantity competition, quantity q_j is given. This means that irrespective of the price he will achieve, the competitor sells quantity q_j. Then firm i's residual demand is $\widetilde{Q}_i(p_i) = \max\{Q(p_i) - q_j, 0\}$. Here, firm i's residual demand curve reacts less sensitively to price changes.

To choose between these two basic models of competition, we have to choose between two different ways in which firms behave in the marketplace: they either stick to a price and sell any quantity at this price or they stick to a quantity and sell this quantity at any price. The former option (i.e., price competition) appears to be the appropriate choice in case of unlimited capacity or when prices are more difficult to adjust in the short run than quantities. For instance, in the mail-order business, it is costly to print new catalogues or price lists and, therefore, over some period of time, prices will remain fixed and quantities will adjust accordingly.

In contrast, the latter option (i.e., quantity competition) may be the more appropriate choice in case of limited capacities, even if firms are price-setters. A formal explanation of this latter insight has been provided above in the capacity-then-price model, where quantities (seen as capacities) are more difficult to adjust than prices. For instance, this is the case in the package-holiday industry: hotel rooms or aircraft seats are usually booked more than one year before a given touristic season and, therefore, prices adjust to sell the available capacities (using, e.g., 'last-minute discounts'). As illustrated in Case 3.4, technological progress may change the way firms behave in the marketplace and, thereby, the appropriate model to represent it.

Case 3.4 Digital revolution in the publishing industry

Digital technologies have turned 'publish on demand' (or print on demand, POD) into a common and accessible alternative (or supplement) to traditional publishing methods. POD systems allow publishers to print economically very small print runs of materials in book form, which make them particularly suitable for publications with low or fairly unpredictable demand. Compared with the traditional 'batch printing' approach, which requires books to be printed in large numbers so as to reduce unit cost, POD transfers costs from the fixed category to the variable category. Indeed, POD provides a cost-effective way of keeping the backlist going as books never go out of print (in contrast, batch printing induces lost sales or large reprint costs when books are out of print); furthermore, POD saves on storage or warehousing costs. The downside is that the actual cost of producing each individual book is rather more than the batch printing cost per unit. Comparing the publishing industry under the two technologies, it seems that the quantity competition model fits better with the batch printing technology (because prices will adjust to sell the existing capacity) and the price competition model with the POD technology (because quantity can be adjusted immediately at the announced prices).

Note that a similar 'digital revolution' is currently transforming the cinema industry. Digital technologies deeply change the way that films are made, produced and distributed. In particular, they allow the cost of distributing movies to be reduced dramatically, since moving a multi-gigabyte file containing pictures and sound is much cheaper than shipping spools of 35 mm film prints. As in the publishing industry, this makes it cheaper and easier to set up small cinemas and to let capacities quickly adjust to the fluctuations of the audience.

3.4 Strategic substitutes and strategic complements

The comparison of price and quantity competition with differentiated products has revealed that the two models lead to different conclusions about price–cost margins and thus market power. This comparison has also highlighted another difference between price and quantity competition models, namely in terms of how firms 'react' to actions taken by their competitors. Here, we provide a general presentation of the previous finding, which extends beyond the simple oligopoly models discussed so far, and which will be helpful in many situations with strategic interaction.[12] The analysis is based on a firm's reactions to the actions of its competitors, which are captured by the best-response (or reaction) function. We are concerned with the slope of these best-response functions.

Suppose that firm i has the objective function π_i which depends on some unspecified variables x_i $(i = 1, \ldots, n)$, where x_i is in the control of firm i and $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ is in the control of the other firms. Suppose that the variable is chosen from some compact interval of the real line. We say that the variables are *strategic complements* if, in the continuous and differentiable case, an increase in x_{-i} leads to a higher marginal product $\partial\pi_i/\partial x_i$. Formally, variables x_i are strategic complements if, for all i, we have $\partial\pi_i(x_i, x'_{-i})/\partial x_i \geq \partial\pi_i(x_i, x_{-i})/\partial x_i$ for

[12] We follow here the seminal analysis of Bulow, Geanakoplos and Klemperer (1985).

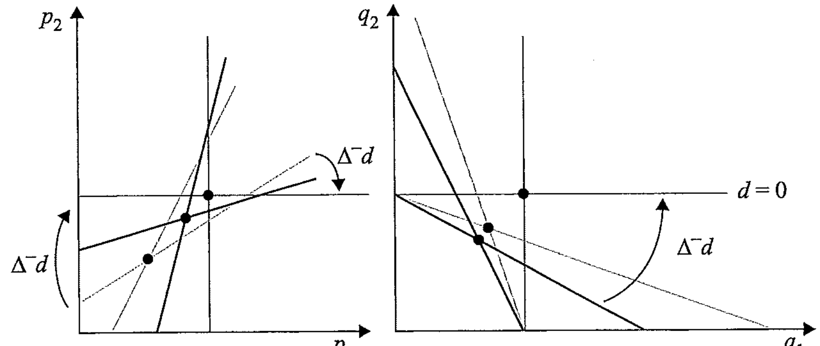

Figure 3.6 Reaction functions for price vs. quantity competition (when firms produce substitutable goods)

all $x'_{-i} \geq x_{-i}$. If profits are twice differentiable, this is equivalent to $\partial^2 \pi_i(x_i, x_{-i})/\partial x_i \partial x_j \geq 0$ for all i. Strategic complementarity implies that the best-response functions are upward-sloping.[q]

We can also consider discrete variables. For simplicity, suppose that each variable x_i can only take two values, $x_i \in \{0, 1\}$. Formally, variables x_i are strategic complements if, for all i, we have $\pi_i(1, x'_{-i}) - \pi_i(0, x'_{-i}) \geq \pi_i(1, x_{-i}) - \pi_i(0, x_{-i})$ for all $x'_{-i} \geq x_{-i}$. The condition is formally equivalent to $\pi_i(1, x'_{-i}) + \pi_i(0, x_{-i}) \geq \pi_i(1, x_{-i}) + \pi_i(0, x'_{-i})$; that is, for $x_{-i} = 0$ and $x'_{-i} = 1$, variables are strategic complements if the sum of diagonal elements dominates the sum of off-diagonal elements.

In both the continuous and the discrete version, variables are *strategic substitutes* if the reverse inequalities hold. In particular, in the continuous version, strategic substitutability implies that best-response functions are downward-sloping. The two panels of Figure 3.6 illustrate the cases of strategic complements and substitutes in the model of price vs. quantity competition with differentiated products.[r]

The analysis of games with strategic complements is generally useful for three reasons. First, the existence of equilibrium is guaranteed (although there may exist multiple equilibria). Second, the set of equilibria has a smallest and a largest equilibrium. Third, games with strategic complements exhibit unambiguous comparative statics properties. While the first reason is of little relevance in models in which profits are quasiconcave, the second gives some structure on the equilibrium set in the presence of multiple equilibria. In the specification of the models that we are considering, we always have a unique equilibrium so that the second reason does not apply. However, the third reason is relevant: even if we do not obtain explicit solutions for equilibrium value, we are interested in the effect of changes in market environments on market outcomes. Let us return to the case with continuous variables. Consider a policy parameter γ and assume that an increase in this parameter globally increases marginal profits, $\partial^2 \pi_i(x_i, x_{-i}; \gamma)/\partial x_i \partial \gamma \geq 0$. Then, if variables are strategic complements, the smallest and largest

[q] In general, the continuity and differentiability of best-response functions is not required: strategic complementarity corresponds to a situation in which best-response functions are upward-sloping (possibly with upward jumps).

[r] A health warning: while in many specifications of the Cournot model (e.g., the one with linear demand), quantities are strategic substitutes, this is not necessarily the case; correspondingly, for models with price competition. Furthermore, with linear demand we have seen in the previous section that the reverse applies if firms produce complementary instead of substitutable goods.

Nash equilibria have the property that a policy change from γ to γ', with $\gamma' > \gamma$, leads to an equilibrium in which both firms weakly increase their choices, $x^*(\gamma') \geq x^*(\gamma)$. Clearly, if there is a unique Nash equilibrium, we have the unique prediction that x^* weakly increases in γ.

Lesson 3.12 **If the firms' choices are strategic complements (i.e., if best-response functions slope upwards) and if an increase in some parameter of the market environment raises marginal profits, then an increase in this parameter leads firms to increase their strategic choice at equilibrium.**

The intuition comes directly from the second partial derivatives. Strategic complementarity implies that for a given market environment γ, firm i optimally reacts to an increase in x_{-i} by increasing x_i. The condition $\partial^2\pi_i(x_i, x_{-i}; \gamma)/\partial x_i \partial\gamma \geq 0$ says that given the behaviour of competitors x_{-i}, firm i optimally reacts to an increase in the policy parameter γ by increasing x_i. This means that an increase in γ leads to an outward shift of the best-response function. Strategic interaction amplifies the effect of the policy change on x.

As an illustration, take the linear demand model of product differentiation that we analysed above. We showed that for substitutable products (i.e., for $d > 0$), prices are strategic complements. Now, if we let the degree of product differentiation increase (i.e., if we let the parameter d decrease), we observe that firms set higher prices at equilibrium. This is depicted in the left panel of Figure 3.6. Analytically, assuming that both firms have zero marginal costs, the profit function under price competition can be written as

$$\pi_i^B(p_i, p_j; d) = p_i(\widetilde{a} - \widetilde{b}p_i + \widetilde{d}p_j) = p_i\tfrac{1}{b^2-d^2}((b-d)a - bp_i + dp_j).$$

Then, we check that, around symmetric prices, an increase in product differentiation (i.e., a lower d) increases marginal profits:

$$\left.\frac{\partial^2\pi_i^B(p_i, p_j; d)}{\partial p_i \partial d}\right|_{p_i=p_j=p} = \left.\frac{-a(b-d)^2 + (b^2+d^2)p_j - 4bdp_i}{(b-d)^2(b+d)^2}\right|_{p_i=p_j=p}$$
$$= -\frac{(b-d)^2(a-p) + 2bd}{(b-d)^2\,(b+d)^2} < 0.$$

We should thus have that a decrease in d leads the firms to increase their equilibrium price choice: $p_i^B(d') > p_i^B(d)$ for $d' < d$. In this simple model, one can directly check the explicit solution for equilibrium prices and observe indeed that $p_i^B = a(b-d)/(2b-d)$ is a decreasing function of d. For more complex models, monotone comparative statics results are powerful tools.[13]

3.5 Estimating market power

Markets differ in their degree of competitiveness. This may depend on the characteristics of the industry, on the conduct of firms (e.g., in their degree of collusion) and on the particular point in

[13] On the theory of supermodular games, see Vives (1990) and Milgrom and Roberts (1990). For recent applications in industrial organization see for example Vives (2005).

time the analysis is carried out. In general, a large price–cost margin is associated with a lack of competitive pressure. Following the theoretical models presented above, we consider markets in which products can be approximated to be homogeneous. Furthermore, we maintain the symmetry assumption that we invoked at various instances above.

Market demand is characterized by the demand equation

$$p = P(q, x),$$

where q is the total quantity in the market and x is a vector of exogenous variables that affects demand (but not costs). Marginal costs are supposed to be given by a function $c(q, w)$, where w is a vector of exogenous variables that affect (variable) costs.

One approach to empirically estimating market power is to postulate that various market structures can be nested in a single model.[14] Let marginal revenue be written as a function depending on a conduct parameter λ,

$$MR(\lambda) = p + \lambda \frac{\partial P(q, x)}{\partial q} q.$$

If $\lambda = 0$, the market is competitive, as would happen in the symmetric model of pure Bertrand competition. If $\lambda = 1$, we are in a monopoly situation and the firm fully takes into account the effect of a change of total output on price. In the n-firm symmetric Cournot model, we have $\lambda = 1/n$.

At equilibrium, marginal revenue is equal to marginal cost,

$$MR(\lambda) = p + \lambda \frac{\partial P(q, x)}{\partial q} q = c(q, w). \tag{3.10}$$

The basic model then consists of the demand equation and the equilibrium condition. It can be estimated non-parametrically (allowing for a flexible cost function).

How can we interpret the parameter λ? First, we may interpret λ literally as the firm's conjecture on how strongly price reacts to its change in output. Clearly, in a monopoly world in which the firm knows the demand curve, it has to attribute $\lambda = 1$. However, in an oligopoly environment, it may expect competitors to adjust output so that $dq_{-i}/dq_i \neq 0$, where q_{-i} denotes the aggregate output of competitors. This is the basic property of the *conjectural variations approach*. It implies that in the n-firm symmetric quantitative competition model, λ may be different from $1/n$. Note that such conjectures are incompatible with Nash behaviour, which takes output of competitors as given. Consider a constant conjecture $\gamma = dq_{-i}/dq_i$. The first-order condition of profit maximization in a market in which all firms hold constant conjectures γ can then be written as

$$p + (1 + \gamma) \frac{\partial P(q, x)}{\partial q} q_i = c(q, w).$$

Since in 'equilibrium' $q_i = q/n$, we obtain a relationship between the conjectural variation parameter γ and the conduct parameter λ: $\lambda = (1 + \gamma)/n$.

Thus we can infer from the observed or estimated conduct parameter λ and the observed number of firms n the conjectural variation parameter. We can thus see this exercise as the

[14] This approach has been suggested by Just and Chern (1980), Bresnahan (1982) and Lau (1982). For an in-depth analysis of this and alternative approaches, we refer the reader to Perloff, Karp and Golan (2007), from which we borrow heavily. See also Bresnahan (1989).

estimation of a static conjectural variation model. However, as pointed out above, conjectural variations different from zero are at odds with a game-theoretic static analysis. Alternatively, we can see conjectural variations as a shortcut for an explicit dynamic specification. In one such dynamic specification, firms compete in prices for two periods (products are symmetric but differentiated). Firms only observe their own realized demand; the demand intercept is unknown. Here, the first-period price can be used to manipulate the competitor's perception. This implies that a higher first-period price (leading to a lower first-period quantity q_i^1) makes the competitor choose a higher price in the second period (resulting in a lower second-period quantity q_j^2). Hence, $dq_j^2/dq_i^1 > 0$.[15]

A second interpretation is to be agnostic about the precise game being played. Rewriting the equilibrium condition, we have $p - c(q, w) = -\lambda(\partial P/\partial q)q$ or that the Lerner index satisfies

$$L = \frac{p - c(q, w)}{p} = -\lambda \frac{\partial P(q, x)}{\partial q} \frac{q}{p} = \frac{\lambda}{\eta},$$

where η is the price elasticity of demand. Here, λ can be interpreted as an index of market power.

Lacking cost data, the question is whether we can identify and thus estimate our index of market power λ. Suppose we have many observations of our endogenous variables p and q and our exogenous variables w and x. We can then write output as a function of exogenous variables w and x, $q = g_1(w, x)$. This equation is always identified. Since price is given by $p = P(q, x)$, we have $p = P(g_1(w, x), x) = g_2(w, x)$, which is also identified.[s] The equilibrium condition (3.10) is identified if there is a single marginal cost function $c(q, w)$ and a single index λ that satisfy this condition. Identification here is only a problem for some particular functional forms of demand, which, however, are often used in theoretical models (namely that demand is linear or has a constant elasticity).

In the empirical estimation, one has to take care of two issues: the estimation of endogenous variables on the right-hand side and the feature that the index λ is a ratio of two estimated parameters. Once these two issues are properly taken care of, we have to interpret results. Following the second interpretation, we can simply treat λ as a continuous variable. Alternatively, we may explicitly test, for example, the Cournot model and thus accept or reject the hypothesis that a particular market can properly be described as a Cournot market (so that λ takes a particular value).

Empirical estimates of λ or the Lerner index L have been obtained for a number of industries including textiles and tobacco. In his econometric study, Applebaum (1982) finds that textiles are priced close to marginal cost ($L = 0.07$), whereas tobacco enjoys a larger markup

[15] For a formal analysis of this point see Riordan (1985). In a signalling context with private information about costs, Mailath (1989) shows qualitatively similar results.

[s] Recall that the identification problem in econometrics has to do with being able to solve for unique values of the parameters of the structural model from the values of the parameters of the reduced form of the model (the reduced form of a model being the one in which the endogenous variables are expressed as functions of the exogenous variables).

($L = 0.65$) in his data set.[t] We return to the empirical estimation of market power in imperfectly competitive markets in later chapters, in particular when considering differentiated product markets.

Review questions

1. How does product differentiation relax price competition? Illustrate with examples.
2. How does the number of firms in the industry affect the equilibrium of quantity competition?
3. When firms choose first their capacity of production and next the price of their product, this two-stage competition sometimes looks like (one-stage) Cournot competition. Under which conditions?
4. Using a unified model of horizontal product differentiation, one comes to the conclusion that price competition is fiercer than quantity competition. Explain the intuition behind this result.
5. Define the concepts of strategic complements and strategic substitutes. Illustrate with examples.
6. What characteristics of a specific industry will you look for to determine whether this industry is better represented by price competition or by quantity competition? Discuss.

Further reading

The original literature on oligopoly theories dates back to the nineteenth and early twentieth centuries. The seminal treatments of quantity and price competition are due, respectively, to Cournot (1838) and Bertrand (1883). Edgeworth (1897) introduced the notion of capacity constraints in price competition. Hotelling (1929) extended price competition by considering a spatial-differentiation model. This literature might be difficult to read as it does not make use of the same vocabulary and concepts (in particular, the game-theoretical concepts) that are current nowadays. We prefer thus to refer you to Shapiro (1989), who provides a good and exhaustive introduction to oligopoly theory. For more on specific issues, see Spulber (1995) for price competition with uncertain costs, Kreps and Scheinkman (1983) on the capacity-then-price game and Bulow, Geanakoplos and Klemperer (1985) for the concepts of strategic complements and substitutes.

[t] The author uses econometric production theory techniques in a framework which enables him to estimate the conjectural variation. The data are obtained from the US *Survey of Current Business*, over the period 1947–71.

Exercises

3.1 Price competition

Consider a duopoly in which homogeneous consumers of mass 1 have unit demand. Their valuation for good $i = 1, 2$ is $v(\{i\}) = v_i$ with $v_1 > v_2$. Marginal cost of production is assumed to be zero. Suppose that firms compete in prices.

1. Suppose that consumers make a discrete choice between the two products. Characterize the Nash equilibrium.
2. Suppose that consumers can now also decide to buy both products. If they do so they are assumed to have a valuation $v(\{1, 2\}) = v_{12}$ with $v_1 + v_2 > v_{12} > v_1$. Firms still compete in prices (each firm sets the price for its product – there is no additional price for the bundle). Characterize the Nash equilibrium.
3. Compare regimes from parts (1) and (2) with respect to consumer surplus. Comment on your results.

3.2 Asymmetric duopoly

Consider two quantity-setting firms that produce a homogeneous good and choose their quantities simultaneously. The inverse demand function for the good is given by $P = a - q_1 - q_2$, where q_1 and q_2 are the outputs of firms 1 and 2 respectively. The cost functions of the two firms are $C_1(q_1) = c_1 q_1$ and $C_2(q_2) = c_2 q_2$, where $c_1 < a$ and $c_2 < (a + c_1)/2$.

1. Compute the Nash equilibrium of the game. What are the market shares of the two firms?
2. Given your answer to (1), compute the equilibrium profits, consumer surplus and social welfare.
3. Prove that if c_2 decreases slightly, then social welfare increases if the market share of firm 2 exceeds $1/6$, but decreases if the market share of firm 2 is less than $1/6$. Give an economic interpretation of this finding.

3.3 Price-setting in a market with limited capacity

Suppose that two identical firms in a homogeneous-product market compete in prices. The capacity of each firm is 3. The firms have constant marginal cost equal to 0 up to the capacity constraint. The demand in the market is given by $Q(p) = 9 - p$. If the firms set the same price, they split the demand equally. If the firms set a different price, the demand of each one of the firms is calculated according to the efficient rationing rule. Show that $p_1 = p_2 = 3$ can be sustained as an equilibrium. Calculate the equilibrium profits.

3.4 Essay question: Industries with price or quantity competition

Which model, the Cournot or the Bertrand model, would you think provides a better first approximation to each of the following industries/markets: the oil refining industry, farmers' markets, cleaning services. Discuss.