

# Part II. Market power

## Chapter 3. Static imperfect competition



## Oligopolies

- Industries in which a few firms compete
- Market power is collectively shared.
- Firms can't ignore their competitors' behaviour.
- **Strategic interaction** → Game theory

## Oligopoly *theories*

- *Cournot* (1838) → quantity competition
- *Bertrand* (1883) → price competition
- Not competing but complementary theories
  - Relevant for different industries or circumstances

## Organization of Part II

- Chapter 3
  - Simple settings: unique decision at single point in time
  - How does the nature of strategic variable (price or quantity) affect
    - strategic interaction?
    - extent of market power?
- Chapter 4
  - Incorporates time dimension: sequential decisions
  - Effects on strategic interaction?
  - What happens before and after strategic interaction takes place?

## Case. DVD-by-mail industry

- Facts

- < 2004: *Netflix* almost only active firm
- 2004: entry by *Wal-Mart* and *Blockbuster* (and later *Amazon*), not correctly foreseen by *Netflix*

- Sequential decisions

- Leader: *Netflix*
- Followers: *Wal-Mart*, *Blockbuster*, *Amazon*

- Price competition

- *Wal-Mart* and *Blockbuster* undercut *Netflix*
- *Netflix* reacts by reducing its prices too.

- Quantity competition?

- Need to store more copies of latest movies

## Chapter 3. Learning objectives

- Get (re)acquainted with basic models of oligopoly theory
  - Price competition: Bertrand model
  - Quantity competition: Cournot model
- Be able to compare the two models
  - Quantity competition may be mimicked by a two-stage model (capacity-then-price competition)
  - Unified model to analyze price & quantity competition
- Understand the notions of strategic complements and strategic substitutes
- See how to measure market power empirically

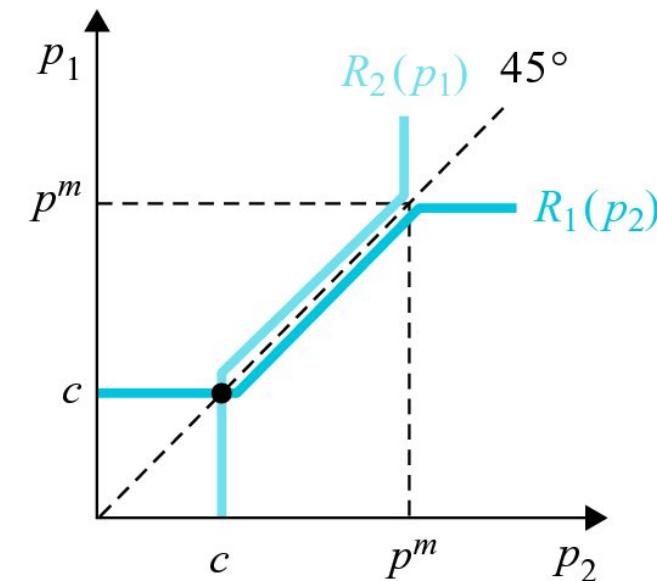
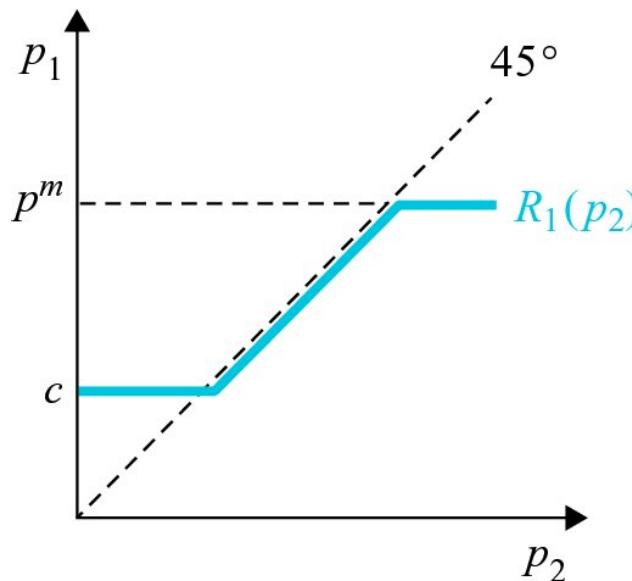
# The standard Bertrand model

- 2 firms
  - Homogeneous products
  - Identical constant marginal cost:  $c$
  - Set price simultaneously to maximize profits
- Consumers
  - Firm with lower price attracts all demand,  $Q(p)$
  - At equal prices, market splits at  $\alpha_1$  and  $\alpha_2=1-\alpha_1$
- → Firm  $i$  faces demand

$$Q_i(p_i) = \begin{cases} Q(p_i) & \text{if } p_i < p_j \\ \alpha_i Q(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

## The standard Bertrand model (cont'd)

- Unique Nash equilibrium
  - Both firms set price = marginal cost:  $p_1 = p_2 = c$
  - *Proof*
    - For any other  $(p_1, p_2)$ , a profitable deviation exists.
    - Or: unique intersection of firms' *best-response functions*



## The standard Bertrand model (cont'd)

- 'Bertrand Paradox'
  - Only 2 firms **but** perfectly competitive outcome
  - Message: there exist circumstances under which duopoly competitive pressure can be very strong
- **Lesson:** In a homogeneous product Bertrand duopoly with identical and constant marginal costs, the equilibrium is such that
  - firms set price equal to marginal costs;
  - firms do not enjoy any market power.

## The standard Bertrand model (cont'd)

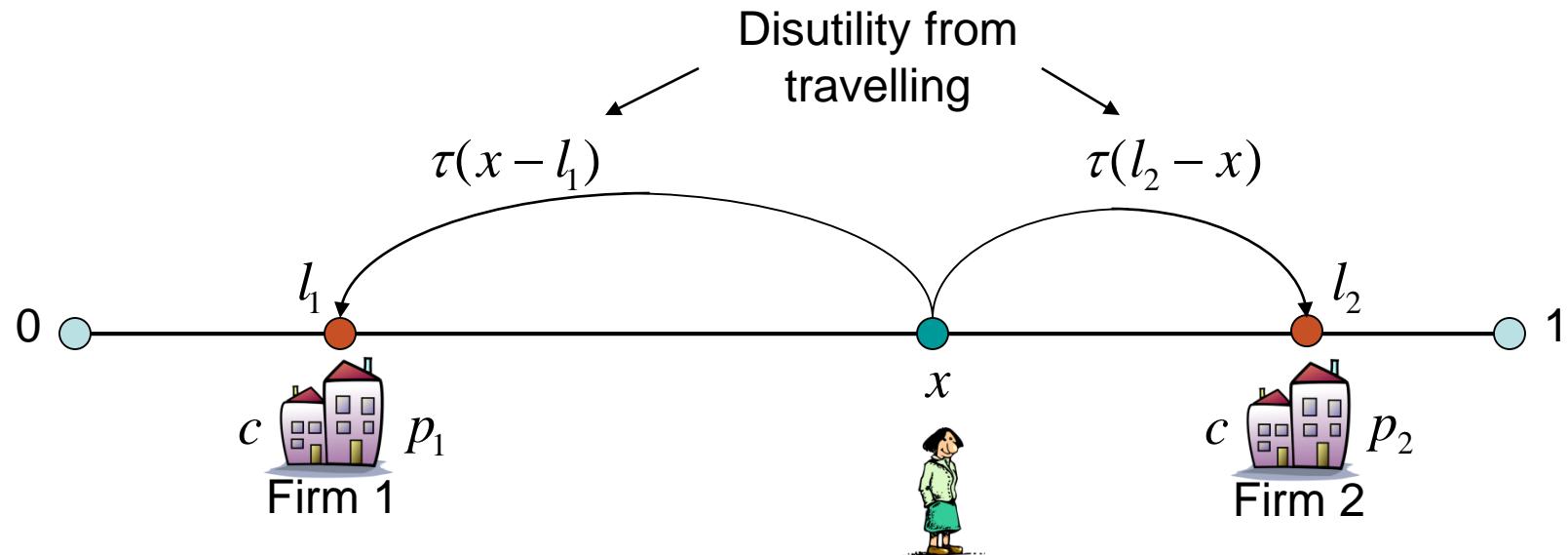
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  - firms set price equal to marginal costs;
  - firms do not enjoy any market power.
- Cost asymmetries:  $n$  firms,  $c_i < c_{i+1}$ 
  - Equilibrium: any price  $p_i = p_j = p \in [c_1, c_2]$
  - Select  $p^* = c_2$

## Bertrand competition with uncertain costs

- Each firm has private information about its costs
  - Trade-off between margins and likelihood of winning the competition
  - See particular model in the book.
- **Lesson:** In the price competition model with homogeneous products and private information about marginal costs, at equilibrium,
  - firms set price above marginal costs;
  - firms make strictly positive expected profits;
  - more firms  $\rightarrow$  price-cost margins  $\downarrow$ , output  $\uparrow$ , profits  $\downarrow$ ;
  - number of firms explodes  $\rightarrow$  competitive limit.

## Price competition with differentiated products

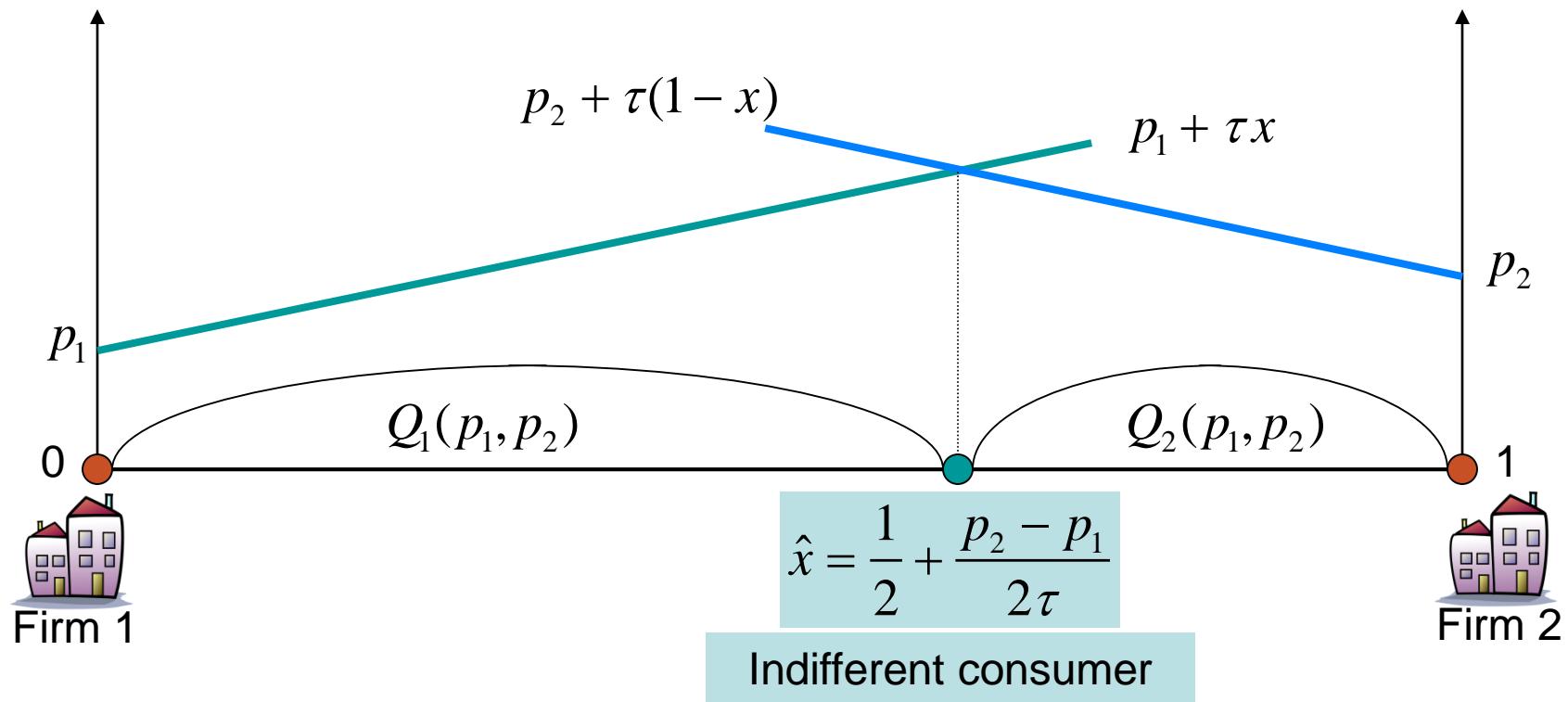
- Firms may avoid intense competition by offering products that are imperfect substitutes.
- Hotelling model (1929)



Mass 1 of consumers, uniformly distributed

## Hotelling model (cont'd)

- Suppose location at the extreme points



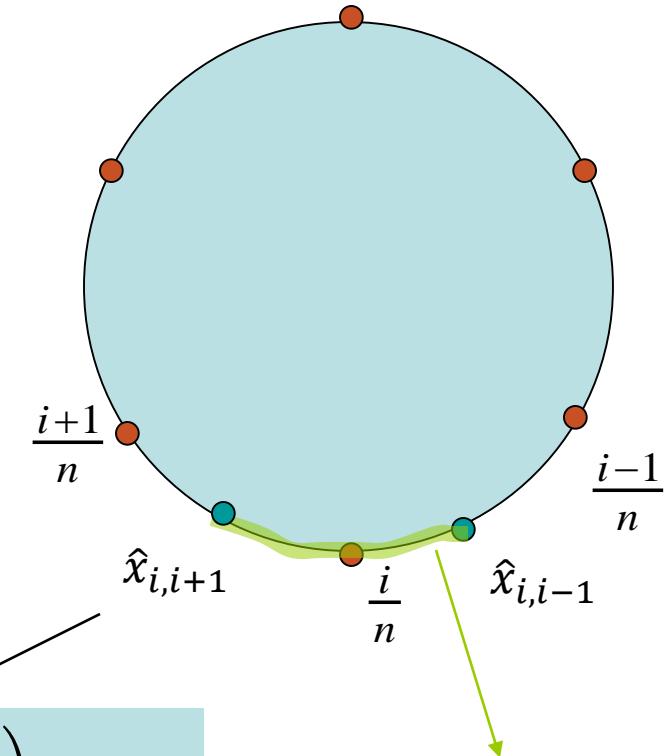
## Hotelling model (cont'd)

- Resolution
  - Firm's problem:  $\max_{p_i} (p_i - c) \left( \frac{1}{2} + \frac{p_j - p_i}{2\tau} \right)$
  - From FOC, best-response function:  $p_i = \frac{1}{2}(p_j + c + \tau)$
  - Equilibrium prices:  $p_i = p_j = c + \tau$
- **Lesson:** If products are more differentiated, firms enjoy more market power.
- Extensions
  1. Localized competition with  $n$  firms: **Salop** (circle) model
  2. Asymmetric competition with differentiated products

## Extension 1: Salop model

- Setting

- Firms equidistantly located on circle with circumference 1
- Consumers uniformly distributed on circle
- They buy at most one unit, from firm with lowest 'generalized price'
- Unit transportation cost,  $\tau$



$$\begin{aligned}
 r - \tau \left( \hat{x}_{i,i+1} - \frac{i}{n} \right) - p_i &= r - \tau \left( \frac{i+1}{n} - \hat{x}_{i,i+1} \right) - p_{i+1} \\
 \Leftrightarrow \hat{x}_{i,i+1} &= \frac{2i+1}{2n} + \frac{p_{i+1} - p_i}{2\tau}
 \end{aligned}$$

Firm  $i$ 's demand

## Extension 1: Salop model (cont'd)

- Focus on symmetric equilibrium
- Firm  $i$ 's problem:

$$\max_{p_i} (p_i - c)Q(p_i, p) = (p_i - c) \left( \frac{1}{n} + \frac{p - p_i}{\tau} \right)$$

- FOC:  $1/n + (p - 2p_i + c)/\tau = 0$
- Setting  $p_i = p$  yields:  $p^* = c + \tau/n$ 
  - $n \uparrow \rightarrow$  closer substitutes on the circle  
 $\rightarrow$  competitive pressure  $\uparrow \rightarrow p^* \downarrow$
  - If  $n \rightarrow \infty$ , then  $p^* \rightarrow c$  (perfect competition)

## Extension 2: Asymmetric competition with differentiated products

- Same setting as Hotelling model
- Only difference: product 1 is of superior quality
  - Consumer's indirect utility:

$$\begin{cases} r_1 - \tau x - p_1 & \text{if buy 1} \\ r_2 - \tau(1-x) - p_2 & \text{if buy 2} \end{cases} \text{ with } r_1 > r_2$$

- Assume:  $r_2 + \tau > r_1 \rightarrow$  product 2 more attractive for some consumers
- Indifferent consumer

$$\hat{x} = \frac{1}{2} + \frac{(r_1 - r_2) - (p_1 - p_2)}{2\tau} = Q_1(p_1, p_2)$$

## Extension 2: Asymmetric competition with differentiated products (cont'd)

- Firm 1 chooses  $p_1$  to maximize  $(p_1 - c)Q_1(p_1, p_2)$
- Similarly for firm 2.
- Solving for the two FOCs:

$$\begin{cases} p_1^* = c + \tau + \frac{1}{3}(r_1 - r_2) \\ p_2^* = c + \tau - \frac{1}{3}(r_1 - r_2) \end{cases}$$

$$Q_1(p_1^*, p_2^*) = \frac{1}{2} + \frac{r_1 - r_2}{6\tau}$$

- High-quality firm sets a higher price and sells more.

## Extension 2: Asymmetric competition with differentiated products (cont'd)

- Welfare maximization → sell at marginal cost

$$Q_1(c, c) = \frac{1}{2} + \frac{r_1 - r_2}{2\tau} > Q_1(p_1^*, p_2^*) = \frac{1}{2} + \frac{r_1 - r_2}{6\tau}$$

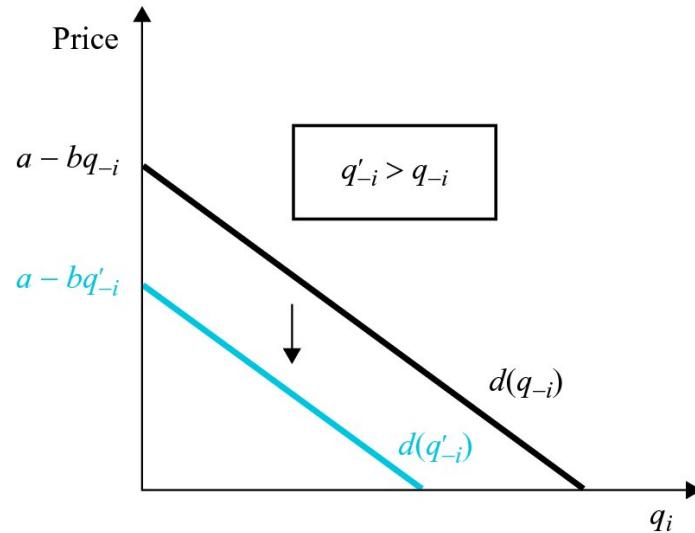
- Firm 1's equilibrium demand is too low from a social point of view.
- Same analysis if  $r_1 = r_2 = r$ , but  $c_1 < c_2$
- Lesson:** Under imperfect competition, the firm with higher quality or lower marginal cost sells too few units from a welfare perspective.

# The linear Cournot model

- Model
  - Homogeneous product market with  $n$  firms
  - Firm  $i$  sets quantity  $q_i$
  - Total output:  $q = q_1 + q_2 + \dots + q_n$
  - Market price given by  $P(q) = a - bq$
  - Linear cost functions:  $C_i(q_i) = c_i q_i$
  - Notation:  $q_{-i} = q - q_i$

- Residual demand

$$\begin{aligned}
 P(q) &= (a - bq_{-i}) - bq_i \\
 &\equiv d(q_i; q_{-i})
 \end{aligned}$$



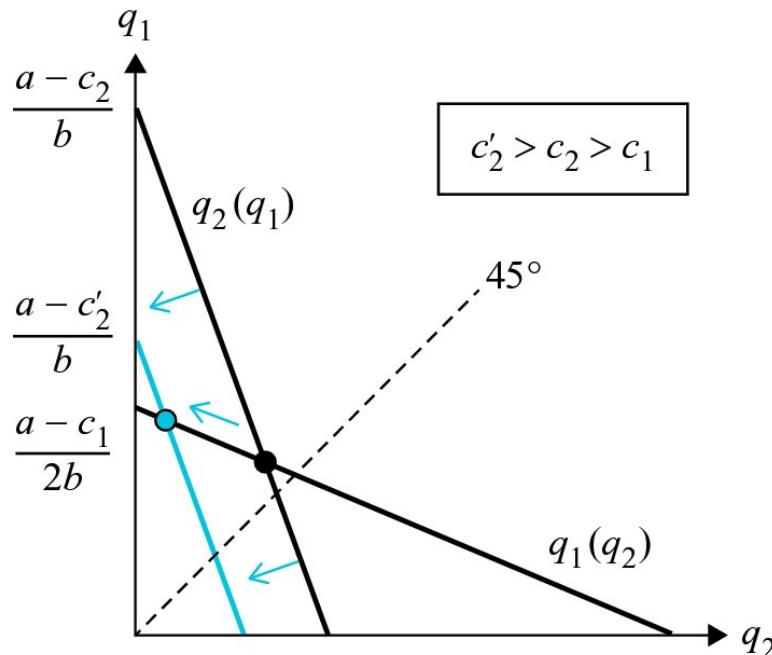
## The linear Cournot model (cont'd)

- Firm's problem
  - Cournot conjecture: rivals don't modify their quantity
  - Firm  $i$  acts as a monopolist on its residual demand: 
$$\max_{q_i} (P(q) - c_i)q_i$$
  - FOC: 
$$a - c_i - 2bq_i - bq_{-i} = 0$$
  - Best-response function: 
$$q_i(q_{-i}) = \frac{1}{2b}(a - c_i - bq_{-i})$$
- Nash equilibrium in the duopoly case
  - Assume:  $c_1 \leq c_2$  and  $c_2 \leq (a + c_1)/2$
  - Then, 
$$q_1^* = \frac{1}{3b}(a - 2c_1 + c_2)$$
 and 
$$q_2^* = \frac{1}{3b}(a - 2c_2 + c_1)$$

$$q_1^* \geq q_2^* \Rightarrow \pi_1^* \geq \pi_2^*$$

## The linear Cournot model (cont'd)

- Duopoly



- **Lesson:** In the linear Cournot model with homogeneous products, a firm's equilibrium profit increases when the firm becomes relatively more efficient than its rivals.

## Symmetric Cournot oligopoly

- Suppose that  $c_i = c$  for all  $i = 1 \square n$
- Then

$$q^*(n) = \frac{a - c}{b(n + 1)} \rightarrow L(n) = \frac{p^*(n) - c}{p^*(n)} = \frac{a - c}{a + nc}$$

- If  $n \uparrow \rightarrow$  individual quantity  $\downarrow$ , total quantity  $\uparrow$ , market price  $\downarrow$ , markup  $\downarrow$
- If  $n \rightarrow \infty$ , then markup  $\rightarrow 0$
- **Lesson:** The (symmetric linear) Cournot model converges to perfect competition as the number of firms increases.

## Implications of Cournot competition

- General demand and cost functions
- Cournot pricing formula (details see next slide)

$$\frac{P(q) - C'_i(q_i)}{P(q)} = \frac{\alpha_i}{\eta} \text{ with } \alpha_i = q_i / q$$

- **Lesson:** In the Cournot model, the markup of firm  $i$  is larger the larger is the market share of firm  $i$  and the less elastic is market demand.
- If marginal costs are constant

$$\frac{p - \sum_{i=1}^n \alpha_i c_i}{p} = \frac{I_H}{\eta} \text{ with } I_H = \sum_{i=1}^n \alpha_i^2, \text{ Herfindahl index}$$

Average Lerner index

## Details: Cournot pricing formula

- F.O.C. of profit maximization for Cournot firm

$$P'(q)q_i + P(q) - C'_i(q_i) = 0 \Leftrightarrow$$

$$P(q) - C'_i(q_i) = -P'(q)q_i \Leftrightarrow$$

$$\frac{P(q) - C'_i(q_i)}{P(q)} = \frac{-P'(q)q}{P(q)} \frac{q_i}{q} = \frac{1}{\eta} \alpha_i$$

- Suppose constant marginal costs:  $C_i(q_i) = c_i q_i$

$$\frac{p - c_i}{p} = \frac{\alpha_i}{\eta} \rightarrow \sum_{i=1}^n \pi_i = \sum_{i=1}^n (p - c_i) \alpha_i q = \begin{cases} (p - \sum_{i=1}^n \alpha_i c_i)q \\ \frac{pq}{\eta} \sum_{i=1}^n \alpha_i^2 \end{cases}$$

$$\Rightarrow \frac{p - \sum_{i=1}^n \alpha_i c_i}{p} = \frac{\sum_{i=1}^n \alpha_i^2}{\eta} = \frac{I_H}{\eta}$$

→ Lerner index (weighted by market shares) is proportional to Herfindahl index

## Price versus quantity competition

- Comparison of previous results
  - Let  $Q(p)=a-p$ ,  $c_1=c_2=c$
  - Bertrand:  $p_1=p_2=c$ ,  $q_1=q_2=(a-c)/2$ ,  $\pi_1=\pi_2=0$
  - Cournot:  $q_1=q_2=(a-c)/3$ ,  $p=(a+2c)/3$ ,  $\pi_1=\pi_2=(a-c)^2/9$
- **Lesson:** Homogeneous product case  $\rightarrow$  higher price, lower quantity, higher profits under quantity than under price competition.
- To refine the comparison
  - Limited capacities of production
  - Direct comparison within a unified model
  - Identify characteristics of price or quantity competition

## Limited capacity and price competition

- Edgeworth's critique (1897)
  - Bertrand model: no capacity constraint
  - But capacity may be limited in the short run.
- Examples
  - Retailers order supplies well in advance
  - DVD-by-mail industry
    - Larger demand for latest movies → need to hold extra stock of copies → higher costs and stock may well be insufficient
  - Flights more expensive around Xmas
- To account for this: **two-stage model**
  1. Firms precommit to capacity of production
  2. Price competition

## Capacity-then-price model (Kreps & Scheinkman)

- Setting

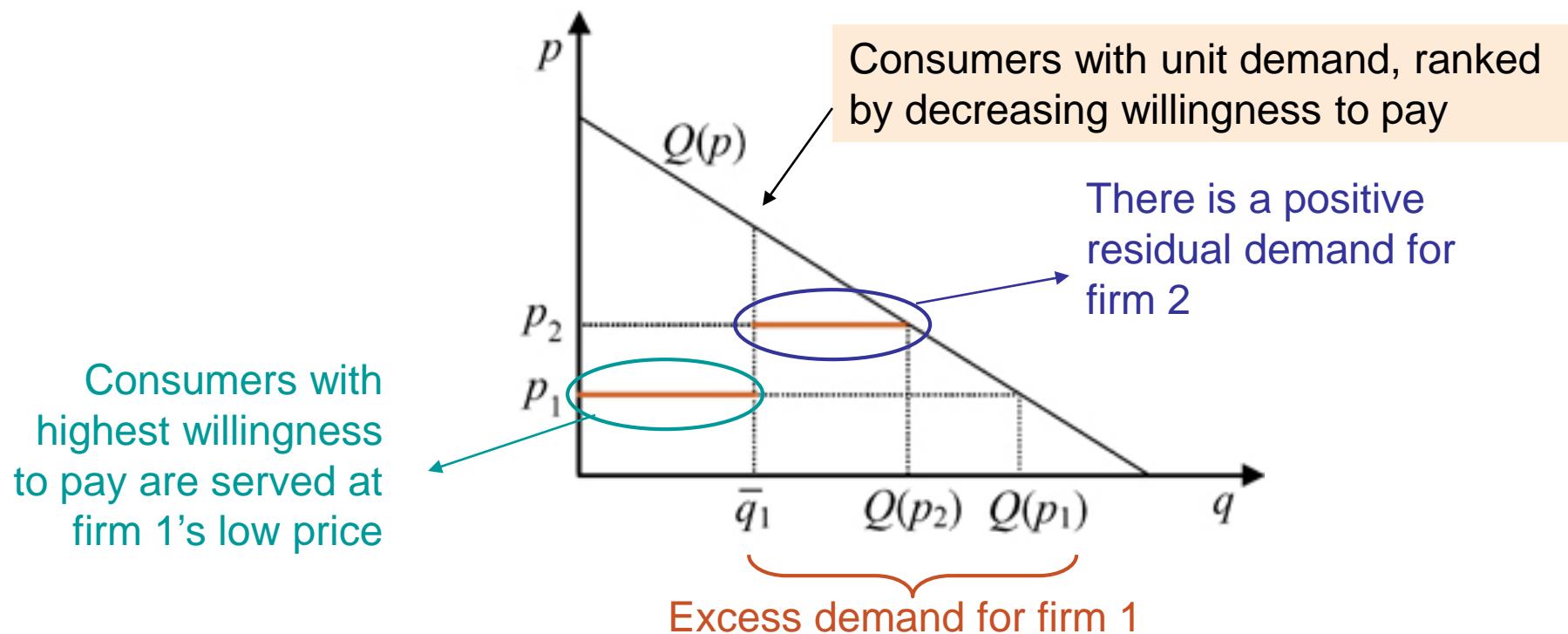
- Stage 1: firms set capacities  $\bar{q}_i$  and incur cost of capacity,  $c$
- Stage 2: firms set prices  $p_i$ ; cost of production is 0 up to capacity (and infinite beyond capacity); demand is  $Q(p) = a - p$ .
- Subgame-perfect equilibrium: firms know that capacity choices may affect equilibrium prices

- Rationing

- If quantity demanded to firm  $i$  exceeds its supply...
- ... some consumers have to be rationed...
- ... and possibly buy from more expensive firm  $j$ .
- Crucial question: Who will be served at the low price?

## Capacity-then-price model (cont'd)

- Efficient rationing
  - First served: consumers with higher willingness to pay.
  - Justification: queuing system, secondary markets



## Capacity-then-price model (cont'd)

- Equilibrium (details next slides)
  - Stage 2. If  $p_1 < p_2$  and excess demand for firm 1, then demand for 2 is:

$$Q(p_2) = \begin{cases} Q(p_2) - \bar{q}_1 & \text{if } Q(p_2) - \bar{q}_1 \geq 0 \\ 0 & \text{else} \end{cases}$$

Claim: if  $c < a < (4/3)c$ , then both firms set the market-clearing price:  $p_1 = p_2 = p^* = a - \bar{q}_1 - \bar{q}_2$

- Stage 1. Same reduced profit functions as in Cournot:

$$\bar{\pi}_1(\bar{q}_1, \bar{q}_2) = (a - \bar{q}_1 - \bar{q}_2)\bar{q}_1 - c\bar{q}_1$$

- **Lesson**: In the capacity-then-price game with efficient consumer rationing (and with linear demand and constant marginal costs), the chosen capacities are equal to those in a standard Cournot market.

## Details: Capacity-then-price model

- Setting
  - Stage 1: firms set capacities  $\bar{q}_i$  and incur cost of capacity,  $c$
  - Stage 2: firms set prices  $p_i$ ; cost of production is 0 up to capacity (and infinite beyond capacity); demand is  $Q(p) = a - p$ .
  - Subgame-perfect equilibrium: firms know that capacity choices may affect equilibrium prices
  - Efficient rationing
- Upper bound on capacity at stage 1

$$c\bar{q}_i \leq \max_q (a - q)q = a^2 / 4 \Leftrightarrow \bar{q}_i \leq a^2 / (4c)$$

## Details: Capacity-then-price model (cont'd)

- Claim: if  $c < a < (4/3)c$ , then both firms set the market-clearing price:  $p_1 = p_2 = p^* = a - \bar{q}_1 - \bar{q}_2$
- Proof
  - Let  $p_1 = p^*$  and show that 2's best-response is  $p_2 = p^*$ .
  - $p_2 < p^*$  doesn't pay: same quantity (because firm 2 sells all its capacity) sold at lower price
  - $p_2 > p^*$  could pay as firm 1 is capacity constrained... For this, revenues should be increasing at  $p^*$  ...
  - Firm 2's revenues:

$$p_2 Q(p_2) = \begin{cases} p_2(a - p_2 - \bar{q}_1) & \text{if } a - p_2 \geq \bar{q}_1, \\ 0 & \text{else} \end{cases}$$

## Details: Capacity-then-price model (cont'd)

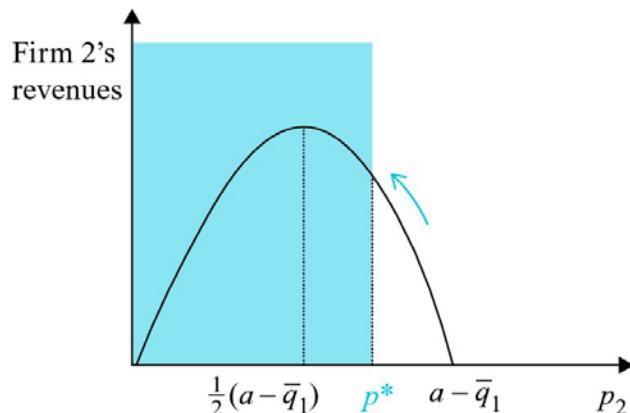
- Proof (cont'd)
  - Max reached at  $\bar{p}_2 = (a - \bar{q}_1) / 2$
  - Revenues are decreasing at  $p^*$  if

$$p^* > \bar{p}_2 \Leftrightarrow a - \bar{q}_1 - \bar{q}_2 > \frac{a - \bar{q}_1}{2} \Leftrightarrow a > \bar{q}_1 + 2\bar{q}_2$$

Since  $\bar{q}_1, \bar{q}_2 \leq a^2/(4c)$ ,  $\bar{q}_1 + 2\bar{q}_2 \leq (3/4)(a^2/c)$

Assumption  $a < (4/3)c \Leftrightarrow (3/4)(a/c) < 1$

- Hence, not profitable to set  $p_2 > p^*$ . QED



## Differentiated products: Cournot vs. Bertrand

### • Setting

- Duopoly, substitutable products ( $b > d > 0$ )
- Consumers maximize linear-quadratic utility function

$$U(q_0, q_1, q_2) = aq_1 + aq_2 - (bq_1^2 + 2dq_1q_2 + bq_2^2)/2 + q_0$$

under budget constraint

$$y = q_0 + p_1q_1 + p_2q_2$$

- Inverse demand functions

$$\begin{cases} P_1(q_1, q_2) = a - bq_1 - dq_2 \\ P_2(q_1, q_2) = a - bq_2 - dq_1 \end{cases}$$

- Demand functions

$$\begin{cases} Q_1(p_1, p_2) = \bar{a} - \bar{b}p_1 + \bar{d}p_2 \\ Q_2(p_1, p_2) = \bar{a} - \bar{b}p_2 + \bar{d}p_1 \end{cases} \quad \text{with} \quad \bar{a} = a / (b + d), \bar{b} = b / (b^2 - d^2), \bar{d} = d / (b^2 - d^2)$$

## Differentiated products

$$\max_{p_i} (p_i - c_i)(\bar{a} - \bar{b}p_i + \bar{d}p_j)$$

- Maximization program

- Cournot:  $\max_{q_i} (a - bq_i + dq_j - c_i)q_i$

- Bertrand:  $\max_{p_i} (p_i - c_i)(\bar{a} - \bar{b}p_i + \bar{d}p_j)$

- Best-response functions

- Cournot:  $q_i(q_j) = (a - dp_j - c_i)/(2\bar{b})$

Downward-sloping → Strategic **substitutes**

- Bertrand:  $p_i(p_j) = (\bar{a} + \bar{d}p_j + \bar{b}c_i)/(2\bar{b})$

Upward-sloping → Strategic **complements**

- Comparison of equilibria

- Lesson:** Price as the strategic variable gives rise to a more competitive outcome than quantity as the strategic variable.

## Appropriate modelling choice: price or quantity?

- Monopoly: it doesn't matter.
- Oligopoly: price and quantity competitions lead to different residual demands
  - Price competition
    - $p_j$  fixed  $\rightarrow$  rival willing to serve any demand at  $p_j$
    - $i$ 's residual demand: market demand at  $p_i < p_j$ ; zero at  $p_i > p_j$
    - So, residual demand is very sensitive to price changes.
  - Quantity competition
    - $q_j$  fixed  $\rightarrow$  irrespective of price obtained, rival sells  $q_j$
    - $i$ 's residual demand: "what's left" (i.e., market demand –  $q_j$ )
    - So, residual demand is less sensitive to price changes.

## Appropriate modelling choice (cont'd)

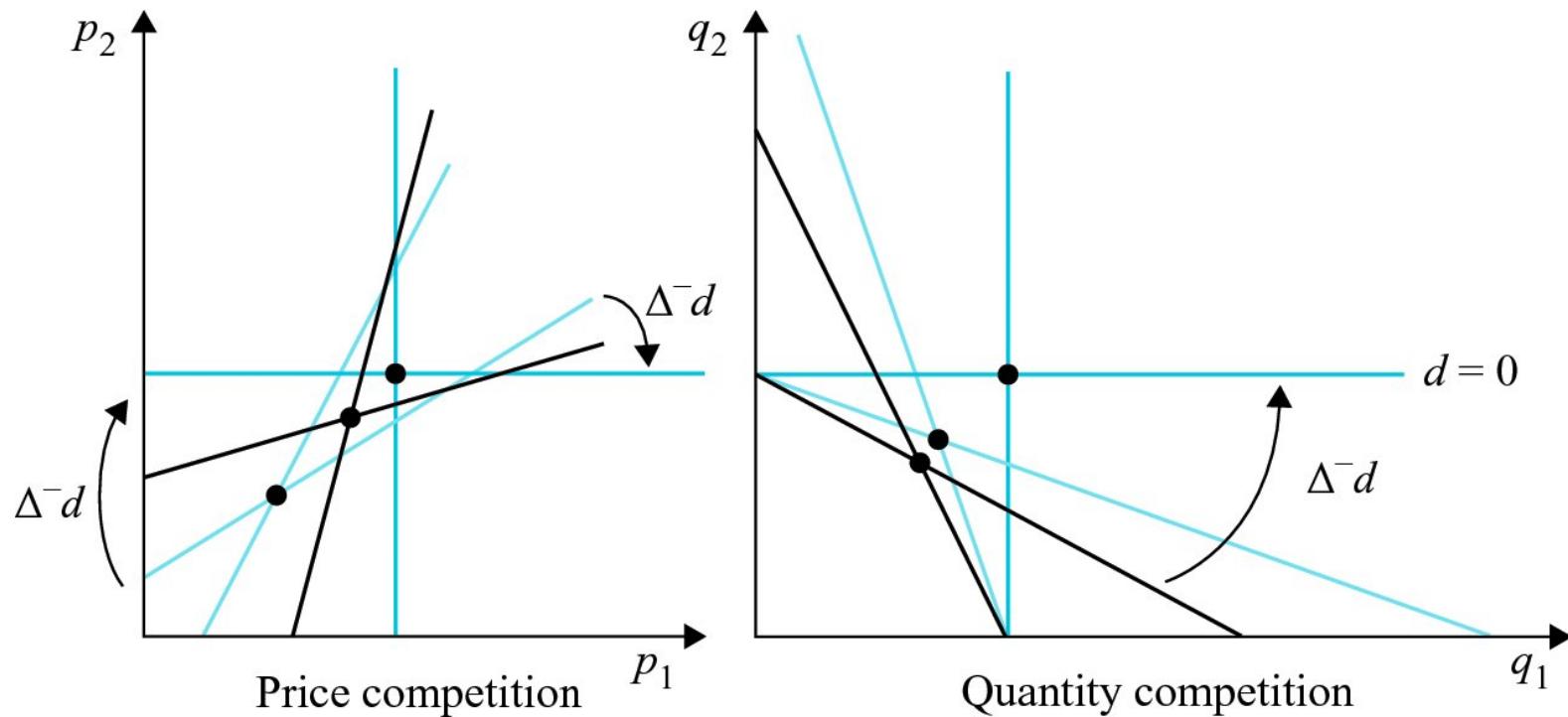
- How do firms behave in the market place?
  - Stick to a price and sell any quantity at this price?
    - **price competition**
    - appropriate choice when
      - Unlimited capacity
      - Prices more difficult to adjust in the short run than quantities
      - Example: mail-order business
  - Stick to a quantity and sell this quantity at any price?
    - **quantity competition**
    - appropriate choice when
      - Limited capacity (even if firms are price-setters)
      - Quantities more difficult to adjust in the short run than prices
      - Example: package holiday industry
  - Influence of technology (e.g. Print-on-demand vs. batch printing)

## Strategic substitutes and complements

- How does a firm react to the rivals' actions?
- Look at the slope of reaction functions.
  - Upward sloping: competitor  $\uparrow$  its action  $\rightarrow$  marginal profitability of my own action  $\uparrow$   
 $\rightarrow$  variables are strategic **complements**
    - Example: price competition (with substitutable products);  
See Bertrand and Hotelling models
  - Downward sloping: competitor  $\uparrow$  its action  $\rightarrow$  marginal profitability of my own action  $\downarrow$   
 $\rightarrow$  variables are strategic **substitutes**
    - Example: quantity competition (with substitutable products);  
see Cournot model

## Strategic substitutes and complements (cont'd)

- Linear demand model of product differentiation  
(with  $d$  measuring the degree of product substitutability)



# Estimating market power

- Setting
  - Symmetric firms producing homogeneous product
  - Demand equation:  $p = P(q, x)$  (1)
    - $q$ : total quantity in the market
    - $x$ : vector of exogenous variables affecting demand (not cost)
  - Marginal costs:  $c(q, w)$ 
    - $w$ : vector of exogenous variables affecting (variable) costs
- Interpretation 1. Nest various market structures in a single model

$$MR(\lambda) = p + \lambda \frac{\partial P(q, x)}{\partial q} q$$

$\lambda = 0$  competitive market  
 $\lambda = 1$  monopoly  
 $\lambda = 1/n$   $n$ -firm Cournot

Firm's *conjecture* as to how strongly price reacts to its change in output

## Estimating market power (cont'd)

- Interpretation 1 (cont'd)
  - Basic model to be estimated non-parametrically: demand equation (1) + equilibrium condition (2)

$$MR(\lambda) = p + \lambda \frac{\partial P(q, x)}{\partial q} q = c(q, w)$$

- Interpretation 2. Be agnostic about precise game being played
  - From equilibrium condition (2), Lerner index is

$$L = \frac{p - c(q, w)}{p} = -\lambda \frac{\partial P(q, x)}{\partial q} \frac{q}{p} = \frac{\lambda}{\eta}$$

- (2) is identified if single  $c(q, w)$  and single  $\lambda$  satisfy it

## Review questions

- How does product differentiation relax price competition? Illustrate with examples.
- How does the number of firms in the industry affect the equilibrium of quantity competition?
- When firms choose first their capacity of production and next, the price of their product, this two-stage competition sometimes looks like (one-stage) Cournot competition. Under which conditions?
- Using a unified model of horizontal product differentiation, one comes to the conclusion that price competition is fiercer than quantity competition. Explain the intuition behind this result.

## Review questions (cont'd)

- Define the concepts of strategic complements and strategic substitutes. Illustrate with examples.
- What characteristics of a specific industry will you look for to determine whether this industry is better represented by price competition or by quantity competition? Discuss.