EconS 503 - Advanced Microeconomics II
Handout on Moral Hazard

1. Macho-Stadler, Ch. 3 #6

Consider a relationship between a principal and an agent in which only two results, valued at 50,000 and 25,000 are possible. The agent must choose between three possible efforts. The probability of each of the results contingent on the efforts is given below:

		Results	
		25,000	50,000
Efforts	e^1	0.25	0.75
	e^2	0.50	0.50
	e^3	0.75	0.25

Assume that the principal is risk-neutral and that the agent is risk-averse, with their respective preferences described by the following functions:

$$B(x, w) = x - w \qquad U(w, e) = \sqrt{w} - v(e)$$

with $v(e^1) = 40$, $v(e^2) = 20$, and $v(e^3) = 5$. The reservation utility level of the agent is $\underline{U} = 120$.

(a) Write down the optimal contracts under symmetric information for each effort level and the profits obtained by the principal in each case. What effort level does the principal prefer?

Answer:

Under symmetric information, the agent receives a fixed pay-off, determined by the participation constraints. For each type, we have

$$\begin{aligned} \sqrt{w^1} - 40 &\geq 120 \\ \sqrt{w^2} - 20 &\geq 120 \\ \sqrt{w^3} - 5 &\geq 120 \end{aligned}$$

In equilibrium, all three of these constrants will bind, yielding solutions $(w^1, w^2, w^3) = (25,600,\ 19,600,\ 15,625)$. We can then plug each of these wages into the firm's expected utility

$$p(e^i)(25000 - w^i) + (1 - p(e^i))(50,000 - w^i)$$

to obtain expected utilities of the firm $(B^1, B^2, B^3) = (18,150,\ 17,900,\ 15,625)$ Since e^1 gives the highest expected utility for the firm, it will be the preferred contract.

(b) Write down the optimal contracts when there exists a moral hazard problem. What is the optimal effort level and the contract chosen by the principal?

Answer:

As in part (a), to determine the optimal contract, we will want to evaluate each effort level seperately.
Case 1: $e = e^1$.
For high effort, the firm solves the following objective function

$$\max_{w^h, w^l} \quad \frac{1}{4}(25000 - w^l) + \frac{3}{4}(50000 - w^h)$$

$$\text{subject to} \quad \frac{1}{4}\sqrt{w^l} + \frac{3}{4}\sqrt{w^h} - 40 \geq 120 \qquad (PC^1)$$

$$\frac{1}{4}\sqrt{w^l} + \frac{3}{4}\sqrt{w^h} - 40 \geq \frac{1}{2}\sqrt{w^l} + \frac{1}{2}\sqrt{w^h} - 20 \qquad (IC_2^1)$$

$$\frac{1}{4}\sqrt{w^l} + \frac{3}{4}\sqrt{w^h} - 40 \geq \frac{3}{4}\sqrt{w^l} + \frac{1}{4}\sqrt{w^h} - 5 \qquad (IC_3^1)$$

we can rearrange the constraints to

$$\sqrt{w^l} + 3\sqrt{w^h} \geq 640 \qquad (PC^1)$$

$$\sqrt{w^h} \geq \sqrt{w^l} + 80 \qquad (IC_2^1)$$

$$\sqrt{w^h} \geq \sqrt{w^l} + 70 \qquad (IC_3^1)$$

It is trivial to show that if IC_2^1 holds, IC_3^1 definitely holds. Thus, we can eliminate IC_3^1. Taking Kuhn-tucker first-order conditions yields

$$-\frac{1}{4} + \lambda_1 \frac{1}{2\sqrt{w^l}} - \lambda_2 \frac{1}{2\sqrt{w^l}} = 0 \qquad (1)$$

$$-\frac{3}{4} + 3\lambda_1 \frac{1}{2\sqrt{w^h}} + \lambda_2 \frac{1}{2\sqrt{w^h}} = 0 \qquad (2)$$

Now, we consider cases on the values of our Lagrange multipliers. First, if $\lambda_1, \lambda_2 = 0$ (neither constraint binds), neither first-order condition can yield a solution, so at least one must bind for sure. If $\lambda_1 = 0$, and $\lambda_2 > 0$ (i.e., only IC_2^1 binds), then we would have a contradiction in equation (1), as $\lambda_2 < 0$. If $\lambda_1 > 0$, and $\lambda_2 = 0$ (i.e., only PC^1 binds) the first-order conditions reduce to

$$w^l = w^h$$

which violates IC_2^1. Hence, our only option remaining is $\lambda_1, \lambda_2 > 0$ (both constraints bind). Rearranging our constraints gives

$$\sqrt{w^l} + 3\sqrt{w^h} = 640 \qquad (PC^1)$$

$$\sqrt{w^h} = \sqrt{w^l} + 80 \qquad (IC_2^1)$$

and solving, we have $(w^h, w^l, \lambda_1, \lambda_2, B^1) = (32,400,\ 10,000,\ 80,\ 30,\ 16,950)$.

Case 2: $e = e^3$ (Skipping ahead, will do e^2 next).

For low effort, the firm solves the following objective function

$$\max_{w^h,w^l} \quad \frac{3}{4}(25000 - w^l) + \frac{1}{4}(50000 - w^h)$$

$$\text{subject to} \quad \frac{3}{4}\sqrt{w^l} + \frac{1}{4}\sqrt{w^h} - 5 \geq 120 \qquad (PC^3)$$

$$\frac{3}{4}\sqrt{w^l} + \frac{1}{4}\sqrt{w^h} - 5 \geq \frac{1}{4}\sqrt{w^l} + \frac{3}{4}\sqrt{w^h} - 40 \qquad (IC_1^3)$$

$$\frac{3}{4}\sqrt{w^l} + \frac{1}{4}\sqrt{w^h} - 5 \geq \frac{1}{2}\sqrt{w^l} + \frac{1}{2}\sqrt{w^h} - 20 \qquad (IC_2^3)$$

we can rearrange the constraints to

$$3\sqrt{w^l} + \sqrt{w^h} \geq 500 \qquad (PC^3)$$

$$\sqrt{w^l} \geq \sqrt{w^h} - 70 \qquad (IC_1^3)$$

$$\sqrt{w^l} \geq \sqrt{w^h} - 60 \qquad (IC_2^3)$$

It is trivial to show that if IC_2^3 holds, IC_1^3 definitely holds. Thus, we can eliminate IC_1^3. Taking Kuhn-tucker first-order conditions yields

$$-\frac{3}{4} + 3\lambda_1 \frac{1}{2\sqrt{w^l}} + \lambda_2 \frac{1}{2\sqrt{w^l}} = 0 \qquad (1)$$

$$-\frac{1}{4} + \lambda_1 \frac{1}{2\sqrt{w^h}} - \lambda_2 \frac{1}{2\sqrt{w^h}} = 0 \qquad (2)$$

Now, we consider cases on the values of our Lagrange multipliers. First, if $\lambda_1, \lambda_2 = 0$ (neither constraint binds), neither first-order condition can yield a solution, so at least one must bind for sure. If $\lambda_1 = 0$, and $\lambda_2 > 0$ (i.e., only IC_2^3 binds), then we would have a contradiction in equation (2), as $\lambda_2 < 0$. If $\lambda_1 > 0$, and $\lambda_2 = 0$ (i.e., only PC^1 binds) the first-order conditions reduce to $w^l = w^h$. Substituting into PC^3 gives

$$4\sqrt{w^l} = 500$$

which implies that $w^l = w^h = 15,625$, yielding a utility for the firm of $15,625$ (note that this is the same solution as in complete information).
Lastly, we must check when $\lambda_1, \lambda_2 > 0$ (both constraints bind). Rearranging our first-order conditions gives

$$3\lambda_1 + \lambda_2 = \frac{3}{2}\sqrt{w^l} \qquad (1)$$

$$\lambda_1 - \lambda_2 = \frac{1}{2}\sqrt{w^h} \qquad (2)$$

and combining them yields

$$4\lambda_1 = \frac{3}{2}\sqrt{w^l} + \frac{1}{2}\sqrt{w^h} > 0$$

$$4\lambda_2 = \frac{3}{2}\sqrt{w^l} - \frac{3}{2}\sqrt{w^h} < 0$$

which is a violation. Hence, only the solution under complete information is valid.
Case 3: $e = e^2$.
For medium effort, the firm solves the following objective function

$$\max_{w^h,w^l} \quad \frac{1}{2}(25000 - w^l) + \frac{1}{2}(50000 - w^h)$$

$$\text{subject to} \quad \frac{1}{2}\sqrt{w^l} + \frac{1}{2}\sqrt{w^h} - 20 \geq 120 \qquad (PC^2)$$

$$\frac{1}{2}\sqrt{w^l} + \frac{1}{2}\sqrt{w^h} - 20 \geq \frac{1}{4}\sqrt{w^l} + \frac{3}{4}\sqrt{w^h} - 40 \qquad (IC_1^2)$$

$$\frac{1}{2}\sqrt{w^l} + \frac{1}{2}\sqrt{w^h} - 20 \geq \frac{3}{4}\sqrt{w^l} + \frac{1}{4}\sqrt{w^h} - 5 \qquad (IC_3^2)$$

we can rearrange the constraints to

$$\sqrt{w^l} + \sqrt{w^h} \geq 280 \qquad (PC^2)$$

$$\sqrt{w^l} \geq \sqrt{w^h} - 80 \qquad (IC_1^2)$$

$$\sqrt{w^h} \geq \sqrt{w^l} + 60 \qquad (IC_3^2)$$

At this point, it is uncertain which of the incentive compatibility constraints binds, but it can be trivially shown that if one constraint binds, the other will not. Taking Kuhn-tucker first-order conditions yields

$$-\frac{1}{2} + \lambda_1 \frac{1}{2\sqrt{w^l}} + \lambda_2 \frac{1}{2\sqrt{w^l}} - \lambda_3 \frac{1}{2\sqrt{w^l}} = 0 \qquad (1)$$

$$-\frac{1}{2} + \lambda_1 \frac{1}{2\sqrt{w^h}} - \lambda_2 \frac{1}{2\sqrt{w^h}} + \lambda_3 \frac{1}{2\sqrt{w^h}} = 0 \qquad (2)$$

Since we know that one of λ_2 and λ_3 is positive and the other is zero, we can use the same logic in parts (a) and (b) to show that $\lambda_1 > 0$ (**Practice**: Work it out!). All that remains is to determine which of the two incentive compatibility constraints binds. First, we will consider the case where $\lambda_2 > 0$. Our first-order conditions become

$$-\frac{1}{2} + \lambda_1 \frac{1}{2\sqrt{w^l}} + \lambda_2 \frac{1}{2\sqrt{w^l}} = 0 \qquad (1)$$

$$-\frac{1}{2} + \lambda_1 \frac{1}{2\sqrt{w^h}} - \lambda_2 \frac{1}{2\sqrt{w^h}} = 0 \qquad (2)$$

Rearranging terms gives

$$\lambda_1 + \lambda_2 = \sqrt{w^l} \qquad (1)$$

$$\lambda_1 - \lambda_2 = \sqrt{w^h} \qquad (2)$$

Combining,

$$2\lambda_1 = \sqrt{w^l} + \sqrt{w^h} > 0$$

$$2\lambda_2 = \sqrt{w^l} - \sqrt{w^h} < 0$$

which is a violation. Thus, $\lambda_2 = 0$. Let's check $\lambda_3 > 0$. Our first-order conditions become

$$-\frac{1}{2} + \lambda_1 \frac{1}{2\sqrt{w^l}} - \lambda_3 \frac{1}{2\sqrt{w^l}} = 0 \quad (1)$$
$$-\frac{1}{2} + \lambda_1 \frac{1}{2\sqrt{w^h}} + \lambda_3 \frac{1}{2\sqrt{w^h}} = 0 \quad (2)$$

Rearranging terms gives

$$\lambda_1 - \lambda_3 = \sqrt{w^l} \quad (1)$$
$$\lambda_1 + \lambda_3 = \sqrt{w^h} \quad (2)$$

Combining,

$$2\lambda_1 = \sqrt{w^l} + \sqrt{w^h} > 0$$
$$2\lambda_3 = \sqrt{w^h} - \sqrt{w^l} > 0$$

Hence, IC_3^2 is our binding constraint. Updating our constraints,

$$\sqrt{w^l} + \sqrt{w^h} = 280 \quad (PC^2)$$
$$\sqrt{w^h} = \sqrt{w^l} + 60 \quad (IC_3^2)$$

and solving, we have $(w^h, w^l, \lambda_1, \lambda_2, \lambda_3, B^2) = (28,900,\ 12,100,\ 140,\ 0,\ 30,\ 17,000)$.

Summarizing, the table below shows the utilities that the firm will receive by designing contracts for each effort level

Effort	Utility
e^1	16,950
e^2	17,000
e^3	15,625

Hence, the firm will prefer effort level e^2 and design its contract accordingly.

2. Macho-Stadler Ch. 3 # 9

Consider a relationship between a principal and an agent in which there are only two possible results, one high, x_2, and the other low, x_1. The frequency with which each result occurs depends on the agent's effort, $e \in [0, 1]$, and a random state variable. Assume that the probability of the high result is the same as the effort, i.e., $\Pr(x = x_2|e) = e$, so that $\Pr(x = x_1|e) = 1 - e$.
The agent's utility is of the form $U(w, e) = u(w) - v(e)$, where $u(\cdot)$ is increasing and concave, and $v(\cdot)$ is increasing and convex. The principal's objective function is $B(x - w)$, which is increasing and concave (that is, she could be risk averse).
(a) Write down the constrained maximization problem of the principal, and find the conditions that determine the optimal contract.

Answer:

The principal will maximize her expected utility, subject to the participation constraint, i.e.,

$$\max_{w_h,w_l,e} \quad eB(x_2-w_h)+(1-e)B(x_1-w_l)$$
$$\text{subject to} \quad e(u(w_h)-v(e))+(1-e)(u(w_l)-v(e)) \geq \bar{U}$$

Taking first-order conditions with respect to w_h and w_l,

$$\begin{aligned} -eB'(x_2-w_h)+\lambda eu'(w_h) &= 0 \\ -(1-e)B'(x_1-w_l)+\lambda(1-e)u'(w_l) &= 0 \end{aligned}$$

Combining these two equations yields our condition for the optimal contract,

$$\frac{B'(x_2-w_h)}{B'(x_1-w_l)}=\frac{u'(w_h)}{u'(w_l)}$$

(b) Now we assume that the agent's effort is not publicly known. Write down the constrained maximization problem that defines the optimal contract in this case. Is the first-order approach valid in this example? Describe the relationship between the optimal contract's wages and the differences in this contract compared to part (a).

Answer:

The most challenging part of this problem is figuring out the incentive compatibility constraints. For each effort type, there are an infinite amount of IC's, but they reduce to the form of

$$e \in \arg\max_{\hat{e}} \quad \hat{e}u(w_h)+(1-\hat{e})u(w_l)-v(\hat{e})$$

taking a first-order condition with respect to $\hat{e}$ yields

$$u(w_h)-u(w_l)-v'(\hat{e})=0$$

where e solves this equation with equality. Since the agent's function is concave in e (since $v(e)$ is convex) the first-order condition is both necessary and sufficient for a maximum. Thus, the first-order approach is valid.
We now use our derived incentive compatibility constraint as a new constraint in the objective function, which becomes

$$\max_{w_h,w_l,e} \quad eB(x_2-w_h)+(1-e)B(x_1-w_l)$$
$$\text{subject to} \quad e(u(w_h)-v(e))+(1-e)(u(w_l)-v(e) \geq \bar{U}$$
$$u(w_h)-u(w_l)-v'(\hat{e})=0$$

Taking first-order conditions with respect to w_h and w_l,

$$\begin{aligned} -eB'(x_2-w_h)+\lambda eu'(w_h)+\mu u'(w_h) &= 0 \\ -(1-e)B'(x_1-w_l)+\lambda(1-e)u'(w_l)-\mu u'(w_l) &= 0 \end{aligned}$$

Combining these two equations yields our condition for the optimal contract,

$$\frac{B'(x_2 - w_h)}{B'(x_1 - w_l)} = \frac{u'(w_h)}{u'(w_l)} \underbrace{\left(\frac{\lambda + \frac{\mu}{e}}{\lambda - \frac{\mu}{1-e}} \right)}_{>1}$$

This new condition implies that $\frac{B'(x_2-w_h)}{B'(x_1-w_l)} > \frac{u'(w_h)}{u'(w_l)}$ (assuming that both constraints bind). This implies that the principal makes the agent carry more than the efficient level of risk. That is, the optimal contract makes the agent more interested in the result than what is really optimal. Hence, this is a generalization of what we have studied for a risk-neutral principal.

3. 22.7 Business Application

[**Competitive Provision of Health Insurance**]. Consider the challenge of providing health insurance to a population with different probabilities of getting sick.
A: Suppose that, as in our car insurance example, there are two consumer types-consumers of type 1 that are likely to get sick, and consumers of type 2 that are relatively healthy. Let x represent the level of health insurance, with $x = 0$ with no insurance and higher levels of x indicating in curves (equal to marginal willingness to pay), with d^1 representing the demand curve for a single consumer of type 1 and d^2 representing the demand curve for a single consumer of type 2. Suppose further that the marginal cost of providing additional health coverage to an individual is constant, with $MC^1 > MC^2$.

[**Section A**]: **This exercise attempts to formalize a key intuition we covered in the textbook with a different type of model for insurance.**
(a) For simplicity, suppose throughout that d^1 and d^2 have the same slope. Suppose further, unless otherwise stated, that d^1 has higher intercept than d^2. Do you think it is reasonable to assume that type 1 has higher demand for insurance?

Answer:

It seems reasonable to assume that those who are more likely to get sick have higher demand for health insurance-which is what we are assuming when we assume that the intercept of d^1 is higher than the intercept of d^2.

(b) Begin by drawing a graph with d^1, d^2, MC^1 and MC^2 assuming that the vertical intercepts of both demand curves lie above MC^1. Indicate the efficient level of insurance $\bar{x}^1$ and $\bar{x}^2$ for the two types.

Answer:

This is done in figure 1 where the efficient level of insurance for type 1 consumers occurs where MC^1 intersects d^1 and the efficient level of insurance for type 2 consumers occurs where MC^2 intersects d^2. (Note: There is no particular reason for $\bar{x}^1$ to lie to the left of $\bar{x}^2$- had we draw the difference between d^1 and d^2 larger-or the difference between MC^1 and MC^2 smaller, the reverse would hold. Nothing fundamental changes in the analysis regardless of how the graph is drawn.)

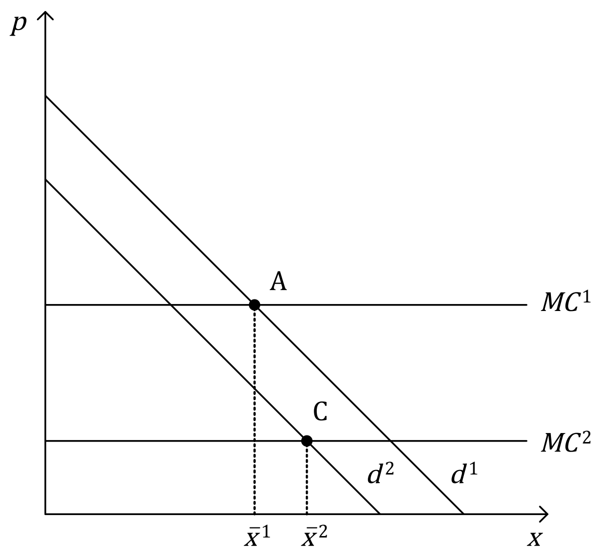

Figure 1: Equilibrium in the Insurance Market.

(c) Suppose the industry offers any level of x at price $p = MC^1$. Illustrate on your graph the consumer surplus that type 1 individuals will get if this were the only way to buy insurance and they buy there optimal policy A. How much consumer surplus will type 2 individuals get?

Answer:

Type 1 consumers will buy $x^A = \bar{x}^1$ and thus get consumer surplus $(a + b + c)$ as shown in figure 2. Consumer of type 2 will buy only up to the point where MC^1 crosses d^2−thus getting consumer surplus (a).

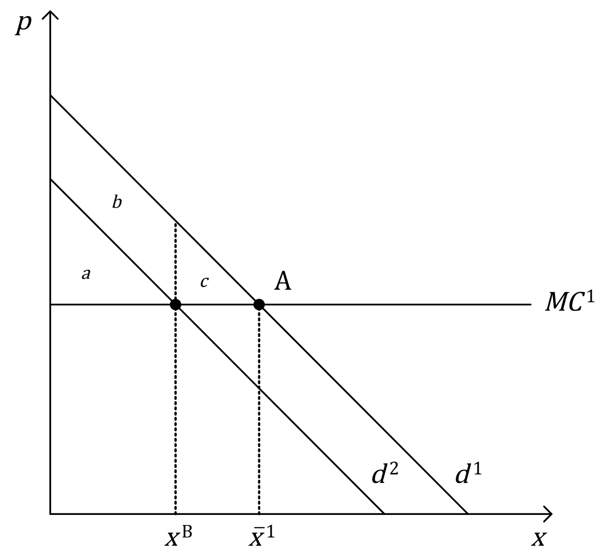

Figure 2: Only $p = MC^1$ is offered.

(d) Next, suppose you want to offer an individual insurance contract B that earns zero profit if bought only by type 2 consumers, that is preferred by type 2 individuals to A and that makes type 1 consumers just as well off as they are under the options from part (c). Identify B in your graph. *[Hint: Comparing consumer surplus]*

Answer:

This can be seen in figure 3. Note that there is no particular reason that B lies vertically underneath the intersection of MC^1 and d^2- it could lie to the right or left. It must be, however, that B lies on the MC^2 curve - otherwise firms offering it would not make zero profits. In order for type 1 individuals to be indifferent between A and B, it must be that their consumer surplus is the same under both contracts. Since their consumer surplus at A is $(a + b + c)$ and their consumer surplus as B is $(a + b + d)$, this implies that (c) has to be equal to (d). Notice that (c) gets larger and (d) gets smaller as we move B to the left, with (c) small and (d) large when B is horizontally close to A. Thus, starting B vertically underneath A and moving it to the left, there will come some x^B at which (d) is exactly equal to (c). Finally, it has to be the case that type 2 consumers are better off at B than would be otherwise - which has to be the case. (It is trivial to see when B lies right below the intersection of d^2 and MC^1 because then consumer surplus simply increases from (a) to $(a + d)$ - but it is also true if B lies to the right or left of that intersection point.)

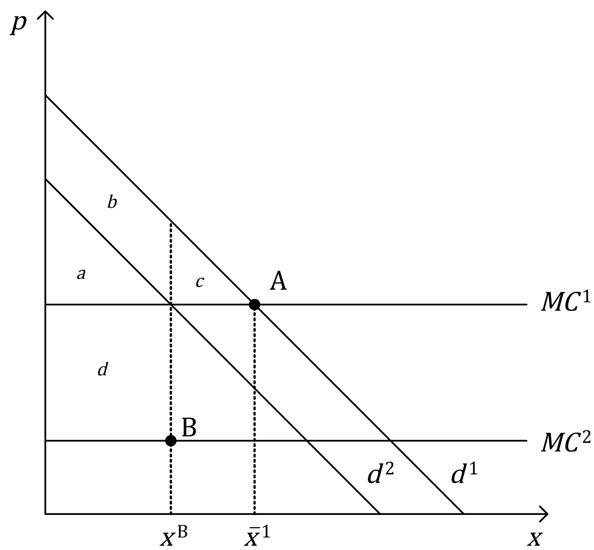

Figure 3: Contracts A and B are offered.

(e) Suppose for a moment that it is an equilibrium for the industry to offer only contacts A and B (and suppose that the actual B is just slightly to the left of the B you identified in part (d)). True or False: While insurance companies do not know what type consumers are when they walk into the insurance office to buy a policy, the companies will know what type of consumer they made a contract with after the consumer leaves.

Answer:

This is true - a consumer of type 1 would be weakly better off buying A while consumers of type 2 would be strictly better off buying B. Thus, you will know that the consumers is of type 1 if he bought A and of type 2 if he bought B.

(f) In order for this to be an equilibrium, it must be the case that it is not possible for an insurance company to offer a "pooling price" that makes at least zero profit while attracting both type 1 and 2 consumers. (Such a policy has a single price p^* that lies between MC^1 and MC^2.) Note that the demand curves graphed thus far were for only one individual of each type. What additional information would you have to know in order to know whether the zero - profit price p^* would attract both types?

Answer:

You would need to know no additional information to know that type 1 individuals would prefer the pooling contract price p^* - because it would be below the price at which they are otherwise buying A. But we don't know if such a price would attract consumers of type 2. It is a higher price, but if it allowed type 2 consumers to buy a larger quantity, that might make up for the loss in consumer surplus from the higher price. This is illustrated in figures 4 and 5 where a low p^* and a high p^* are graphed respectively. At p^*, type 2 consumers will buy where p^* intersects d^2- i.e. at point D in figure 4 and at point E in figure 5. In figure 4, this implies that consumers will lose area (g) in consumer surplus because of the increase in price but will gain area (h) from being able to purchase more insurance. Since (h)>(g), the consumer is better off and thus will choose the pooling contract. But in figure 5, type 2 consumers lose (i) and gain (j) - with the former larger than the latter. Thus, the higher p^*, the less likely it is that a pooling price p^* could attract both consumers.

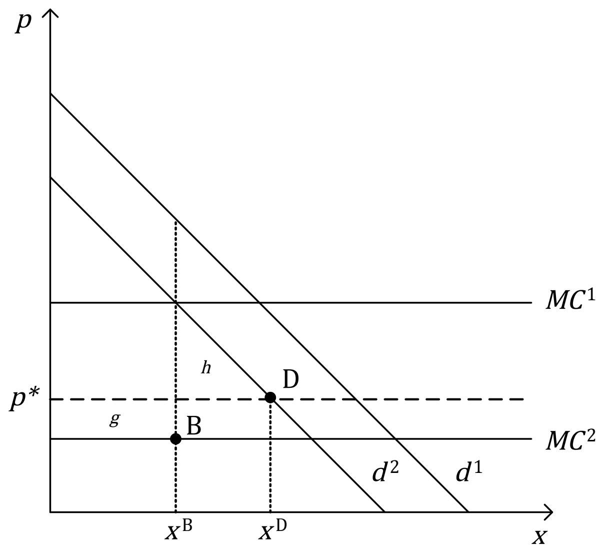

Figure 4: p^* is relatively low.

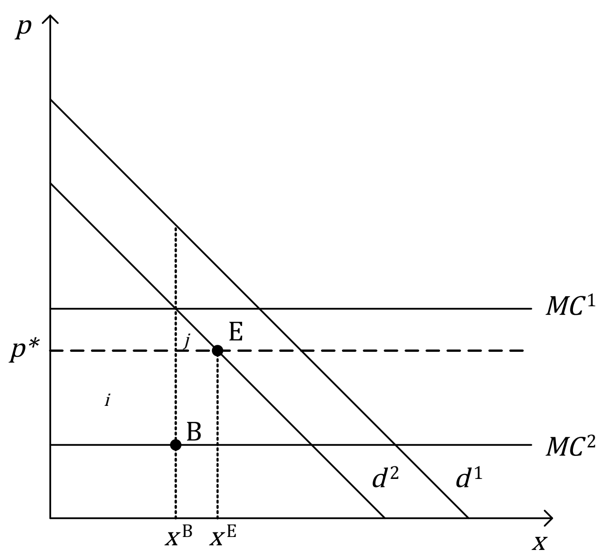

Figure 5: p^* is relatively high.

(g) True or False: The greater the fraction of consumers that are of type 1, the less likely it is that such a "pooling price" exists.

Answer:

This is true - because the greater the fraction of type 1 consumers, the higher the price p^* will have to be in order for firms offering that price to make zero profit.

(h) Suppose that no such pooling price exists. Assuming that health insurance firms cannot observe the health conditions of their customers, would it be a competitive equilibrium for the industry to offer contracts A and B? Would that be a pooling or separating equilibrium?

Answer:

Yes, this would be a separating equilibrium because the two types end up revealing who they are by choosing different contracts. In fact, the equilibrium could simply offer any insurance amount at price $p^A = MC^1$ and any insurance amount up to x^B at price $p^B = MC^2$. But no insurance above x^B can be offered at p^B - otherwise type 1 consumers will buy at p^B - which means p^B would no larger be a zero profit price.

(i) Would you still be able to identify a contract B that satisfies the conditions in (d) if $d^1 = d^2$? What if $d^1 < d^2$?

Answer:

Figure 6 illustrates the case where $d^1 = d^2$. The contract B has to be such that area (a) is equal to area (b) so that type 1 individuals would lose as much (i.e. (b)) as they would gain (i.e. (a)) from switching from A to B. But because $d^1 = d^2$, this implies type 2 individuals will be similarly indifferent - and not strictly prefer B to A. In this borderline case, it is therefore barely possible to find B that satisfies the necessary conditions for a separating equilibrium to emerge. The case where $d^2 > d^1$ is graphed in figure 7. In order for type 1 consumers to be indifferent between A and B, it has to now be that area (c) is equal to area (d). The B that satisfies this is graphed. But type 2 consumers would now prefer A to B - because their surplus under B is $(c + e + f)$ while their surplus under A is $(d + e + f + g)$. Since (c) is equal to (d), the surplus under A can also be written as $(c+e+f+g)$ - implying that consumers of type 2 are better off by area (g) if they pick A. The separating equilibrium can therefore not emerge.

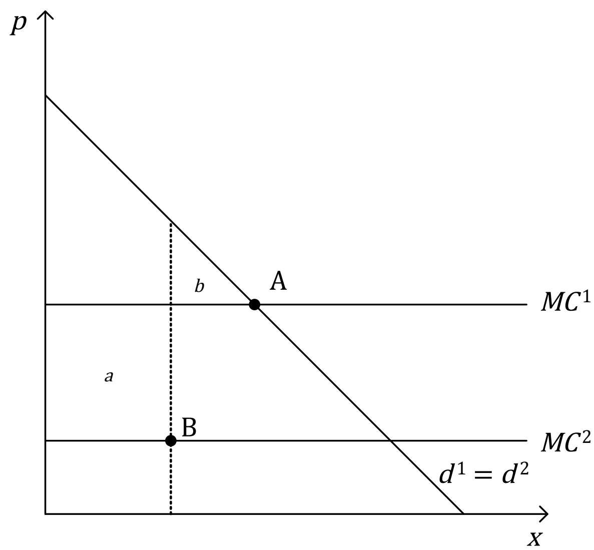

Figure 6: $d^1 = d^2$.

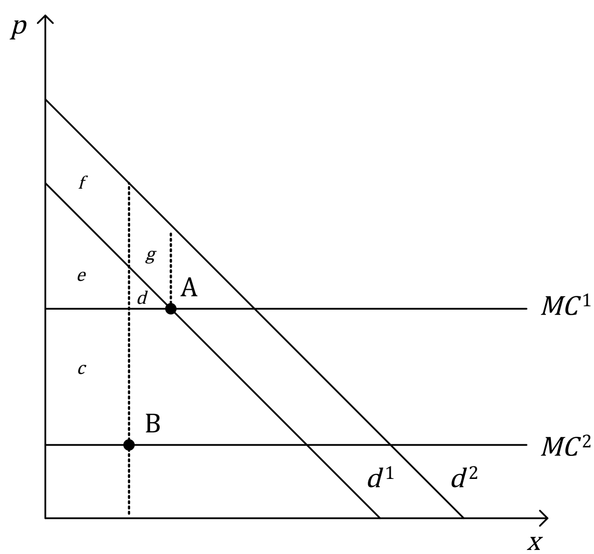

Figure 7: $d^2 > d^1$

[Section B]:

(a) Rather than starting our analysis by distinguishing between marginal costs of different types, our model from section B starts by specifying the probabilities θ and δ that type 1 and type 2 individuals will find themselves in the "bad state" that they are insuring against. Mapping this to our model from part A of this exercise, with type 1 and 2 defined as in part A (consumers of type 2 are relatively healthy), what is the relationship between δ and θ?

Answer:

With θ as the probability of the "bad state" for type 1 and δ the probability of the "bad state" for type 2, it must be that $\theta > \delta$.

(b) To fit the story with the model from section B, we can assume that what matters about bad health shocks is only the impact they have on consumption - and that taste are state independent. (We will relax this assumption in exercise 22.8). Suppose we can, for both types, write taste over risky gambles as von-Neumann Morgenstern expected utility functions that employ the same function $u(y)$ as "utility of consumption" (with consumption denoted y). Write out the expected utility functions for the two types.

Answer:

Let y_1 be consumption when sick and y_2 consumption when healthy (with, presumably, $y_1 < y_2$). We would get

$$U^1(y_1, y_2) = \theta u(y_1) + (1-\theta) u(y_2) \tag{22.11}$$

for type 1 consumers and

$$U^2(y_1, y_2) = \delta u(y_1) + (1-\delta) u(y_2) \tag{22.12}$$

(c) Does the fact that we can use the same $u(y)$ to express expected utilities for both types imply that the two types have the same taste over risky gambles - and thus the same demand for insurance?

Answer:

No, they do not. The expected utility functions U^1 and U^2 differ because the probabilities θ and δ differ. The expected utility functions in fact take the Cobb-Douglas form-with U^1 placing heavier emphasis on y_1 than U^2.

(d) If insurance companies could tell who is what type, they would (in a competitive equilibrium) simply charge a price equal to each type's marginal cost. How is this captured in the model developed in section B of the text?

Answer:

This is captured by the zero-profit (or actuarily fair) contract lines-which differ for the two types. In particular, for type 1, the zero-profit contracts are $p = \theta b$ (where p is the insurance premium and b is the insurance benefit), and type 2 they are $p = \delta b$ (a payment to the insurance company equals the expected value of their future claim against that policy).

(e) In the separating equilibrium we identified in part A, we had insurance companies providing the contract A that is efficient for type 1 individuals-but providing an inefficient contract B to type 2. Draw the model from section B of the text and illustrate the same A and B contracts. How are they exactly analogous to what we derived in part A?

Answer:

Figure 8 illustrates our model from part A and our analogous model from Section B of the text in figure 9. In figure 8, high cost types have higher demand for insurance levels x-and B is structured so that high cost types are indifferent between their efficient insurance choice A and the option intended for low cost types-B. This is done by insuring that the shaded areas in the panel are equal to one another-because that insures that the loss in surplus from switching between A and B is equal to the gain for type 1 consumers. That's exactly what we do in figure 9 for the new model. There, $p = \theta b$ represents the zero profit contract line for type 1 consumers and $p = \delta b$ represents the zero profit line for type 2 consumers who cost less and thus have more generous benefits for any insurance premium. The efficient insurance choice for type 1 consumers is A-the point at which they fully insure (given their risk aversion) at the actuarily fair rate. Type 1 consumers are then indifferent between all insurance contracts that fall on the indifference curve U^1 - which goes through A and B. Thus, B is the actuarily fair insurance contract aimed at type 2 consumers that makes type 1 consumers indifferent to A-just as it is in figure 8. In both cases, type 2 consumers are better off at B than A. The analogy extends even further: In figure 8, all the darkened insurance packages can be offered - any insurance level at price p^A and all insurance levels up to x^B at price p^B. The analogous darkened lines in figure 9 say the same thing: Any actuarily fair (or zero-profit) insurance contract aimed at high cost types can be offered, but only actuarily fair insurance packages aimed at type 2 to the left of B can be offered. If the restriction on what can be offered at the zero-profit rates for type 2 were not included, then type 1 individuals would buy at the type 2 price in both cases.

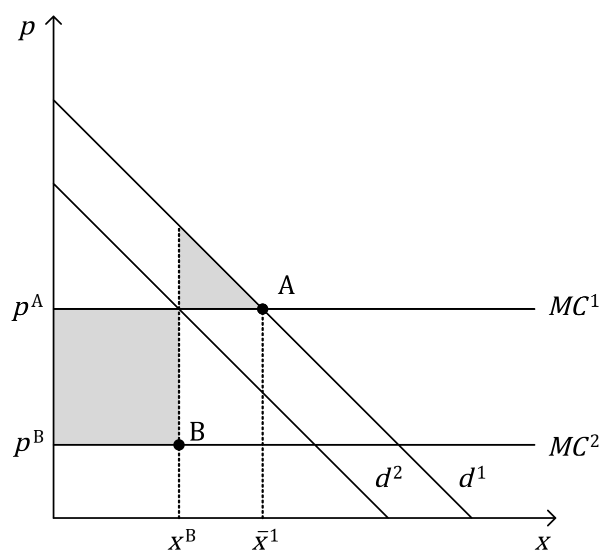

Figure 8. Model from Section A.

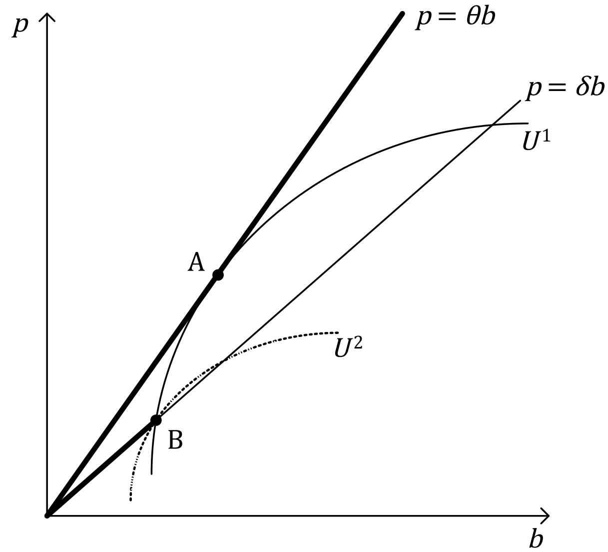

Figure 9: Actuarily Fair Insurance Policy.

(f) In part A we also investigated the possibility of a potential pooling price-or pooling contract-breaking the separating equilibrium in which A and B are offered. Illustrate in the different model here how the same factors are at play in determining whether such a pooling price or contract exists.

Answer:

In the model of figure 8, the crucial factor is whether the pooling price p^* is such that it would in fact attract type 2 consumers away from B. The closer p^* is to p^B, the more likely this is the case, as p^* gets closer to p^B when there are fewer type 1 consumers. In figure 9, the zero-profit pooling line falls between $p = \theta b$ and $p = \delta b$ - getting closer to the former as the fraction of type 1 consumers increases and getting closer to the latter as the fraction of type 1 consumers falls. The separating equilibrium cannot be broken in figure 4 unless the zero profit pooling line crosses the dashed indifference curve U^2, which is more likely to happen when there are fewer type 1 consumers. Once again, the conclusion and intuition is exactly the same.

(g) Evaluate again the True/False statement in section A part (g).

Answer:

This is true as already discussed in the previous part. Again, the two models give exactly the same punch line.

4. 22.8 Policy Application

Expanding Health Insurance Coverage: Some countries are struggling with the problem of expanding the fraction of the population that has good health insurance.

[Section A]:

Continue with the set-up first introduced in exercise 22.7 including the definition of x as the amount of insurance coverage bought by an individual. Assume throughout that demand for health insurance by the relatively healthy (type 2) is lower than demand for health insurance by the relatively sick (type 1) - i.e., $d^1 > d^2$.

(a) Illustrate d^1, d^2, MC^1 and MC^2- and identify the contracts A and B from exercise 22.7.

Answer:

This is done in figure 10.

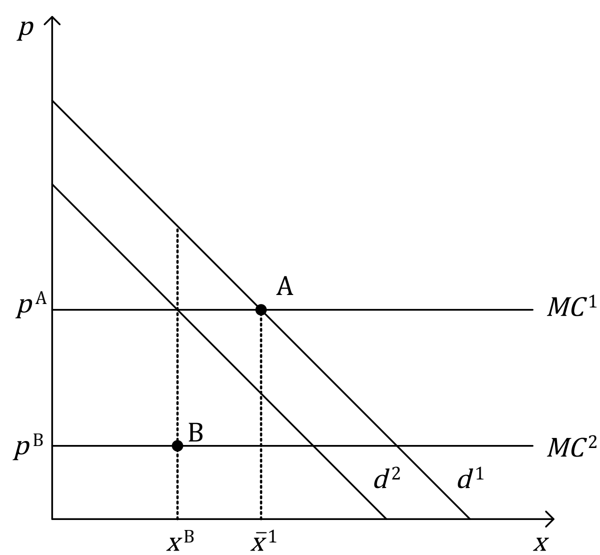

Figure 10: Contracts from Previous Problem.

(b) Suppose that the fraction of relatively sick (type 1) consumers is sufficiently high such that no pooling contract can keep this from being an equilibrium. On the MC^1 line, indicate all the contracts that can be offered in this equilibrium (even though only A is chosen). Similarly, indicate on the MC^2 line all the contracts that can be offered in this equilibrium (even though only B is chosen).

Answer:

This is done in figure 11 by indicating the contracts that can be offered in the separating equilibrium through bold lines. The entire MC^1 line can be offered, and the portion of the MC^2 line up to B can be offered. The pooling price p^* is indicated sufficiently high such that (g) is larger than (e) - with (g) being the consumer surplus that is lost by type 2 individuals if forced to buy at p^* rather than get B at p^B - and (e) is how much type 2 consumers would gain.

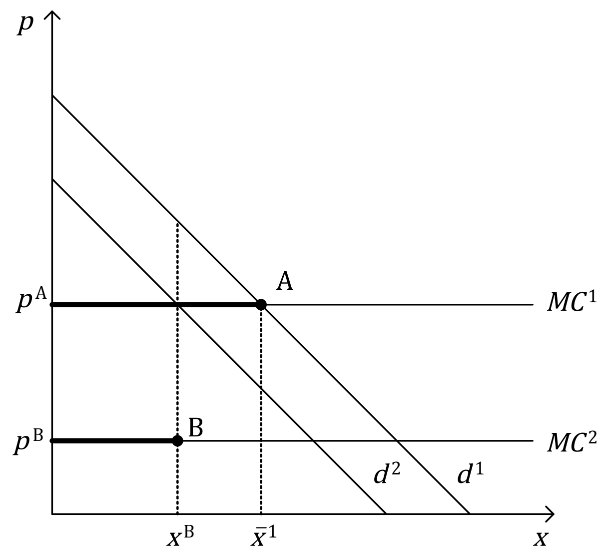

Figure 11: Contracts that can be offered.

(c) True or False: Insurance companies in this equilibrium restrict the amount of insurance that can be bought at the price $p = MC^2$ in order to keep type 1 consumers from buying at that price.

Answer:

This is true - if any policies to the right of B were sold, the type 1 consumers would want to buy at p^B- which would make p^B no longer a zero-profit price.

(d) Why is the resulting separating equilibrium inefficient? How big is the deadweight loss?

Answer:

Efficiency would require that each type buy insurance so long as the marginal benefit is larger than the marginal cost. For type 1 individuals, this happens until we reach A – and for type 2 individuals, this happens until we get to C. At the efficient allocations, type 1 individuals would get consumer surplus of $(a + b + c)$ while type 2 individuals would get consumer surplus (a+d+e+g+h). In the separating equilibrium, type 1 consumers consume exactly at the efficient level but type 2 consumers under-consume. As a result, their surplus is $(a + d + g)$ — or $(e + h)$ less than that would be at the efficient point C. The deadweight loss in the separating equilibrium is then the area $(e + h)$ multiplied by the number of type 2 consumers.

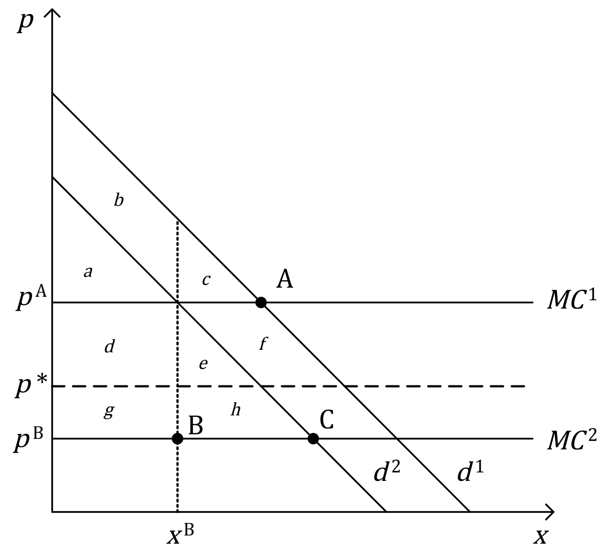

Figure 12: Deadweight Loss Analysis.

(e) Suppose that the government regulates this health insurance market in the following way: It identifies the zero-profit pooling price p^* and requires insurance companies to charge p^* for each unit of x but does not mandate how much x every consumer consumes. Illustrate in your graph how much insurance type 1 and type 2 consumers will consume under this policy? Does overall insurance coverage increase or decrease?

Answer:

This is illustrated in figure 13. Individual of type 1 initially consume x^A insurance at A — and increase consumption to x^1 when their price falls from p^A to p^*. Type 2 consumers initially consume x^B and increase their consumption to x^2 despite the fact that price increases — because previously they were prohibited from buying more at p^B. Thus, both types increase their insurance levels — implying that insurance coverage overall increases as a result of the regulation.

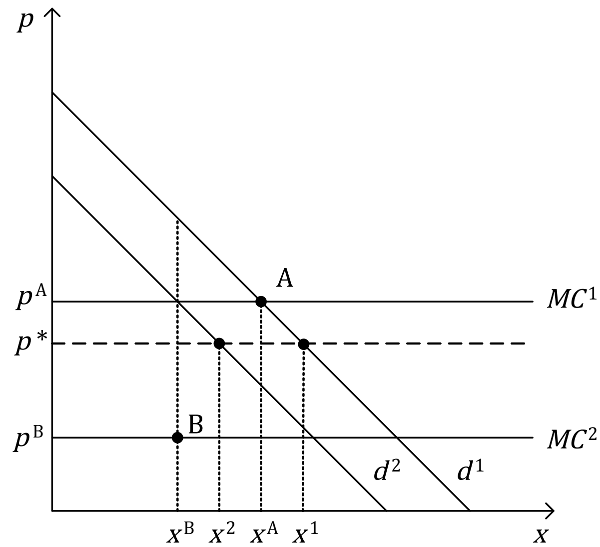

Figure 13: Regulated Pooling Price.

(f) How much does consumer surplus for each type change as a result of this regulation? Does overall surplus increase?

Answer:

Using the letters labeling the areas in figure 14, consumer surplus for type 1 consumers increases from $(a + b + c)$ to $(a + b + c + d + e + i + j)$; and consumer surplus for type 2 consumers falls from $(a+d+g)$ to $(a+d+e)$. (We know that type 2 consumer surplus falls because we know that (g) is greater than (e) — otherwise the separating equilibrium could not have been an equilibrium.)

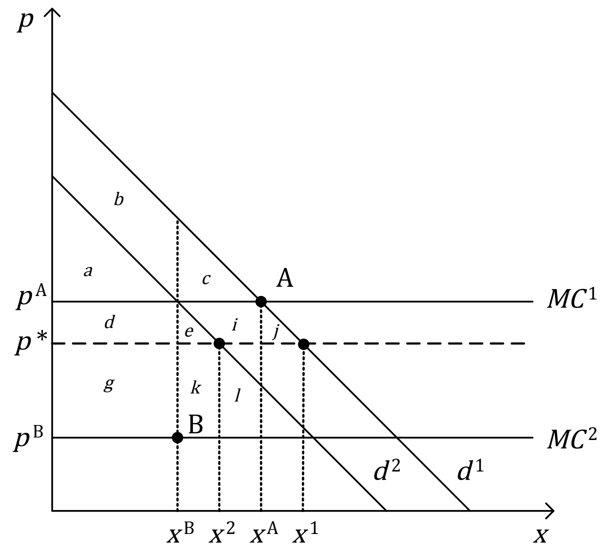

Figure 14: Consumer Surplus Changes.

Overall surplus then changes by

$$N^1 (d + e + i + j) - N^2 (g - e)$$

where N^1 is the number of type 1 consumers and N^2 is the number of type 2 consumers. It is almost certainly a positive change in overall consumer surplus despite the fact that type 2 consumers are made somewhat worse off.

(g) True or False: This policy is efficiency enhancing but does not lead to efficiency.

Answer:

This is likely to be true. Based on our analysis above, overall consumer surplus almost certainly increase — and firms make zero profit before and after. It is not, however, the case that the result is efficient. In fact, type 1 consumers now over consume insurance (since they were initially consume the efficient quantity) — while type 2 consumers are still underconsuming since they are paying more than their marginal cost.

(h) It may be difficult for the government to implement the above price regulation p^* because it does not have enough information to do so. Some have suggested that the government instead set the insurance level to some $\bar{x}$ and then let insurance companies compete on pricing this insurance level. Could you suggest, in a new graph, a level of $\bar{x}$ that will result in greater efficiency than regulating price? (You need to do this on a new graph for the following reason: If the government sets $\bar{x}$ between the amounts consumed by type 1 and 2 under the zero-profit price regulation p^*, the resulting competitive price $\bar{p}$ should be lower than p^*)?

Answer:

This is done in figure 15 where $\bar{x}$ is set between x^2 and x^1 from figure 13. Where exactly $\bar{x}$ ends up will depend on the relationship between the intersection of MC^1 with d^1 (i.e. point A) and the intersection of MC^2 with d^2 (i.e. point C) where the efficient insurance levels for type 1 and 2 consumers lies. It will also depend on the number of each type in the economy. But you should be able to see in the graph that D results in substantially less deadweight loss — equal to the small darkened triangles above and below D for the graph we have drawn. These emerge because, as we have drawn this, D has slightly more than the efficient insurance level for type 1 and slightly less than the efficient insurance level for type 2. In the special case where A lies at the same level of x as C — i.e. where A lies right above C, the government can set $\bar{x}$ to be equal to that level and achieve full efficiency. It is still the case that type 1 consumers are subsidized by type 2 consumers, but that is a simple transfer from one type to the other. The crucial efficiency gain comes from moving both their consumptions closer to the amounts that are efficient for them. (The competitive price $\bar{p}$ when the insurance level is fixed at $\bar{x}$ is lower than p^* from figure 13 because low cost types now buy more insurance while high cost types buy less - thus making it cheaper to service them jointly).

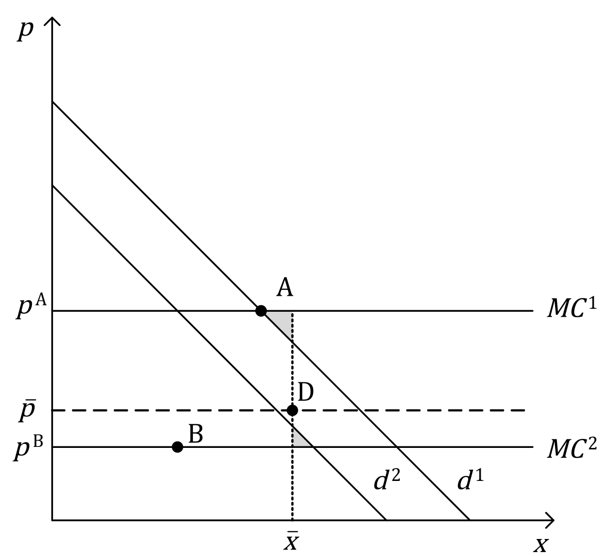

Figure 15: Quantity Regulation.

[Section B]: Now consider again whether we can find analogous conclusions in the model from Section B as modified in exercise 22.7.

(a) Interpreting the model as in exercise 22.7, illustrate the separating equilibrium in a graph with the insurance benefit b on the horizontal axis and the insurance premium p on the vertical axis. Include in your graph a zero-profit pooling contract line that makes the separation of types an equilibrium outcome.

Answer:

This is illustrated in figure 16. The zero profit pooling line lies to the northwest of the dashed indifference curve U^2 for type 2 individuals who consume B — and thus would not be chosen over B by type 2 consumers. This implies we have sufficiently many type 1 consumers to make the pooling contract line lie sufficiently far up in the graph.

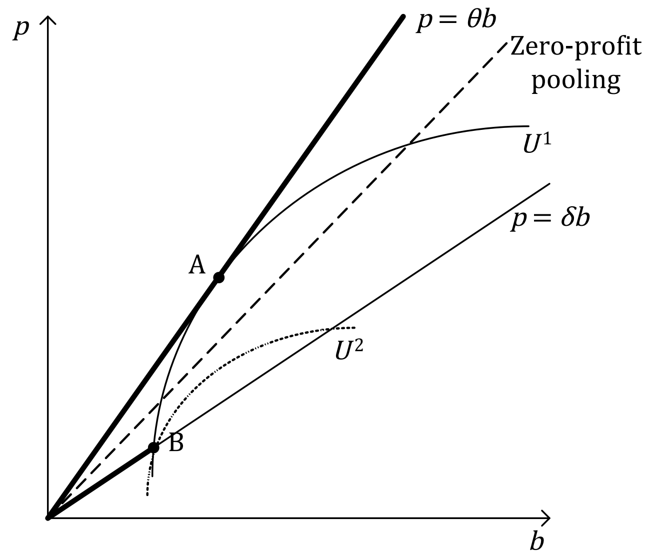

Figure 16: Separating Equilibrium.

(b) How would you interpret the price regulation proposed in Section A, part (e) in the context of this model?

Answer:

The price regulation here would mean that the government restricts the set of contracts that can be sold to those that would result in zero profit if everyone bought at the same insurance terms. Note that this might imply a zero profit line different from what we concluded in the text where we assumed a single pooling contract was offered in the pooling equilibrium. If we allow the two types to choose different contracts that are structured on the same term (i.e. the same relationship between p and b), the zero profit line will depend not only on the fraction of the population that falls into each type category but also the insurance packages that are chosen.

(c) Illustrate in your graph how insurance coverage will increase if the government implements this policy.

Answer:

We illustrate this in figure 17. With typically shaped indifference curves (which are what emerges from expected utility theory), both income and substitution effects suggest that type 1 individuals will move south-east from A — to a contract like D. (The income effect is positive — implying more coverage will be bought, and the substitution effect is also positive since insurance for type 1 consumers has become cheaper.) Individuals of type 2 were previously restricted to B — and it is because of this restriction that their new choice — some contract like C — will have more insurance coverage. But it will be less than full insurance because the terms are not actuarily fair from a type 2 perspective.

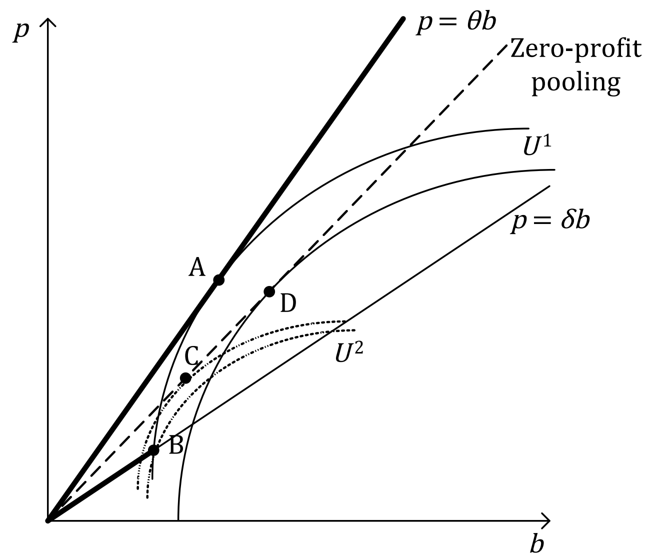

Figure 17: Regulation of Prices.

(d) Now consider the same problem in a graph with y_2 — the consumption level when healthy — on the horizontal axis and y_1 — the consumption level when sick — on the vertical axis. Illustrate the "endowment point" $E = (\bar{y}_1, \bar{y}_2)$ that both types face in the absence of insurance.

Answer:

This is done in figure 18.

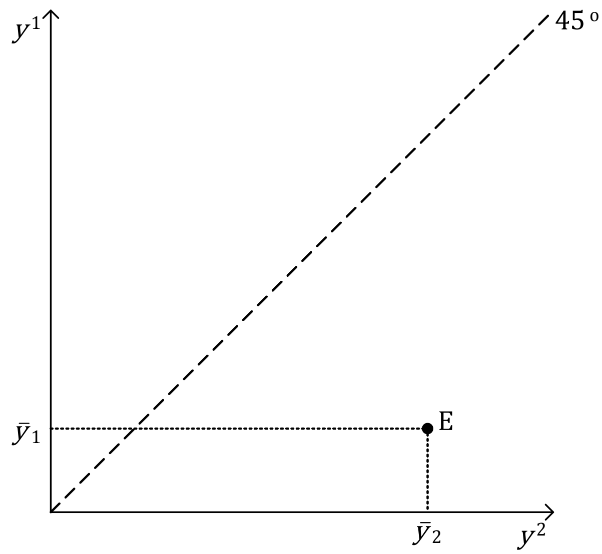

Figure 18: Consumption Levels.

(e) Illustrate the actuarily fair insurance contracts for type 1 and 2 consumers. Then indicate where the separating equilibrium contracts A and B lie assuming state-independent tastes.

Answer:

This is done in figure 19. The shallower solid line through E is the actuarily fair insurance line for type 1 — and the steeper solid line through E is the actuarily fair insurance line for type 2. (It has to be shallower for the high cost types because their premium for the same benefit level are higher — implying that their increase in y_1 for any decrease in y_2 is less.) With state-independent tastes, individuals will fully insure under actuarily fair terms — which implies that type 1 individuals will choose A on the 45 degree line. To keep type 1 consumers from settling on the type 2 actuarily fair contract line, insurance companies cannot offer any contract higher than B on the type 2 actuarily fair line — because B gives type 1's the same utility as A but anything higher would make them switch away from A.

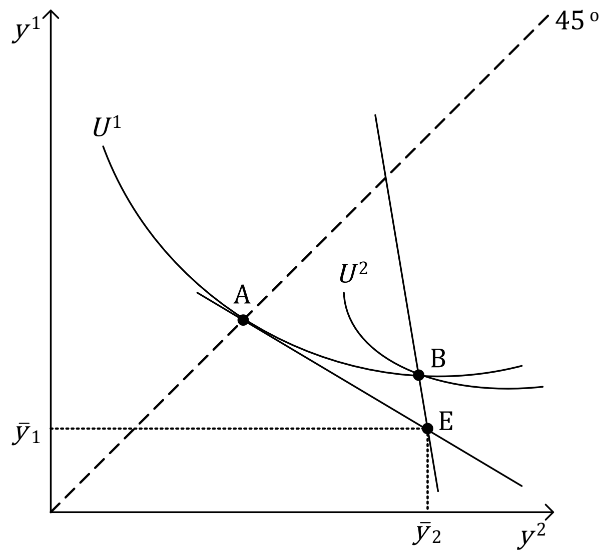

Figure 19. Actuarily Fair Contracts.

(f) Introduce into your graph a zero - profit pooling contract line such that the separating equilibrium is indeed an equilibrium. Then illustrate how the proposed government regulation affects the choices of both types of consumers.

Answer:

This is done in figure 20. The pooling line is the bold dashed line through E between the actuarily fair contract lines for the two types. In order for the separating equilibrium to emerge, it has to be the case that this pooling contract line lies below the indifference curve U^2 for type 2 that passes through B. If so, type 2 consumers will not want to switch to the pooling line from B. If the government regulation goes into effect and only contracts on the pooling line are offered, type 2 individuals will choose some contract C to the right of the 45 degree line — because the dashed indifference curves have the slope of the steep actuarily fair contract line along the 45 degree line and are shallower to the left where they must be tangent to the shallower pooling contracts line. Type 1 individuals, on the other hand, will end up at some contract like D on the opposite side of the 45 degree line where their indifference curves are sufficiently steep (relative to what they are along the 45 degree line). Type 1 individuals thus over-insure while type 2 individuals under-insure — but both get more insurance than they had at A and B before the regulation.

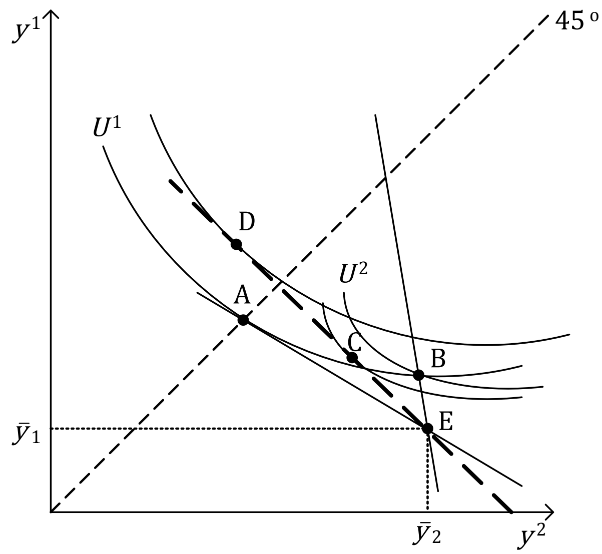

Figure 20: Price Regulation.

(g) Suppose that, instead of regulating price, the government set an insurance benefit level b (as in section A part (h)) and then allowed the competitive price to emerge. Where in your graph would the resulting contract lie if it fully insures both types?

Answer:

Since type 1 consumers would now not over-insure — and type 2 consumers would not under-insure, it should cost less to provide this full insurance to both types with the same contract that is competitively priced. Since the contract has full insurance, it must lie on the 45-degree line, and since it costs less than the price regulation, it must lie above the dashed zero-profit contract line in figure 21. It must also lie below the U^2 indifference curve — otherwise the separating equilibrium could not have been an equilibrium. Contract F in the figure satisfies all these conditions.

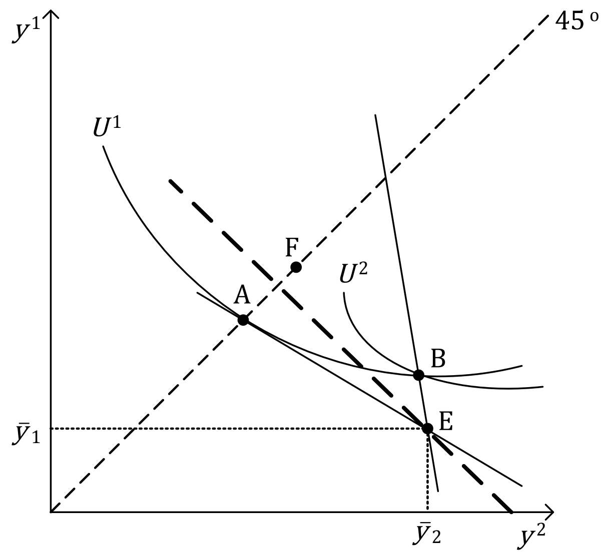

Figure 21: Quantity Regulation.

(h) Suppose next that tastes were state-dependent — with $u_1(y)$ and $u_2(y)$ the functions (for evaluating consumption when sick and when healthy) that we need to use in order to arrive at our expected utility function. If u_1 and u_2 are the same for both consumer types, does our main conclusion that the price regulation will cause an increase in insurance coverage change?

Answer:

No — the logic will be exactly as in figure 20, except that A no longer needs to be on the 45 degree line, and C and D don't necessarily need to be where they are relative to the 45 degree line. But the direction of the changes is the same.