Cheap Talk Games with three types

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- So far, in all signaling games we considered. . .
  - There were two types for the privately informed player,
    - e.g., high and low productivity, beneficial or useless test, etc.,
  - and the privately informed player only had two possible messages to choose from.
    - e.g., acquire/not acquire college education, recommend/not recommend the MRI test, etc.
- What if we extend our analysis to signaling games in which:
  - the privately informed player has **three** possible types, and
  - he/she has **three** available messages to choose from.

Example 2: Stock recommendations

- Let us now examine the transmission of information between:
  - An analyst who privately observes the future prospects of a stock, and
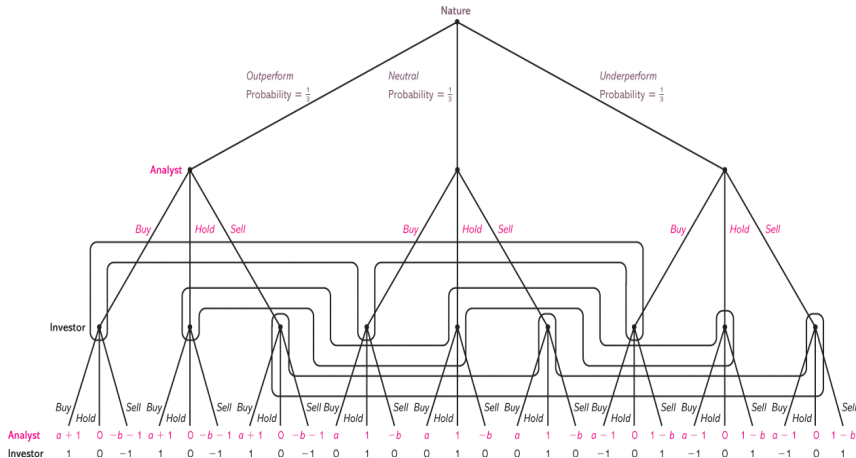 - An investor who does not observe such information.

- We will assume that your analyst is none of these two guys, otherwise you wouldn't be paying attention to his messages (since they are never informative!)

Example 2: Stock recommendations

- First, nature determines whether the stocks of a particular firm will:
    - outperform/underperform/be neutral to average stock market prices.
- This information is only observed by an analyst, after weeks of research, but not by the investor holding the stock.
- The analyst then decides to recommend buy/hold/sell to the investor.
- Finally, the investor observes the recommendation from his analyst, and decides whether to buy/hold/sell.

- Figure.→

Example 2: Stock recommendations

- What about the payoffs?

Example 2: Stock recommendations

- Let's first examine the investor's payoff (last column)

State	Action	Analyst's Payoff	Investor's Payoff
Outperform	Buy	$a + 1$	1
Outperform	Hold	0	0
Outperform	Sell	$-b - 1$	-1
Neutral	Buy	a	0
Neutral	Hold	1	1
Neutral	Sell	$-b$	0
Underperform	Buy	$a - 1$	-1
Underperform	Hold	0	0
Underperform	Sell	$1 - b$	1

Investment banking (column header)

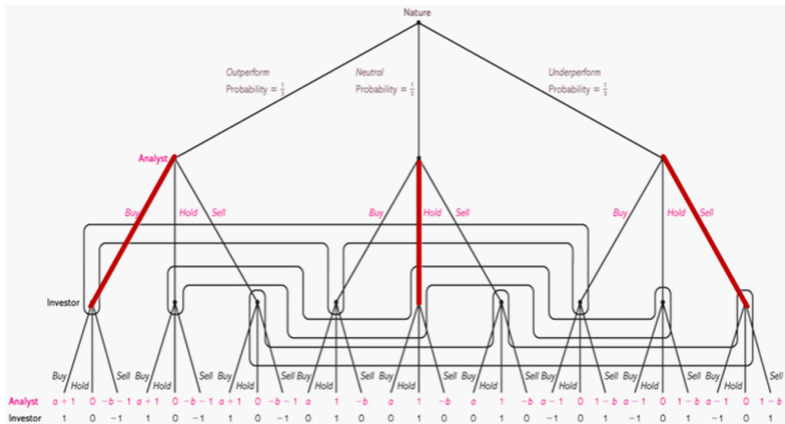Bulls (rows: Outperform), Bears (rows: Underperform)

Payoff of 1 from the best action
Payoff of -1 from the worst action
Payoff of 0 from the neutral action

- a: benefit for your analyst when you buy stock
- b: cost for your analyst when you sell stock

Example 2: Stock recommendations

- Let us check for the existence of a <u>separating</u> equilibrium where:
    - Information is perfectly transmitted from the analyst to the investor.
    - That is, the analyst recommends:
        - Buy only when the prospect of the stock are Good.
        - Hold only when the prospect of the stock are Neutral.
        - Sell only when the prospect of the stock are Bad.
- This strategy profile is represented in the following figure.

Separating equilibrium (fully informative)
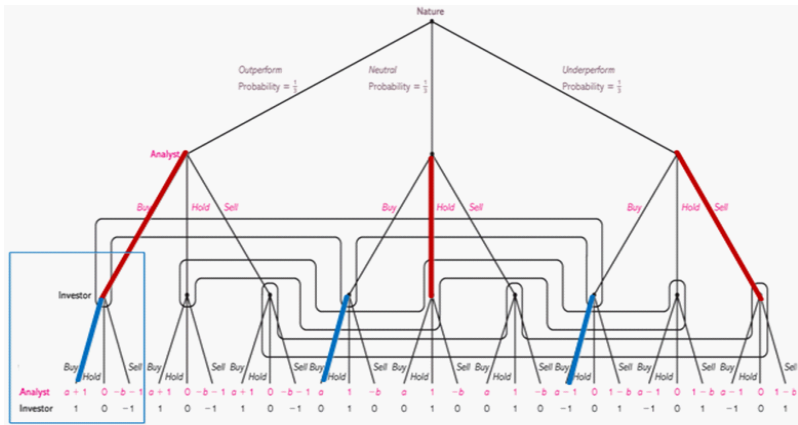
Example 2: Stock recommendations

- **Investor:**
- After observing a recommendation of "Buy" from his analyst, the investor responds Buying since

$$1 \;>\; 0 \text{ (if Hold), and}$$
$$1 \;>\; -1 \text{ (if Sell)}$$

- We hence shade the branch where the investor responds with "Buy" every time he observes a recommendation of Buy.
  - Blue shaded branch in the following tree.

After observing a recommendation of "Buy"

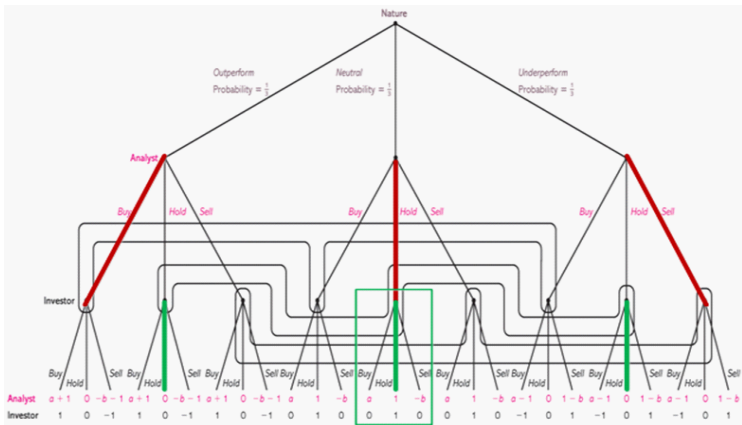# Example 2: Stock recommendations

- **Investor (responder in this game):**
- After observing a recommendation of <u>"Hold"</u> from his analyst, the investor responds Holding since

$$1 > 0 \text{ (if Buy), and}$$
$$1 > 0 \text{ (if Sell)}$$

- We hence shade the branch where the investor responds with "Hold" every time he observes a recommendation of Hold.
    - Green shaded branch in the following tree.

- **Investor (responder in this game):**
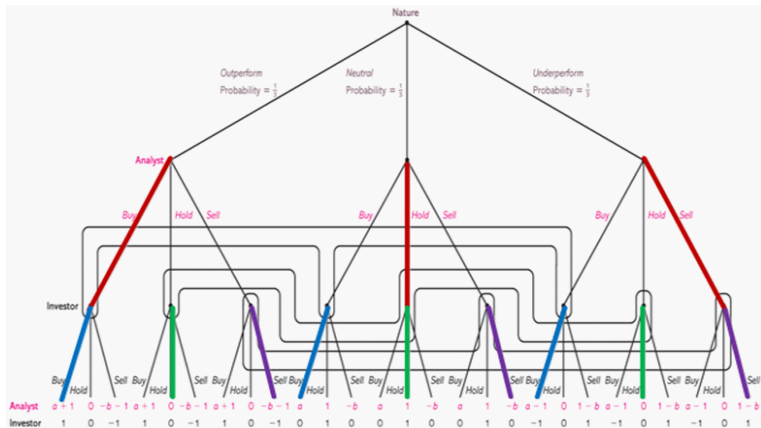- After observing a recommendation of "Sell" from his analyst, the investor responds Selling since

$$1 > 0 \text{ (if Hold), and}$$
$$1 > -1 \quad \text{(if Buy)}$$

- We hence shade the branch where the investor responds with "Sell" every time he observes a recommendation of Sell.
- Purple shaded branch in the following tree.

After observing a recommendation of "Sell"

Summarizing the investor's optimal responses found above...

Example 2: Stock recommendations

- **Analyst:**
- If the stock will *outperform* the market, then the analyst recommends to Buy if

$$a + 1 > 0 \text{ (if Hold), and}$$
$$a + 1 > -b - 1 \text{ (if Sell)}$$

  which simplify to $a > -1$ and $a + b > -2$.
- Then, both conditions hold since $a, b > 0$ by definition.

- **Analyst:**
- If the stock will be *neutral* relative to the market, then the analyst recommends to Hold if

$$\text{Always holds since } b > 0 \;\; \rightarrow \quad \boxed{1 > a} \;\; \text{(if Buy), and}$$
$$1 > -b \;\; \text{(if Sell)}$$

- **Analyst:**
- If the stock will *underperform* the market, then the analyst recommends to Sell if

$$1 - b \quad > \quad a - 1 \implies \boxed{2 > a + b} \text{ (if Buy), and}$$

$$1 - b \quad > \quad 0 \implies \boxed{1 > b} \quad \text{(if Hold)}$$

Example 2: Stock recommendations

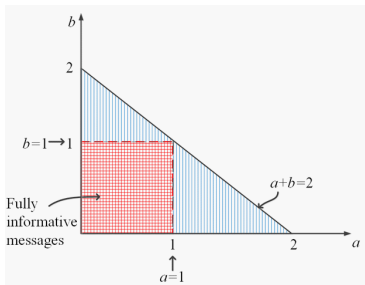- The conditions that must be satisfied for a fully informative separating equilibrium to exist are hence

$$
\begin{aligned}
a + b &< 2 \\
a &< 1, \text{ and} \\
b &< 1
\end{aligned}
$$

Let us represent all three conditions in the following figure.

Conditions for a separating equilibrium in the stock recommendations cheap talk game:
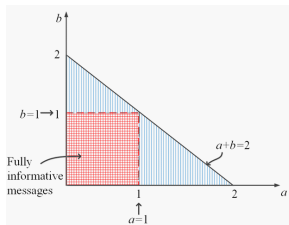

- $a + b < 2$, $a < 1$, $b < 1$
- a: bonus for the analyst if the investor buys shares
- b: penalty for the analyst if the investor sells shares
- a and b being low $\Rightarrow$ preferences of the investor and his analyst (or his investment bank) are very similar.

- This suggests that the fully informative separating equilibrium can be sustained if:
  - a and b are both low.
  - Intuitively, this implies that the preferences of the analyst and investor are very similar, and thus communication is easy.

Example 2: Stock recommendations

- **What happens if, instead, $b > 1$?**
    - Graphically, this occurs in the upper triangle of the figure.
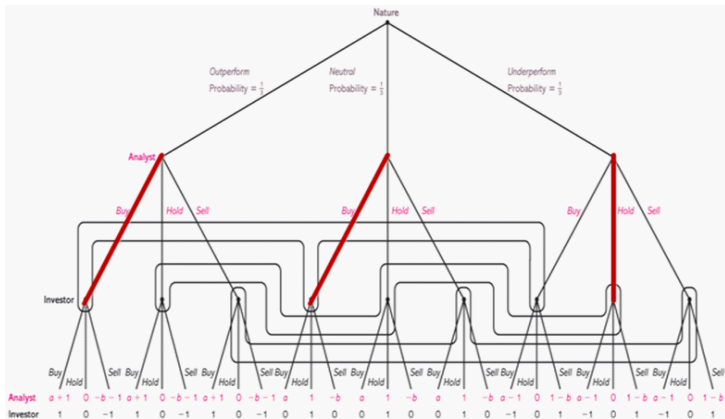

    - Intuitively, $b > 1$ indicates that it is highly detrimental for the analyst to induce clients to sell.
    - We know that the fully informative separating equilibrium cannot be sustained.

- But, can we have at least some information transmission?
  - Yes, we can have a partially separating equilibrium where:
    - The analyst recommends to Buy both when the prospects are Good and Neutral, but
    - Recommends to Hold when the prospects are Bad.
  - (See next figure).

Partially separating equilibrium

Example 2: Stock recommendations

- **Beliefs:**
- Not so immediate!
- Beliefs will often be more involved than in signaling games with only two types and messages. (here we have 3 types and messages)

- **Beliefs:**
- After observing a recommendation of "Buy" from his analyst,

$$\text{prob(outperform|Buy)} = \frac{\frac{1}{3}1}{\frac{1}{3}1 + \frac{1}{3}1} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

since the recommendation of "Buy" may originate from an analyst informed about Good or Neutral prospects.

$$\text{prob(neutral|Buy)} = \frac{\frac{1}{3}1}{\frac{1}{3}1 + \frac{1}{3}1} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$\text{prob(underperform|Buy)} = 0$$

since the recommendation of "Buy" does not originate from an analyst informed about Bad prospects.

- **Beliefs:**
- After observing a recommendation of "Hold" from his analyst,

$$\text{prob(outperform|Hold)} = 0$$

$$\text{prob(neutral|Hold)} = 0$$

$$\text{prob(underperform|Hold)} = 1$$

since the recommendation of "Hold" can only originate from an analyst informed about Bad prospects.

Example 2: Stock recommendations

- **Beliefs:**
- Finally, after observing a recommendation of "Sell" from his analyst (off-the-equilibrium path),

$$\text{prob(outperform}|\text{Hold}) = \gamma_1$$

$$\text{prob(neutral}|\text{Hold}) = \gamma_2$$

$$\text{prob(underperform}|\text{Hold}) = 1 - \gamma_1 - \gamma_2$$

- For simplicity, Harrington assumes that $\gamma_1 = \gamma_2 = 0$
  - (Intuitively, if "Hold" signifies that the stock will underperform, then "Sell" should convey the same information, or worse!).

- Remember that in any case this is an assumption about off-the-equilibrium beliefs.

- **Investor: (Responder in this game)**
- After observing the recommendation of "Buy" from his analyst, the investor obtains

$$\frac{1}{2}1 + \frac{1}{2}0 = \frac{1}{2} \text{ from buying}$$

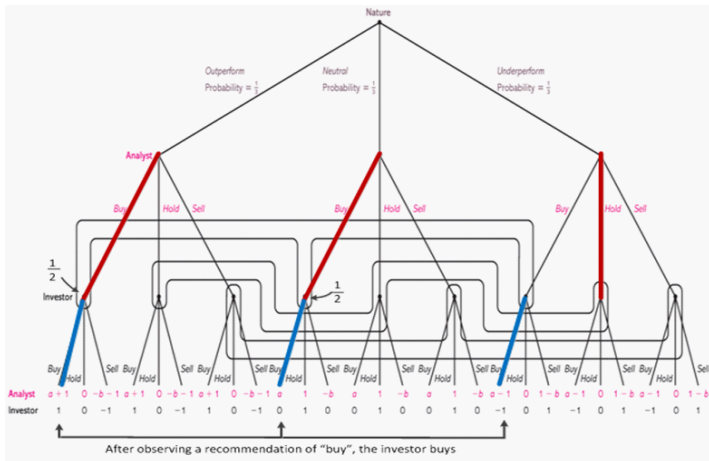$$\frac{1}{2}0 + \frac{1}{2}1 = \frac{1}{2} \text{ from holding}$$

$$\frac{1}{2}(-1) + \frac{1}{2}0 = -\frac{1}{2} \text{ from selling}$$

and hence the investor Buys.

- (Shaded in the next figure).

Partially separating equilibrium

- After observing the recommendation of "buy"...

- **Investor (cont.):**
- After observing the recommendation of "Hold" from his analyst, the investor sells since

$$1 \quad > \quad 0 \text{ (if the investor holds)}$$
$$1 \quad > \quad -1 \text{ (if the investor sells)}$$

  given that the investor puts <u>full probability</u> on "underperform."
- (Shaded in the next figure).

- After observing the recommendation of "hold". . .

- **Investor (cont.):**
- After observing the recommendation of "sell" from his analyst, the investor sells since

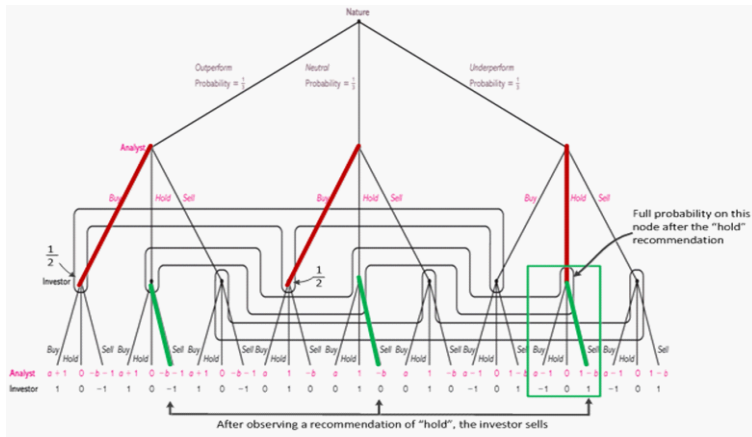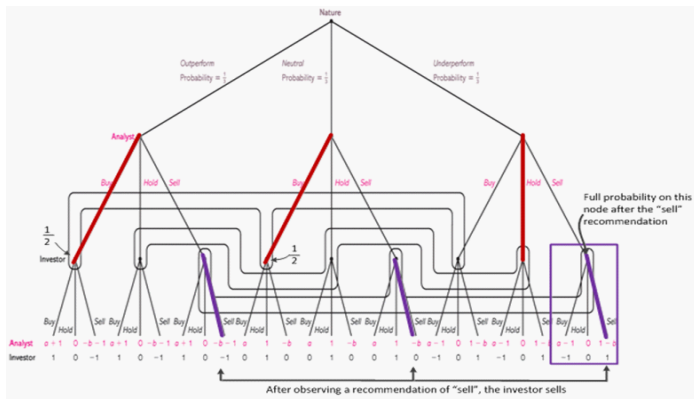$$1 \quad > \quad 0 \text{ (if the investor holds)}$$
$$1 \quad > \quad -1 \text{ (if the investor sells)}$$

  given that the investor puts full probability on "underperform."
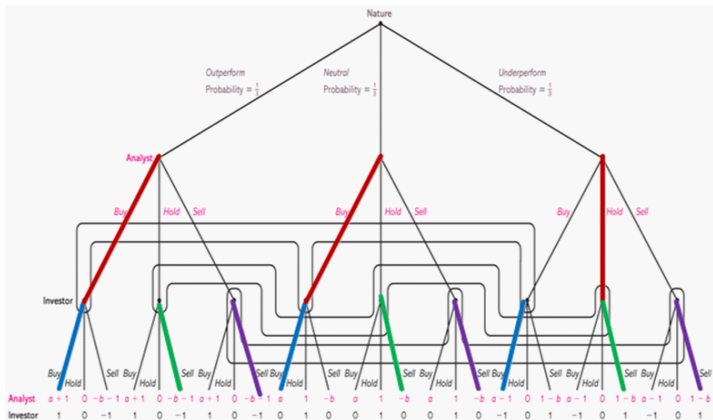- (Shaded in the next figure).

Partially separating equilibrium

- After observing the recommendation of "sell". . .

Partially separating equilibrium

- Summarizing the optimal responses of the investor we found above. . .

Example 2: Stock recommendations

- **Analyst:**
- If the stock will *outperform* the market, then the analyst recommends to Buy if

$$a + 1 > -b - 1 \text{ (if Hold), and}$$
$$a + 1 > -b - 1 \text{ (if Sell)}$$

which simplify to $\not{a} + b > -2$ and $a + b > -2$, respectively.

  - Hence, these two conditions hold given that $a > 0, b > 0$ by definition.

- **Analyst:**
- If the stock will be *neutral* relative to the market, then the analyst recommends to Buy as well if

$$a > -b \ \text{ (if Hold), and} \left.\begin{array}{c} \\ \\ \end{array}\right\} \ \begin{array}{l} \text{satisfied, since} \\ a > 0, b > 0 \end{array}$$

$$a > -b \ \text{ (if Sell)}$$

- **Analyst:**
- If the stock will *underperform* the market, then the analyst recommends to Hold if

$$1 - b > a - 1 \implies \boxed{2 > a + b} \text{ (if Buy), and}$$
$$1 - b = 1 - b \quad \text{(if Hold)}$$

Example 2: Stock recommendations

- As the interests of investor and analyst diverge more (higher a and b). information becomes more difficult to transmit between the parties.

- Only condition now was $a + b < 2$

- We can hence conclude that:
  - When preferences are very similar ($a, b < 1$), a fully informative separating equilibrium can be sustained, whereas...
  - When preferences are not so similar ($a + b < 2$, but either $a > 1$ or $b > 1$), only a partially informative equilibrium can be supported.

- Note that in the latter equilibrium there is no deception:
  - The investor knows that a "Hold" recommendation means that he should sell.
  - However, the information content of recommendations deteriorates:
    - The analyst recommended "Buy," but the investor doesn't know if the stock with be outperforming the market or be neutral.

- Quote from "The effect of investment Banking Relationships on Financial Analysts' Earnings Forecasts and Investment Recommendations," by Amitabh Dugar and Siva Nathan, *Contemporary Accounting Research*, 12(1995), pp. 131-160.
  - Analysts are uncomfortable making sell recommendations on particular stocks. Often the analysis will cop out with a euphemism: the hold rating. But now hold is getting such a bad name that different terminology is gaining favor on the street. Like strong hold... Just what does strong hold mean? Since most investors assume a hold is really a polite way to say well, does strong hold actually mean strong sell?

Example 2: Stock recommendations

Recommendation	Frequency	Cumulative Percentage
Strong buy	38	15.2%
Buy	128	66.4%
Hold	70	94.4%
Sell	14	100.0%
Strong sell	⟶ 0	100.0%

1. No "strong sell", and what does it mean?
2. Few "sell" recommendation
3. This strategy profile resembles the "partially informative separating equilibrium" described above, which emerges when for instance $a > 1$ but $a + b < 2$? Intuition: too big bonuses.