Cheap Talk Games with two types

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So far messages were costly...

- *Example*:
  - Acquiring additional years of education was a costly signal (message) potential employees use to reveal their innate productivity (type).
  - Duel in the Wild West: Drink beer for breakfast as a signal of your strengh. Uff!
- What if messages were costless?
  - Talk is cheap!

Costless messages

- Examples:
  - Your doctor tells you that you should go through an expensive MRI test.
  - Your investment analyst recommends you to buy/sell stocks of a particular company.
  - A lobbyist (expert on a particular topic) informs a politician about the current conditions in a given industry, high schools, natural parks, etc.
- Are there situations in which you would believe that whatever comes out of his/her mouth is the truth? Yes!
- **Reading reference:** Harrington, pp. 359-373.

Costless messages

- We are interested in "information transmission":
- That is, we are searching for separating equilibria where:
  - the doctor tells you to take the test only when necessary;
  - your investment analyst recommends to buy only when the prospects of the firm are good;
  - the lobbyist informs the politician about the true state of the industry, or any other topic.

Stages in a cheap talk game:

- First, nature chooses the sender's type.
- Second, the sender learns her type and chooses a message.
- Third, the receiver observes the sender's message, modifies his beliefs about the sender's type, and chooses an action (response).

Payoffs in a cheap talk game:

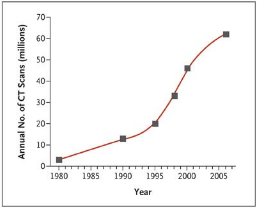
- **Sender:**
  - The payoff of the sender depends on his type θ, and on the action of the receiver (response), a_R,
  - His payoff does not depend on his message,
  - That is, the sender's utility function is of the form $u_S(a_R, \theta)$, where his message m is not an argument.

Payoffs in a cheap talk game:

- **Receiver:**
  - Similarly, the payoff of the receiver depends on his own response to the sender's message and on the sender's type (e.g., it depends on the true state of the world).
  - However, his payoff does not depend on the particular message he receiver from the sender.
  - That is, the receiver's utility function is of the form $u_R(a_R, \theta)$, where the message he received m is not an argument.
  - The message only affects his beliefs (potentially affecting his response) but not his payoff, i.e., $u_R(a_R, \theta)$.

Example 1: Defensive medicine

- "In a recent survey of physicians 93% reported altering their clinical behavior...
  - Of them, 43% reported using imaging technology (such as MRIs) in clinically unnecessary circumstances."



- From the article:
  - "Defensive Medicine Among High-Risk Specialist Physicians in a Volatile Malpractice Environment," *Journal of the American Medical Association*, 293 (2005), pp. 2609-17.

Example 1: Defensive medicine

- Nature moves first determining the value of a test (MRI) to a patient:
  - with prob $\frac{1}{3}$ the test is beneficial,
  - with prob $\frac{2}{3}$ it is useless for his condition.
- The value of the test is only known by the doctor.
- After determining the value of the test for the patient, the doctor decides to recommend/not recommend the test.
- The patient then chooses whether to undertake the test or not, after receiving his doctor's recommendation/non-recommendation of taking the test.

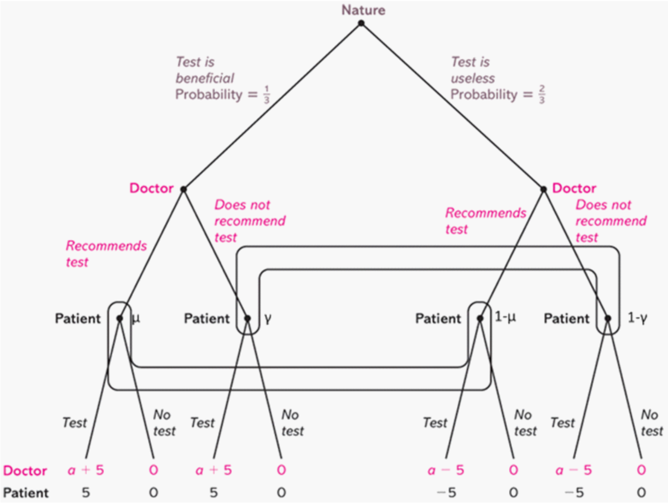
Example 1: Defensive medicine

- **Payoffs:**
- Regarding the *patient*, he only wants to take the test when the test is beneficial:
  - If the patient takes the test when such test was beneficial, his payoff is 5.
  - If the patient takes the test when such test was useless, his payoff is -5 (time, money, etc.).
  - If the patient doesn't take the test, his payoff is 0, regardless of its true benefits.
    - (This is a simplification, but you could modify the game so that the patient's payoff is 0 when he doesn't take the test and it was useless, but -10 when he doesn't take the test but such a test was beneficial).

Example 1: Defensive medicine

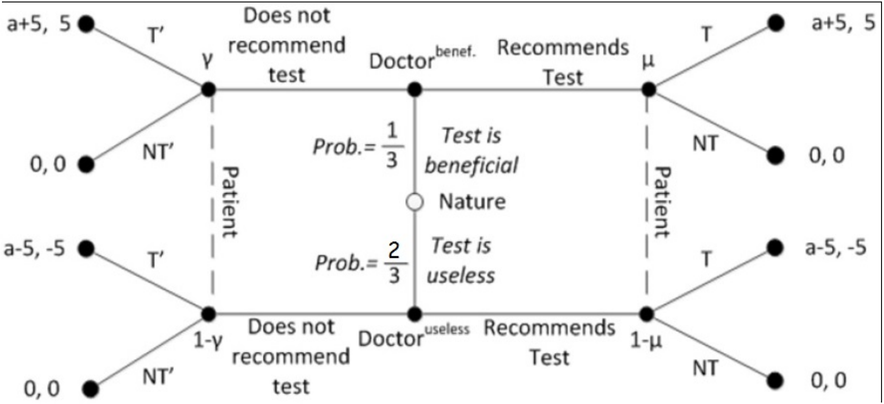
- **Payoffs:**
- Regarding the *doctor*, his payoffs are:
  - If the patient takes the test when such test was beneficial, the doctor's payoff is $a+5$.
  - If the patient takes the test when such test was useless, the doctor's payoff is $a-5$.
  - If the patient doesn't take the test, the doctor's payoff is 0.
- Notice that if $a=0$, then the interests of the doctor and the patient coincide, i.e., preferences are aligned.
- The doctor has a bias towards conducting tests (he obtains a benefit a) even if they are not really necessary.
  - e.g., doing so might avoid potential malpractice suits.
- Figure

Example 1: Defensive medicine



Example 1: Defensive medicine

- We can alternatively depict the game tree in a more familiar way as follows:



Example 1: Defensive medicine

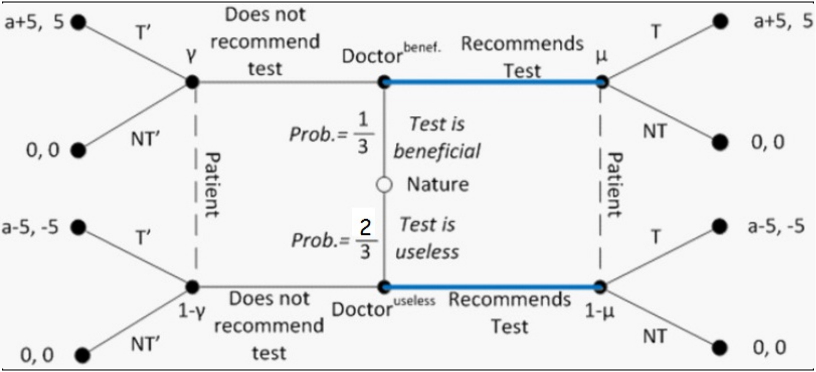
- Let us start checking if a pooling equilibrium can be sustained.
  - In the literature on cheap-talk games, the pooling equilibrium is often referred to as "babbling" equilibrium, since all messages from the sender are uninformative.
  - Your doctor is babbling!: anything that comes out of his mouth is uninformative for you.
  - You can think about him as the Swedish Chef in The Muppets (watch a video in YouTube).

Example 1: Defensive medicine

- Let us check the pooling strategy profile where the doctor recommends to take the test regardless of its true benefits.
  - (See next figure where, as usual, we shaded the appropriate branches).

Example 1: Defensive medicine

- **Pooling (babbling) equilibrium:** The doctor recommends the test regardless of its benefits



Example 1: Defensive medicine

- **Patient (responder):**
- *Beliefs:*
  - After observing that the doctor "Recommends test", the patient's beliefs coincide with the priors, i.e., $\mu = \frac{1}{3}$.

Example 1: Defensive medicine

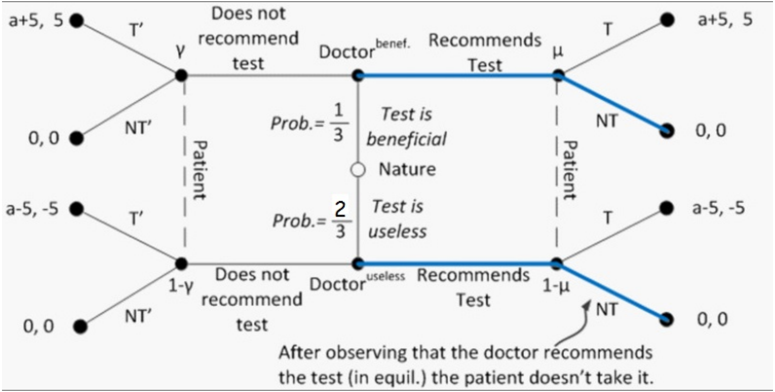
- **Patient (responder):**
- *Response:*
  - Hence, the patient decides whether or not to take the test by comparing the expected utilities:

$$EU_{Patient}\left(T \middle| Recomm.\right) \gtrless EU_{Patient}\left(NT \middle| Recomm.\right)$$

$$\frac{1}{3}5 + \frac{2}{3}(-5) > \frac{1}{3}0 + \frac{2}{3}0 \Longleftrightarrow -\frac{5}{3} < 0$$

  inducing the patient to Not take the test.
  - (This branch is shaded in the next figure)

- **Pooling (babbling) equilibrium** (con't)



Example 1: Defensive medicine

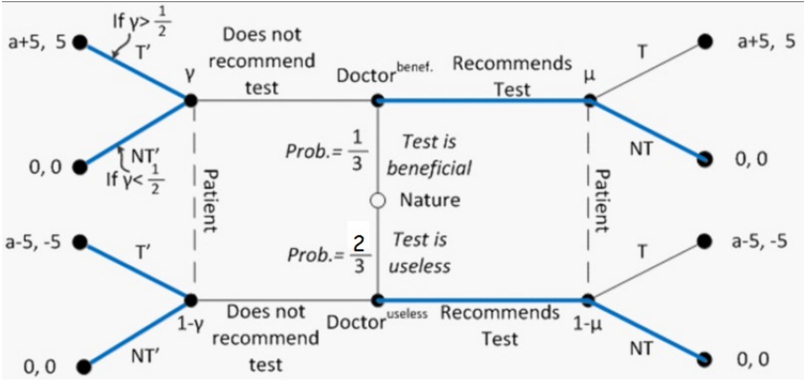
- **Patient (responder):**
- *Response:* After observing that the doctor "Does not recommend the test" (off-the-equilibrium), the patient compares

$$EU_{Patient}\left(T\middle|NoRecomm.\right) \gtrless EU_{Patient}\left(NT\middle|NoRecomm..\right)$$

$$\begin{aligned} \gamma 5 + (1-\gamma)(-5) &> \gamma 0 + (1-\gamma)0 \\ 10\gamma &> 5 \iff \gamma > \frac{1}{2} \end{aligned}$$

  - The patient takes the test after iff $\gamma > \frac{1}{2}$.
  - (Shade this in the figure, one case for $\gamma > \frac{1}{2}$, and another case for $\gamma < \frac{1}{2}$).
  - (Harrington only presents the case in which γ is exactly equal to $\frac{1}{3}$).

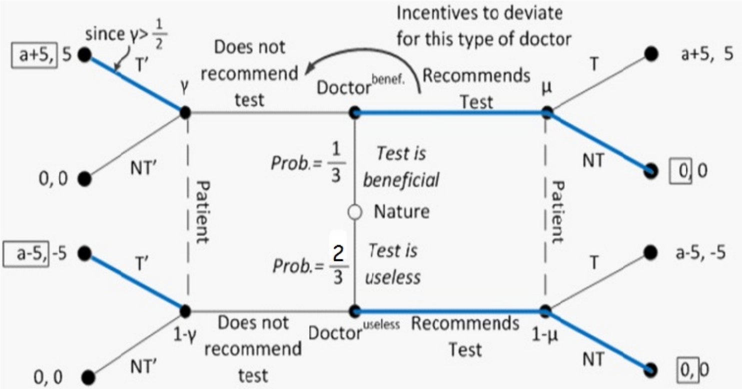
- **Pooling (babbling) equilibrium** (con't)



Example 1: Defensive medicine

- **Doctor (sender):** First case: $\gamma > \frac{1}{2}$.
- If the test is beneficial, he obtains:
  - 0 from recommending it (since the patient responds not taking the test), and
  - $a + 5$ from not recommending it (since the patient takes the test after no recommendation, given that $\gamma > \frac{1}{2}$ in this case).
  - The doctor then prefers to **not** recommend the test...
    - and the pooling strategy profile cannot be sustained as a PBE.
- (No need to check if the doctor wants to recommend the test when such test is useless)

- Summary of the First case: $\gamma > \frac{1}{2}$



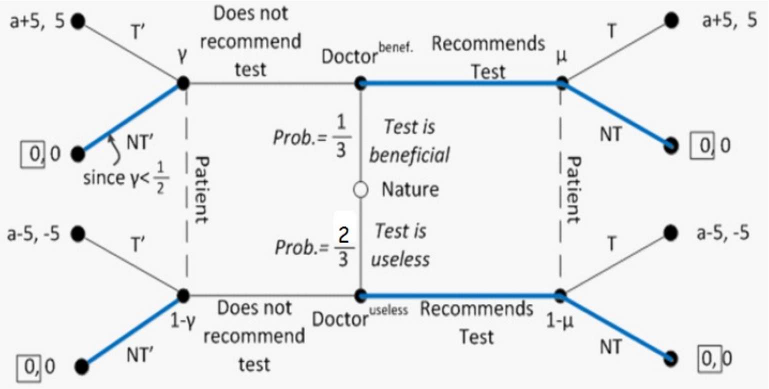
Example 1: Defensive medicine

- **Doctor (sender):** Second case: $\gamma < \frac{1}{2}$ (e.g., $\gamma = \frac{1}{3}$)
- If the test is beneficial, he obtains:
  - 0 from recommending it (since the patient responds by not taking the test), and
  - he also obtains 0 if he doesn't recommend the test (since the patient now does not take the test after no recommendation, given $\gamma < \frac{1}{2}$).
  - The doctor is then indifferent between R/NR the test.

Example 1: Defensive medicine

- **Doctor (sender):** Second case: $\gamma < \frac{1}{2}$ (e.g., $\gamma = \frac{1}{3}$)
- If the test is useless, he obtains:
  - 0 from recommend it (since the patient responds by not taking the test), and
  - he obtains 0 if he doesn't recommend the test (since the patient now does not take the test after no recommendation, given $\gamma < \frac{1}{2}$).
  - The doctor is then indifferent between R/NR the test.
- The pooling (babbling) strategy profile can be sustained as a PBE when $\gamma < \frac{1}{2}$.

- Summary of the Second case: $\gamma < \frac{1}{2}$

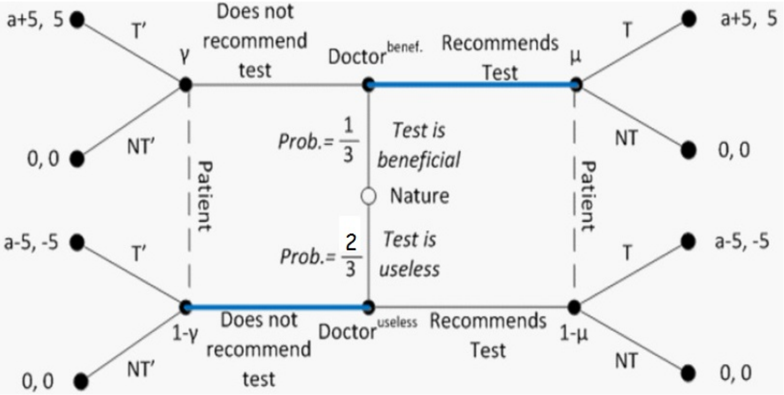


Example 1: Defensive medicine

- Unfortunately, in the pooling (babbling) equilibrium we just found, no information is being transmitted from the doctor to the patient!!
    - There is a Pareto improvement to be made if they communicate better.

- **Can we support some more information transmission in this game?**
- Yes!
- We can find a separating equilibrium where the doctor only recommends the test if it is beneficial for the patient.

- **Separating (informative) equilibrium:** the test is only recommended when it is beneficial.



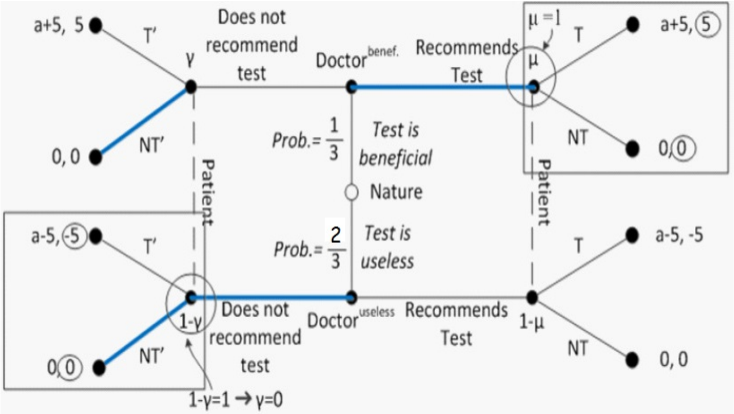
Example 1: Defensive medicine

- **Patient (responder):**
- *Beliefs:*
  - Beliefs are $\mu = 1$ after observing a recommendation, and
  - $\gamma = 0$ after observing no recommendation.

Example 1: Defensive medicine

- **Patient (responder):**
- *Response after Recomm.:*
    - After observing the recommendation of the test, he assigns full probability to the test being beneficial, and he takes it, since $5 > 0$.
- *Response after NoRecomm.:*
    - After observing no recommendation of taking the test, he assigns full probability to the test being useless, and he does not take it, since $-5 < 0$.
- These branches are shaded in the next figure.

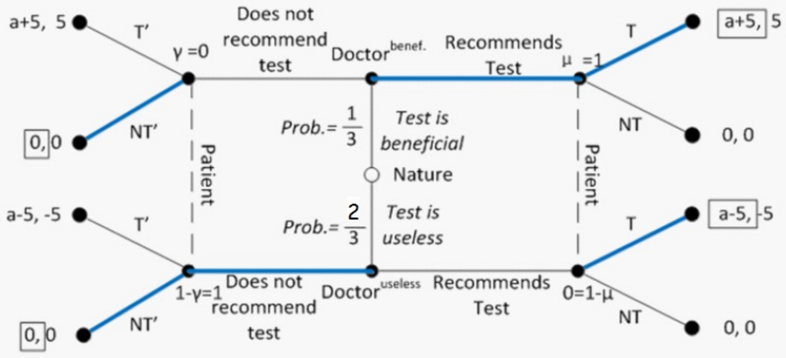
- **Separating (informative) equilibrium (cont.)**



Example 1: Defensive medicine

- **Doctor (sender):**
- When the test is beneficial, he recommends it if and only if $a + 5 > 0$ (which holds since $a > 0$).
- When the test is useless, he doesn't recommend the test if and only if $a - 5 < 0$, i.e., if $a < 5$.
  - Otherwise, he recommends the test, and this separating strategy profile cannot be supported as PBE.

- **Summary of the separating (informative) PBE:**



Example 1: Defensive medicine

- **Intuition:**
  - The difference in the preferences of the patient and the doctor, captured by parameter a, must be relatively small ($a < 5$) for a separating PBE to be supported.
  - Otherwise, only pooling (babbling) PBEs are sustained, in which the doctor recommends the test regardless of its utility for the patient's condition.
    - These are uninformative equilibria, since the uninformed patient cannot infer any information about his condition from observing that his doctor has just recommended the test.
- We can then interpret separating PBEs as equilibria where information is transmitted from the informed to the uninformed party.