Spence's labor market signaling model

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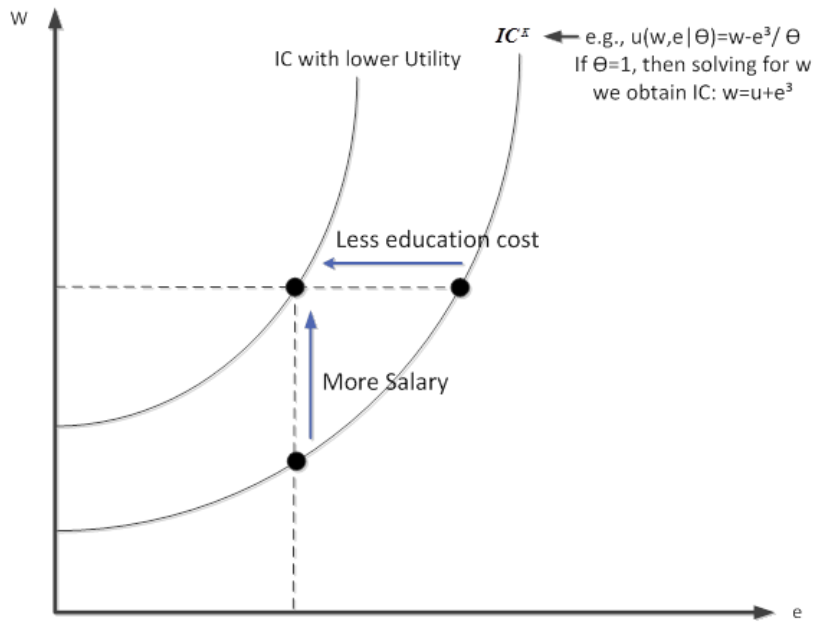
EconS 503 - Advanced Microeconomics II - Washington State University

Readings

- MWG 13.C (You can also read 13.B)
- Macho-Stadler and Perez-Castrillo, Ch. 4 (at least sections 4.1-4.3, and applications in 4.B).
- Bolton and Dewatripont, Ch. 3. (or at least section 3.1).

Spence (1974) Labor market signaling game

Worker's utility function $u(w, e|\theta_K) = w - c(e, \theta)$.



Spence (1974) Labor market signaling game

- Education costs are zero if $e = 0$, that is $c(0, \theta) = 0$ for all types.
- In addition, the marginal cost of acquiring education is positive and increasing, i.e., $c_e > 0$ and $c_{ee} > 0$. That is, the cost of education is convex.
- Furthermore,
  - $c_\theta < 0 \Rightarrow$ a given level of education is less costly to acquire for high than low ability workers, e.g., if $e = 4$ then $c(4, \theta_H) < c(4, \theta_L)$
  - $c_{e\theta} < 0 \Rightarrow$ MC of education is lower for the high than for the low ability worker, that is

$$\frac{\partial c(e, \theta_H)}{\partial e} < \frac{\partial c(e, \theta_L)}{\partial e}$$

Spence (1974) Labor market signaling game

Example

$$u(w, e|\theta_H) = w - \underbrace{\frac{e^3}{\theta}}_{c(e,\theta)}$$

$\Rightarrow$ Check the above four assumptions!

Separating PBE (eL,eH)

Beliefs:

- After observing equilibrium message e_L, beliefs are $\mu(\theta_H|e_L) = 0$, and
- After observing equilibrium message e_H, beliefs are $\mu(\theta_H|e_H) = 1$.
- **Off-the-equilibrium.** What about beliefs after observing off-the-equilibrium message $e \neq e_L \neq e_H$?

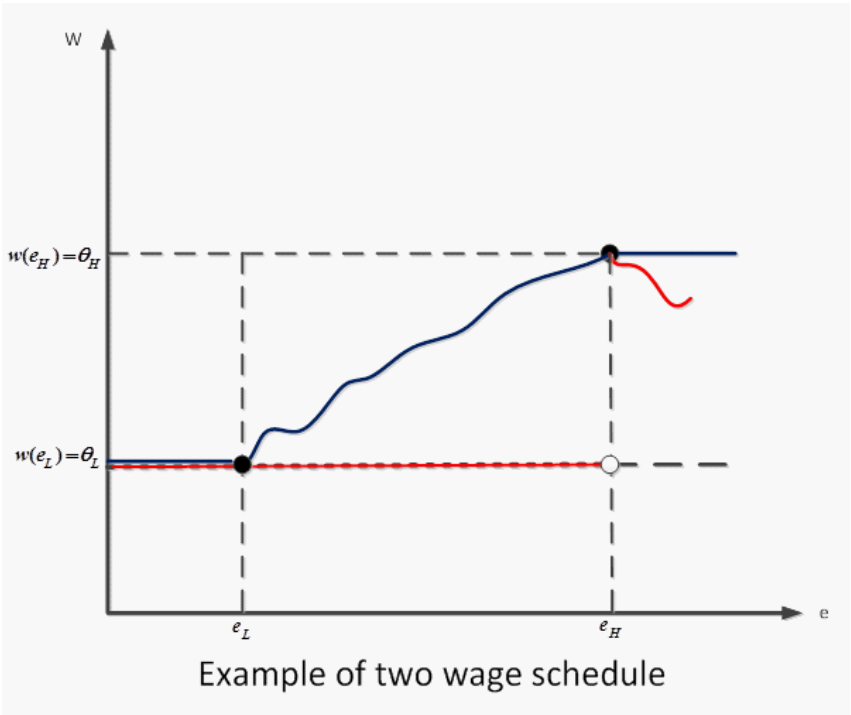
$$\mu(\theta_H|e) \in [0,1] \leftarrow \text{Some restrictions later on}$$

Separating PBE (eL,eH)

Optimal response of the uninformed player given his beliefs

$$w(e_L) = \theta_L \text{ and } w(e_H) = \theta_H$$

$w(e) \in [\theta_L, \theta_H] \ \forall \ e \neq e_L \neq e_H$ $\leftarrow$ We will need more restrictions



Example of two wage schedule

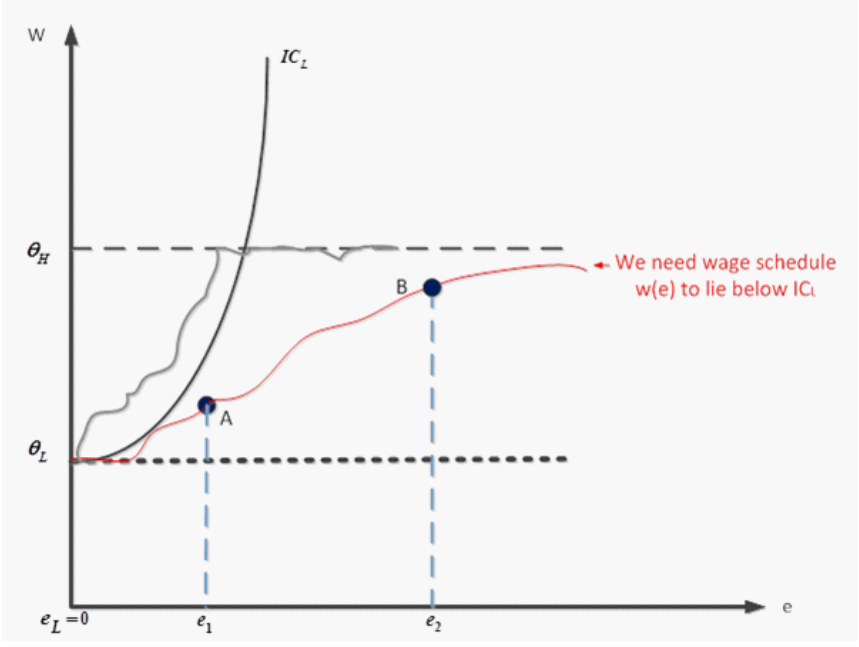
Separating PBE (eL,eH)

Given the responders' responses identified in step (2), which are the optimal messages for each type of sender?

Low type: $e^*(\theta_L) = 0$

- Any other $e \neq e_L \neq e_H$ still identifies him as a low-productivity worker, entailing a wage $w(e) = \theta_L$, but it is more costly to acquire than $e_L = 0$.
- Why doesn't he try to mimic e_H in order to be identified as high-productivity $\Rightarrow w(e_H) = \theta_H$? We will prove that he doesn't do that because it is too costly (with the use of incentive compatibility conditions)

Separating PBE (eL,eH)



- Deviation to education levels e_1 or e_2 are unprofitable.
- Graphically, the indifference curves lying on (w, e)-pairs A and B are associated to a lower utility level than the indifferent curve passing through $(w, e) = (\theta_L, 0)$.

Separating PBE (eL,eH)

High type:

- He chooses the prescribed education level e_H as long as

$$u(\underbrace{\theta_H}_{w}, e_H|\theta_H) \geq u(\underbrace{\theta_L}_{w}, \underbrace{0}_{e_L=0}|\theta_H)$$

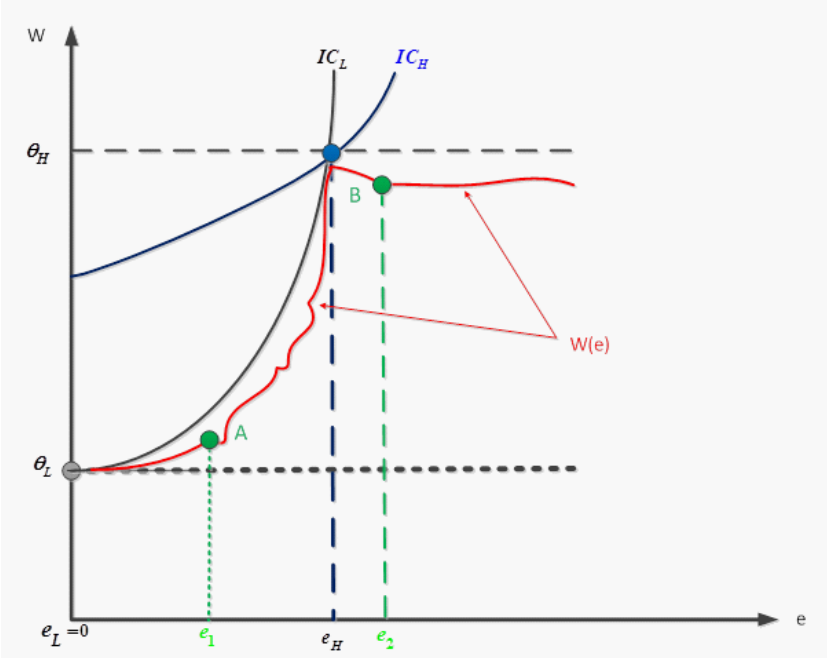
  i.e. no incentives to imitate the low-productive worker.

- In addition,

$$u(\theta_H, e_H|\theta_H) \geq u(w(e), e|\theta_H)$$

  for any off-the-equilibrium education level $e \neq e_L \neq e_H$.

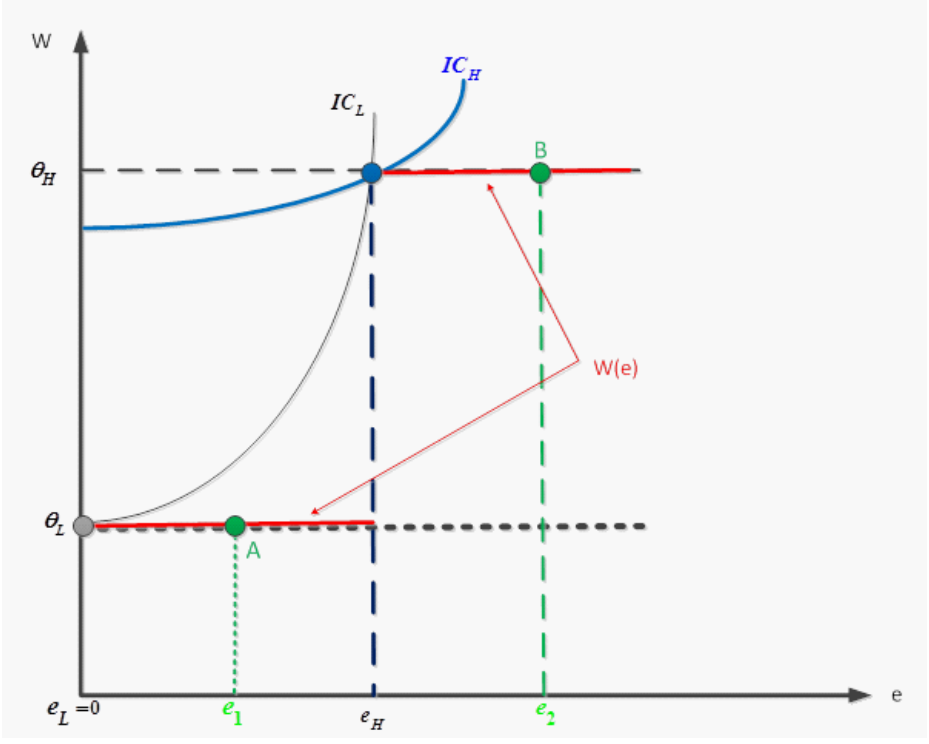
Separating PBE (eL,eH)



- Deviations to e_1 or e_2, given the wage schedule $w(e)$, induce (w, e)-pairs A and B respectively.
- The indifference curve of the high-productive worker passing through these (w, e)-pairs yields a lower utility than passing through the equilibrium pair $(w, e) = (\theta_H, e_H)$.

Separating PBE (eL,eH)

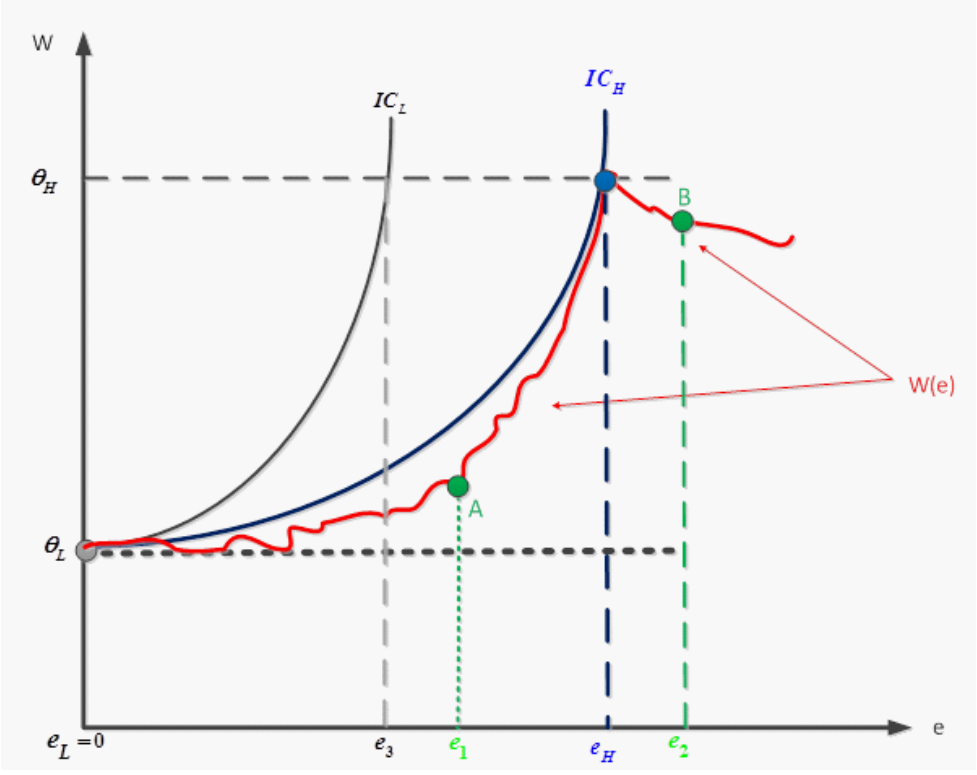
- Another wage schedule $w(e)$ supporting the same separating PBE:



Separating PBE (eL,eH)

- Difference in the wage schedule $w(e)$ only entails distinct responses by the firm to off-the-equilibrium education levels $e \neq e_L \neq e_H$.
- Deviations to education levels e_1 or e_2 is still, of course, unprofitable since it would yield (w, e)-pairs A and B respectively.
- *Nex figure:* Yet, another wage schedule $w(e)$. However, it now gives rise to a different separating PBE.

Separating PBE (eL,eH)



- Deviations to education level e_1 or e_2 is still unprofitable. (Intuition behind this wage schedule in the next slide.)

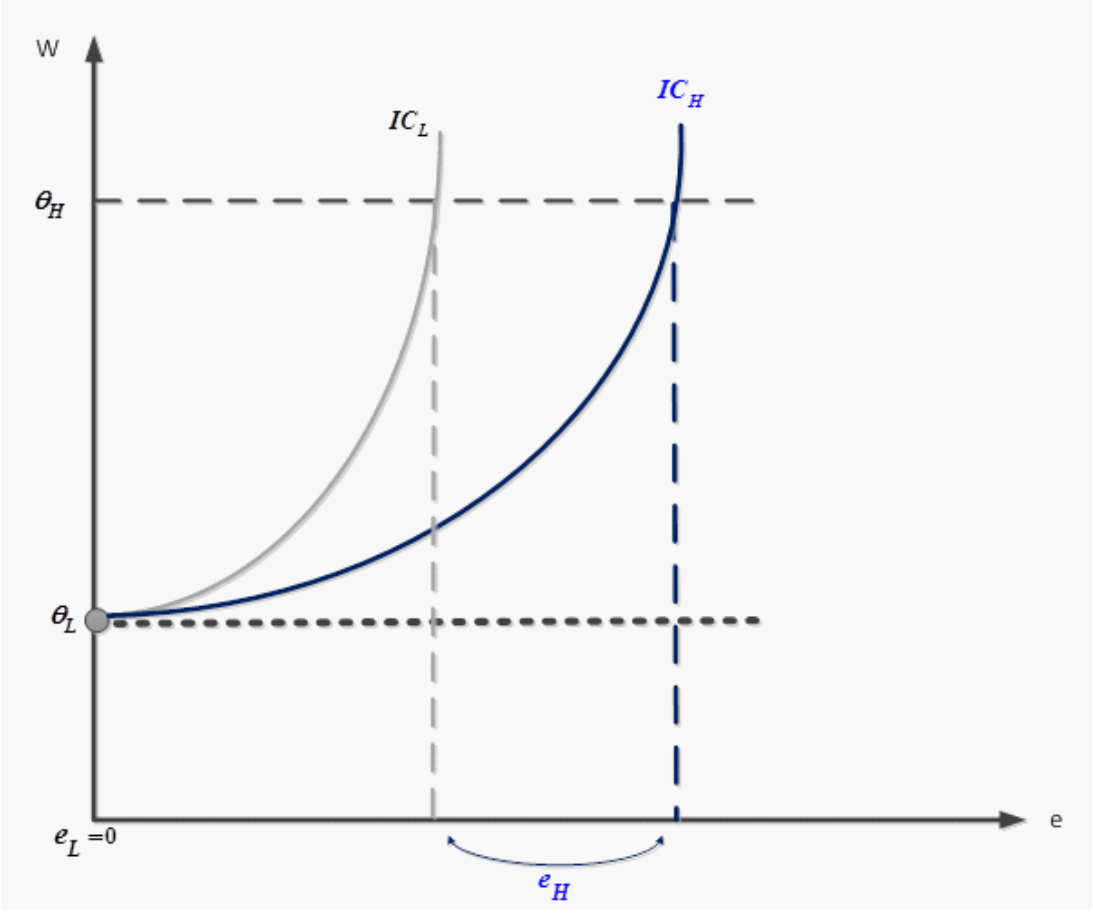
Separating PBE (eL,eH)

Intuition behind this $w(e)$

- *Similarity to all previous wage schedules:* The firm believes $\mu(\theta_H|e_L) = 0$ and $\mu(\theta_H|e_H) = 1$, as usual.
- *Difference:* Any off-the-equilibrium education level $e \in [e_3, e_H)$ is interpreted as not originating from a high-productive worker.
    - Does that make sense? No! The low-productive worker would not benefit from sending such a message. We will confirm this when applying Cho and Kreps' (1987) Intuitive Criterion in order to eliminate all separating PBEs but one.

Separating PBE (eL,eH)

Summary of All Separating PBEs



Pooling PBEs

All workers select the same education level e^*.

Firm's beliefs

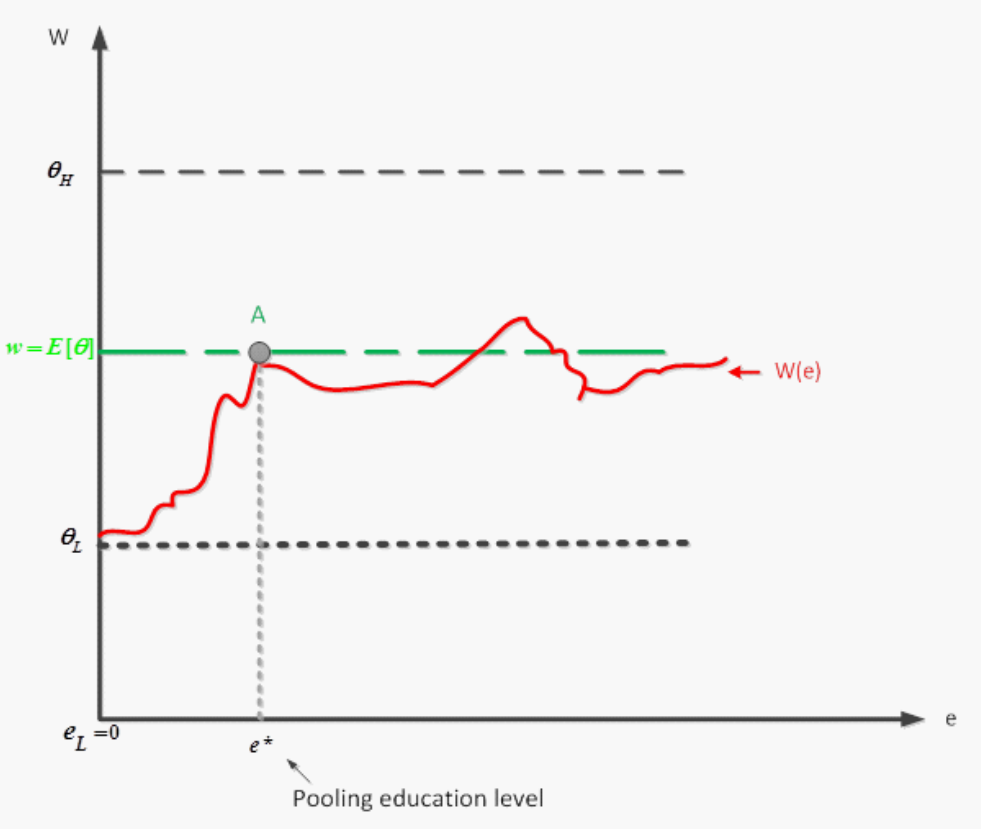
- After observing the pooling message e^*, $\mu(\theta_H|e^*) = p \leftarrow \lambda$ in MWG.
- After observing off-the-equilibrium messages $e \neq e^*$, $\mu(\theta_H|e) \in [0, 1]$.

Pooling PBEs

Firm's response given its above beliefs

- After observing the equilibrium message of e^* $w(e^*) = p\theta_H + (1-p)\theta_L \equiv E[\theta]$, see figure
- After observing off-the-equilibrium messages $e \neq e^*$, $w(e) \in [\theta_L, \theta_H] \leftarrow$ we further restrict this wage schedule below.

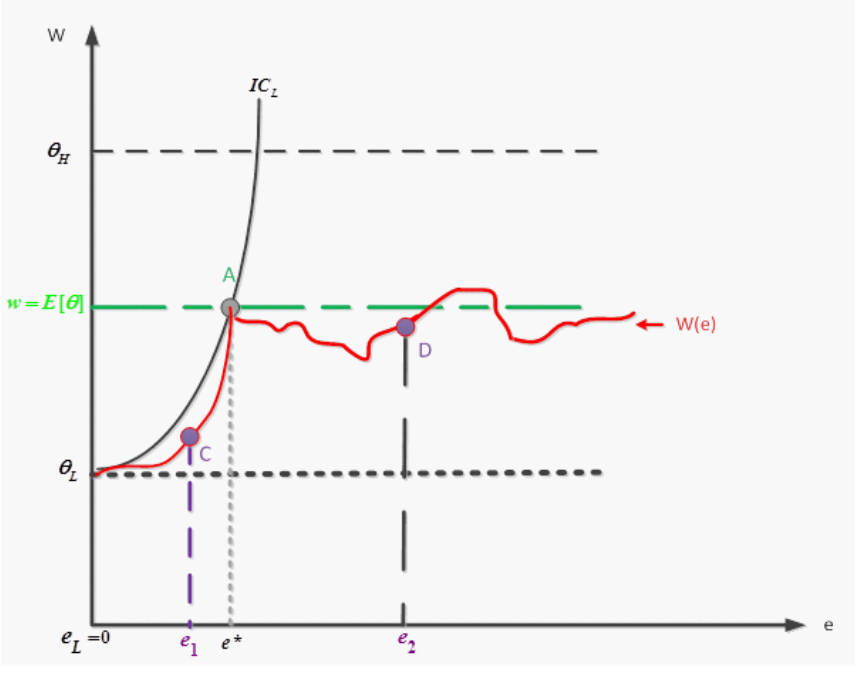
Pooling PBEs



Pooling PBEs

Sender's optimal messages

Low type

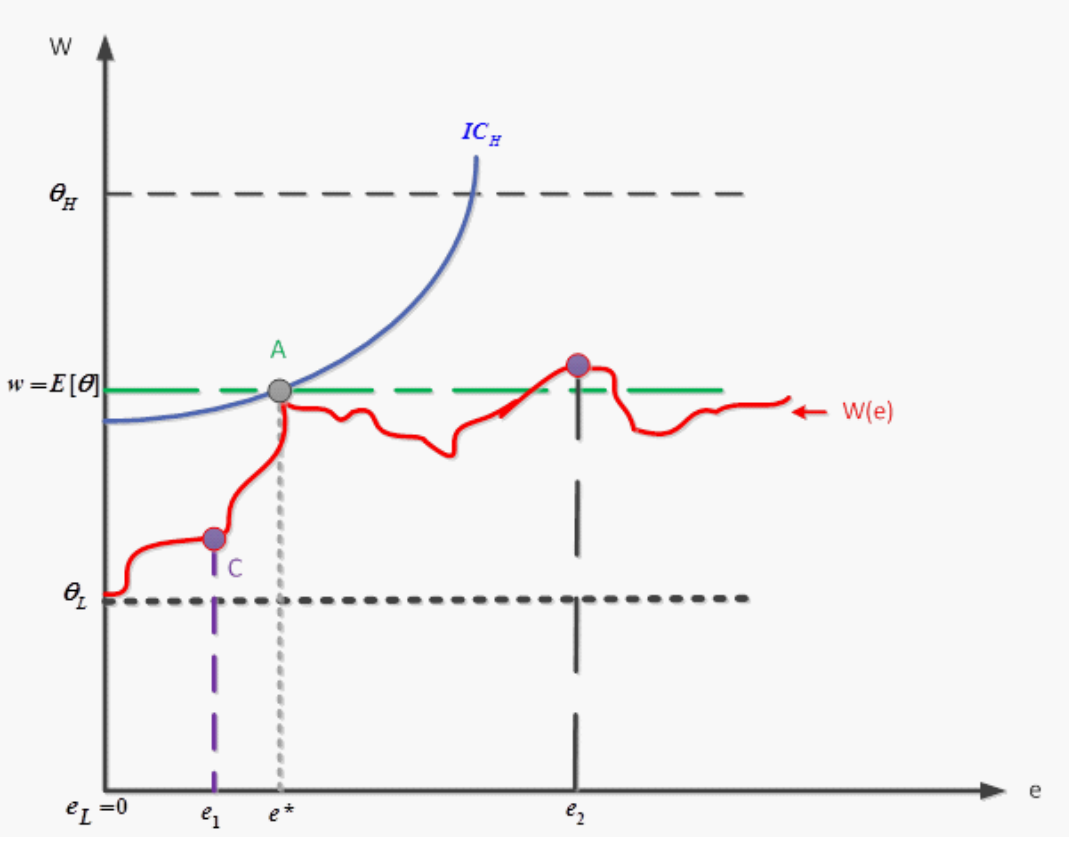


Pooling PBEs

- Deviations away from the pooling education level e^* (such as to e_1 or e_2) yield (w, e)-pairs C or D, which entail a lower utility than at point A, i.e., $(w, e) = (E[\theta], e^*)$
- Why does IC_L originate at θ_L or above?
  - Because any pooling education level higher than e^* would entail a utility level lower than that from selecting no education and receive the lowest possible wage θ_L.
  - Intuition: If the pooling education level is lower than e^*, the low-productive worker would actually be happier: he would still receive a wage $w = E[\theta]$ in equilibrium but incur smaller costs.

Pooling PBEs

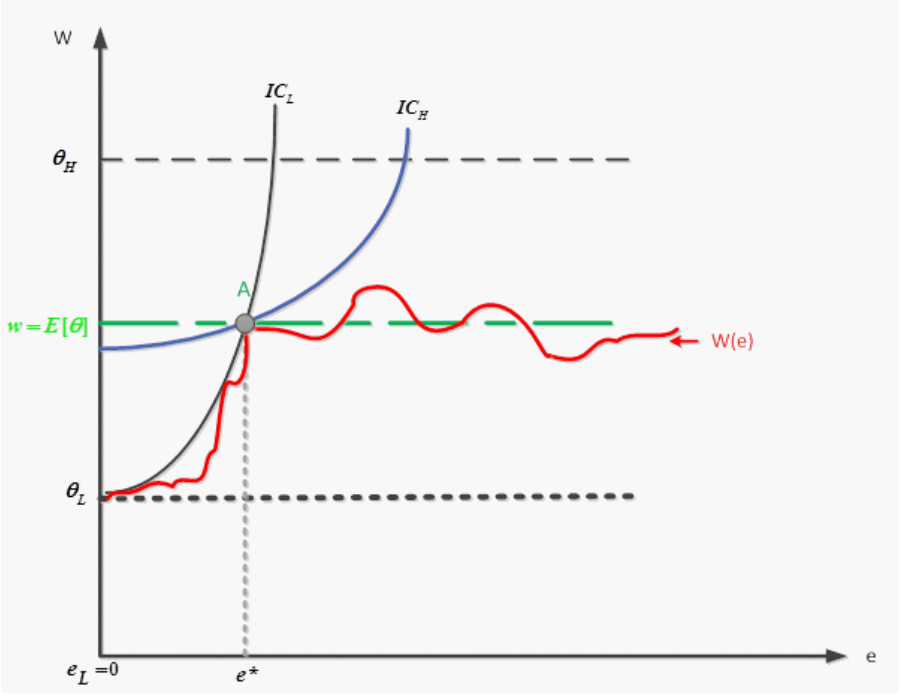
High type



- Deviations from e^* to e_1 or e_2 are also unprofitable for the high-productive workers.

Pooling PBEs

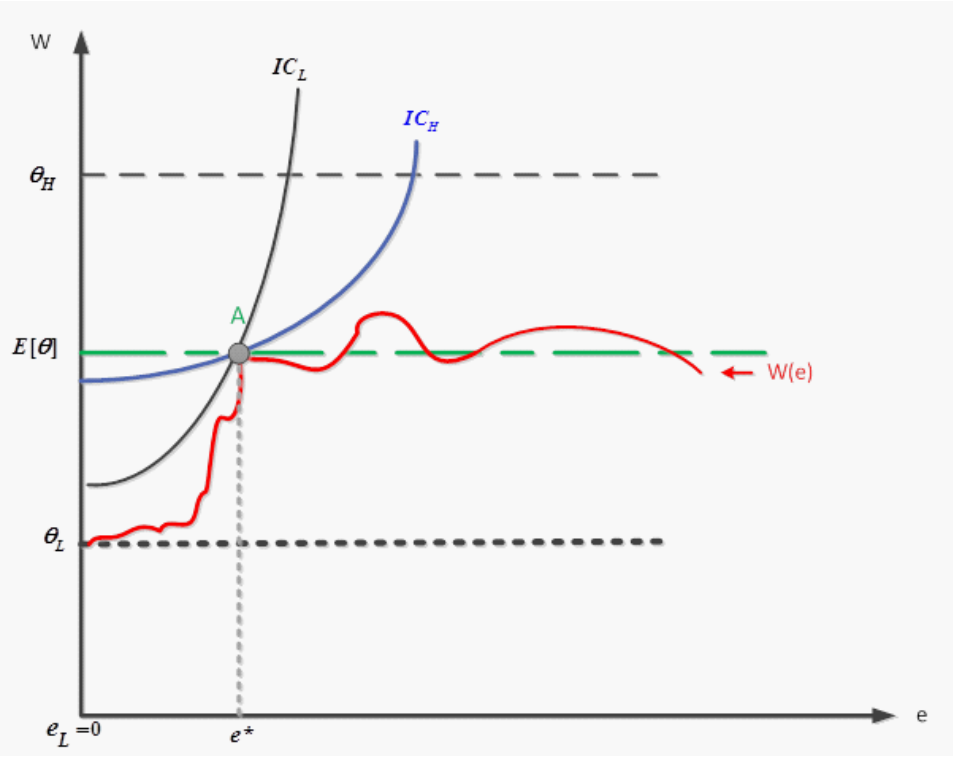
- Putting both figures together:



Pooling PBEs

- Pooling education level e^* is thus the point in which IC_L and IC_H cross each other (point A).
- We cannot sustain a pooling PBEs for education levels above e^*, since the low-productive worker would rather prefer to acquire $e = 0$ (and be recognized as low-productive with $w = \theta_L$) than acquire such a high education level.
- Other pooling PBE (where IC_L originates above θ_L) can also be sustained; as depicted in the next slide.

Pooling PBEs

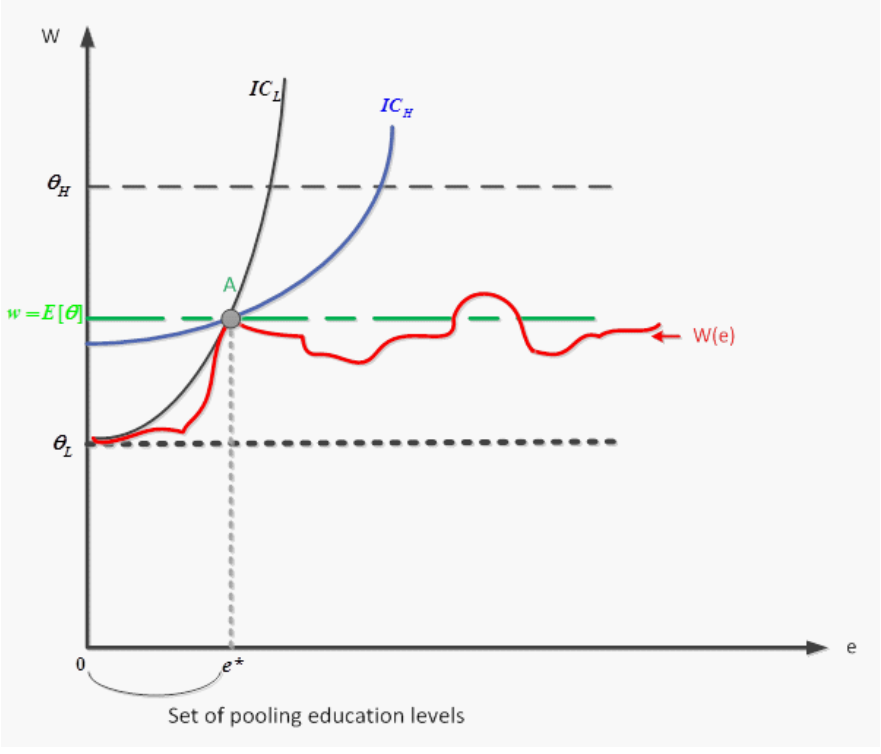


- And similarly for any other education level lower than e^*.

Pooling PBEs

Summary of pooling PBEs

- We can summarize the set of pooling PBE in the next figure:



- However, all of these pooling equilibria violate the Cho and Kreps' (1987) intuitive criterion.

Cho and Kreps' (1987) Intuitive Criterion

Which of all the separating PBEs we identified survive the intuitive criterion?

- Only the least-costly separating equilibrium where $e_L = 0$ for the low-productivity worker, and $e_H = e^*$ for the high-productivity worker, where e^* solves

$$u(w_L = \theta_L, e^* | \theta_L) = u(w_H = \theta_H, e^* | \theta_H)$$

or

$$\begin{aligned} \theta_L - c(e^*, \theta_L) &= \theta_H - c(e^*, \theta_H) \iff \\ c(e^*, \theta_H) - c(e^*, \theta_L) &= \theta_H - \theta_L \end{aligned}$$

- Explanation in slides of EconS 503 website, and associated paper also posted on the website.

Cho and Kreps' (1987) Intuitive Criterion

Which of all the pooling PBEs we found survive the intuitive criterion?

- None!
- **1st Step**: If the firm observes an off-the-equilibrium message e' (see next figure), it can understand that such a message is never equilibrium deviated for the high-productive worker since

$$u^*(E[\theta], e^*|\theta_H) < max_w\, u(w, e'|\theta_H)$$

while this inequality does not held for the low-productive worker

$$u^*(E[\theta], e^*|\theta_L) > max_w\, u(w, e'|\theta_L)$$

Hence, the firm can restrict its beliefs to θ_H after observing the off-the-equilibrium message e', i.e., $H^{**}(e') = \theta_H$.

Cho and Kreps' (1987) Intuitive Criterion

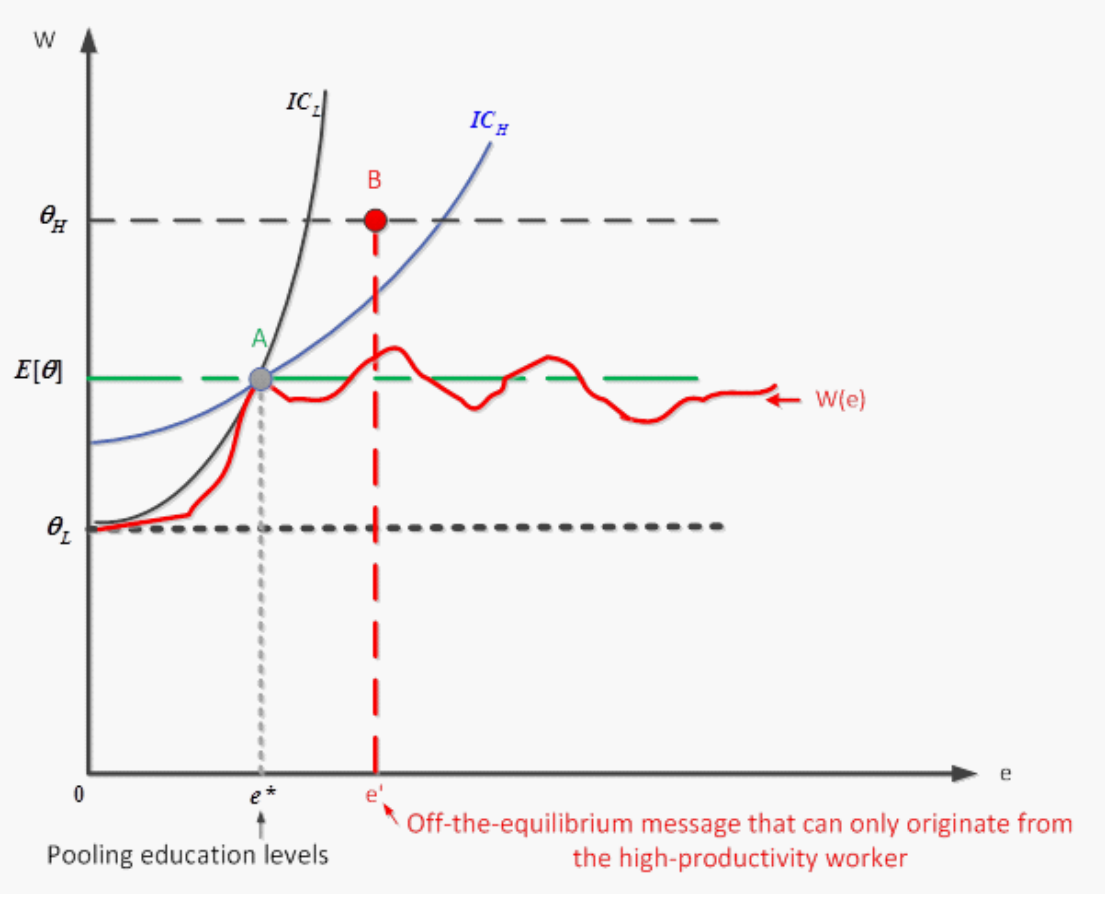
- **2nd Step**: After restricting beliefs to $H^{**}(e^{'}) = \theta_H$, the firm responds with a wage $w(e^{'}) = \theta_H$ after observing $e^{'}$. The high-productive worker has incentives to deviate from the pooling education level e^* to $e^{'}$ since

$$\theta_H - c(e^{'}, \theta_H) > E[\theta] - c(e^*, \theta_H)$$
$$\text{or}$$
$$\theta_H - E[\theta] > c(e^{'}, \theta_H) - c(e^*, \theta_H)$$

- which holds as long as the prior probability p is not extremely close to 1. (You can test this condition for the following parametric example)

$$\theta_H - (p\theta_H + (1-p)\theta_L) > \frac{e^{*3}}{\theta_H} - \frac{e^{'3}}{\theta_H}$$

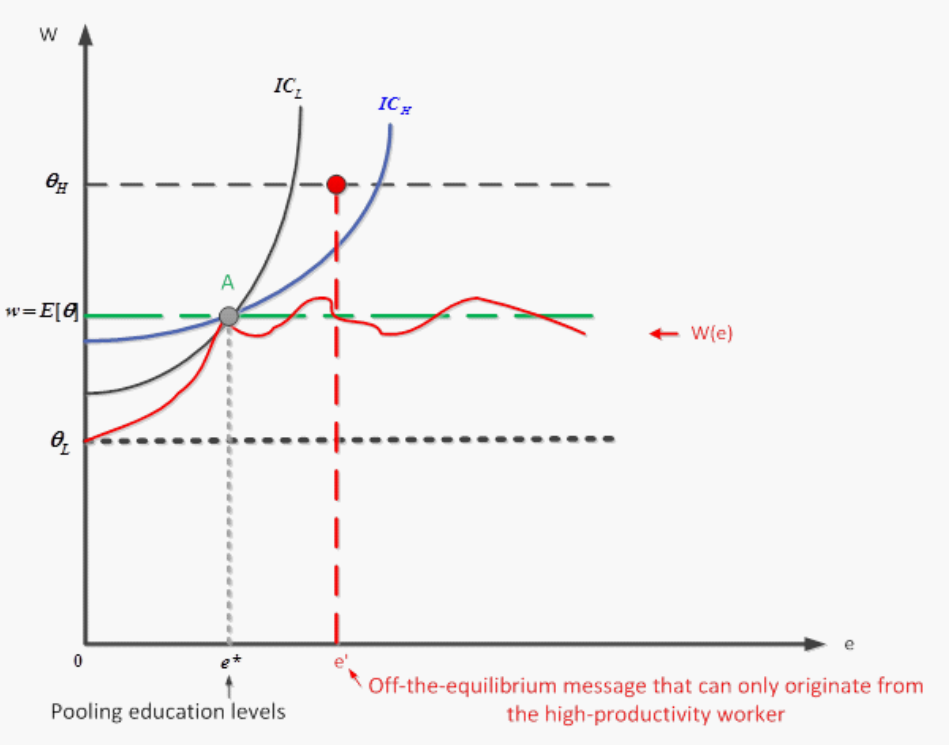
Cho and Kreps' (1987) Intuitive Criterion



Cho and Kreps' (1987) Intuitive Criterion

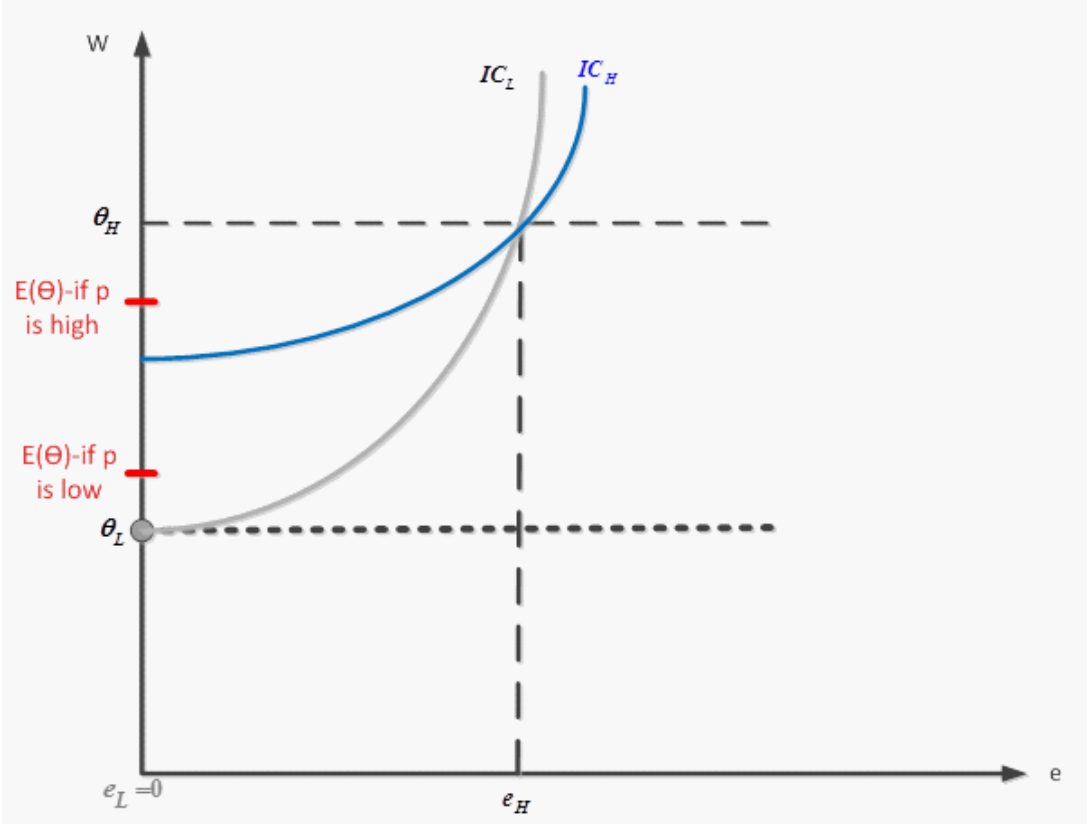
- We have thus found one type of sender (high-prod. worker) and one off-the-equilibrium message he could send (any $e' \in [e_1, e_2]$) that would provide him with a higher utility level than at the pooling PBE in which both types of sender select e^*. Hence, such pooling PBE violates the intuitive criterion.
- A similar argument applies to all other pooling PBEs (practice). Hence, no pooling PBE in the labor market signaling game survives the intuitive criterion. Only the least-costly separating PBE survives the intuitive criterion.
- Of course, all other Pooling PBEs also violate the intuitive criterion.

Cho and Kreps' (1987) Intuitive Criterion



- Hence, the only PBE (separating or pooling) surviving the Cho and Kreps' (1987) Intuitive Criterion is the least-costly separating equilibrium (also referred as Riley outcome), depicted in this figure.

Cho and Kreps' (1987) Intuitive Criterion



Cho and Kreps' (1987) Intuitive Criterion

- Welfare comparison to the setting in which signaling is unavailable:
  - Low type: He is unambiguously worse off with than without signaling. Indeed, without signaling, he would reach the indifference curve passing by $(0, E[\theta])$, which yields a high utility level both when p is low and when p is high, i.e., $E[\theta]$ is close to θ_H.
  - High type: He is better off with signaling only if p is sufficiently low.