Alternatives to majority voting

- In previous sections we have criticized both majority voting and the Condorcet criterion.
  - **Majority rule:**
    - Choose the candidate who is ranked first by more than half of the voters.
  - **Condorcet criterion:**
    - Choose the candidate who defeats all others in pairwise elections using majority rule.

Alternatives to majority voting

- Several other voting procedures have been suggested.
- We will first describe them, and afterwards compare them according to whether they satisfy different properties:
  - **Majority rule, with runoff election:**
    - If one of the m candidates receives a more than half of the votes, then he/she is the winner.
    - Otherwise, a second election is held between the two candidates receiving the most first-place votes on the first ballot. The candidate receiving the most votes on the second ballot is the winner.
  - **Plurality rule:**
    - Each voter ranks the m candidates. Choose the candidate who is ranked first by the largest number of voters.

Alternatives to majority voting

- *An example of plurarity rule:*
  - Consider three candidates A,B, and C; and three voters who rank the candidates as follows:

Voter 1	Voter 2	Voter 3
A	A	C
B	C	B
C	B	A

  - The winner is candidate A, since two of the three candidates rank it first.

Alternatives to majority voting

- Other voting procedures (cont'd):
  - **The Hare system:**
    - Each voter indicates the candidate he ranks highest of the m candidates.
    - Remove from the list of candidates the one ranked highest by the fewest number of voters.
    - Repeat the procedure for the remaining $m - 1$ candidates.
    - Continue until only one candidate remains, who is declared the winner.

Alternatives to majority voting

- *An example of the Hare system:*
  - Consider three candidates A, B, and C; and three voters who rank the candidates as follows:

Voter 1	Voter 2	Voter 3
A	A	C
B	C	B
C	B	A

  - Candidate A is ranked highest by two voters, C is ranked highest by one voter, but B is not ranked highest by any voter.
  - Hence, candidate B is then removed from the list.

Alternatives to majority voting

- *An example of the Hare system (cont'd):*
  - Once candidate B is removed from the list, every voter is asked to ranked the remaining candidates A and C, as follows:

Voter 1	Voter 2	Voter 3
A	A	C
C	C	A

  - Candidate A is ranked highest by two voters, C is ranked highest by one voter.
  - Then, candidate C is then removed from the list.
  - Therefore, A is the winner (coincides with the winner under Plurality voting.)

Alternatives to majority voting

- Other voting procedures (cont'd):
  - **The Coombs system:**
    - Each voter indicates the candidate he ranks *lowest* of the m candidates.
    - Remove from the list of candidates the one ranked *lowest* by the most number of voters.
    - Repeat the procedure for the remaining $m - 1$ candidates.
    - Continue until only one candidate remains, who is declared the winner.

Alternatives to majority voting

- *An example of the Coombs system:*
  - Consider three candidates A, B, and C; and three voters who rank the candidates as follows:

Voter 1	Voter 2	Voter 3
A	A	C
B	C	A
C	B	B

  - Candidate B is ranked lowest by two voters, C is ranked lowest by one voter, but A is not ranked lowest by any voter.
  - Then, candidate B is then removed from the list.

Alternatives to majority voting

- *An example of the Coombs system (cont'd):*
  - After removing candidate B, every voter ranks the remaining candidates as follows:

Voter 1	Voter 2	Voter 3
A	A	C
C	C	A

  - Candidate C is ranked lowest by two voters, while A is ranked lowest by only one voter.
  - Then, candidate C is then removed from the list; and A becomes the winner.

Alternatives to majority voting

- Other voting procedures (cont'd):
  - **Approval voting:**
    - Each voter votes for the k candidates he ranks highest of the m candidates, where k can vary from voter to voter and $k \in (1, m)$.
    - The candidate with the most votes is the winner.

Alternatives to majority voting

- *An example of Approval voting:*
  - Consider three candidates A, B, and C; and three voters who are asked to vote for one, two or all three candidates.
  - A ballot would ask:
    - "In the next list of three candidates, please mark a cross next to the candidate or candidates you want to vote for. You can mark a cross next to one, two or all three of the candidates."

Alternatives to majority voting

- *An example of Approval voting (cont'd):*
  - Here are examples of ballots marked by voters 1 and 2.

Voter 1	
A	*X*
B	*X*
C	

Voter 2	
A	
B	*X*
C	*X*

  - A receives 1 vote, B receives 2 votes, and C receives 1 vote.
  - Therefore, B is the candidate receiving the most votes, and thus it is declared the winner.

Alternatives to majority voting

- Other voting procedures (cont'd):
  - **The Borda count:**
    - Each voter gives a score $s \in (1, m)$ to each of the candidates, i.e., he gives m points to his most preferred candidate, $m-1$ points to the second most preferred candidate, ..., and one point to his least preferred candidate.
    - The candidate receiving the highest number of points is declared the winner.

Alternatives to majority voting

- *An example of the Borda count:*
  - Consider three candidates A, B, and C; and three voters who are asked to score each candidate with a number 1-3.
  - A ballot would ask:
    - "Please give a score 1-3 to each of the three candidates in the following list, writting 3 next to your most preferred candidate, 2 for your second most preferred candidate, and 1 for your least preferred candidate."

Alternatives to majority voting

- *An example of the Borda count (cont'd):*
  - Here are examples of ballots marked by voters 1 and 2.

Voter 1	
A	3
B	2
C	1

Voter 2	
A	1
B	3
C	2

  - A receives 4 points, B receives 5, and C receives only 3 points.
  - Therefore, candidate B is the winner.

Alternatives to majority voting

- *Comment on the Hare and Coombs systems:*
  - If there are 10 candidates, voters would have to go to the polls nine separate times!
  - They could nonetheless be implemented by asking voters to write their ranking of candidates, and then using a computer to determine the winner following the prescribed rule by each system.
  - Thus, the informational requirements in the Hare, Coombs and Borda procedures are identical.
  - These procedures would only differ in how they process this information.

Alternatives to majority voting

- How to compare different voting procedures?
  - One commmon (normative) criterion is to check if they satisfy **decisiveness**:
    - The voting procedure must pick a winner.
  - When $m = 2$, all procedures are decisive.
  - When $m > 2$, majority voting and the Condorcet criterion or not necessarily decisive, but all other voting procedures are.

Alternatives to majority voting

- While the rest of voting procedures pick a winner, it doesn't need to coincide with the Condorcet winner.
  - *Example:*
  - Consider five voters, and four candidates $\{X, Y, Z, W\}$.

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
X	X	Y	Z	W
Y	Y	Z	Y	Y
Z	Z	W	W	Z
W	W	X	X	X

  - X is the winner under plurality voting, but Y is a Condorcet winner. (Check!)
  - What if approval voting is used? X could be the winner (if, for instance, voters 1 and 2 only vote for X, while voters 3-5 vote only for their top candidate, or for their top two or three candidates).

Alternatives to majority voting

- More examples in which the winner picked by some voting procedures doesn't coincide with the Condorcet winner.

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
X	X	X	Y	Y
Y	Y	Y	Z	Z
Z	Z	Z	X	X

  - X is the Condorcet winner, but Y is the winner under Borda count. (Check!)
  - Indeed, X receives 11 points, Y receives 12, and Z receives 7 points. Hence, Y is the winner under the Borda count.

Alternatives to majority voting

- More examples in which the winner picked by some voting procedures doesn't coincide with the Condorcet winner.

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
Y	W	X	Y	W
X	Z	Z	Z	X
Z	X	W	X	Z
W	Y	Y	W	Y

  - X is again the Condorcet winner, but W is the winner under the Hare system.
  - X is ranked the highest by one voter, Y by two voters, W by two voters, and Z by no voter. Hence, Z should be removed from the list.

Alternatives to majority voting

- After removing candidate Z from the list, voters rank the remaining candidates, as follows:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
Y	W	X	Y	W
X	X	W	X	X
W	Y	Y	W	Y

  - X is ranked the highest by one voter, Y by two voters, W by two voters.
  - Hence, we can delete X from the list.

Alternatives to majority voting

- After removing candidate X from the list, voters rank the remaining candidates, as follows:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
Y	W	W	Y	W
W	Y	Y	W	Y

  - Y is ranked the highest by two voters, and W by three voters.
  - Hence, we can delete Y from the list, implying that W is the winner under the Hare system.

Alternatives to majority voting

- Therefore, except for majority rule...
  - the other voting procedures may select a winner that is not necessarily the Condorcet winner (even when one exists).
  - Majority rule would select a Condorcet winner (if one exists).

Alternatives to majority voting

- How can we measure more generally the ability of each procedure to select a Condorcet winner if one exists?
- Merrill (1984, 1985) simulated an electorate of 25 voters allowing for randomly assigned utility functions and several candidates.
    - (The results were insensible to different number of voters.)

Alternatives to majority voting

- Condorcet efficiency

	# of candidates				
Voting system	3	4	5	7	10
Runoff	96.2	90.1	83.6	73.5	61.3
Plurality	79.1	69.4	62.1	52.0	42.6
Hare	96.2	92.7	89.1	84.8	77.9
Coombs	96.3	93.4	90.2	86.1	81.1
Approval	76.0	69.8	67.1	63.7	61.3
Borda	90.8	87.3	86.2	85.3	84.3

Alternatives to majority voting

- *General findings about Condorcet efficiency:*
  - With only $m = 2$ candidates, all procedures choose the Condorcet winner with 100% probability, i.e., "Condorcet efficiency" is 100.
  - Decline in "Condorcet efficiency" when we increase the number of candidates.
  - But only two procedures are mainly in use today: plurality and majority-runoff.
  - An alternative question: What is the gain in Condorcet efficiency if we move from either of these procedures currently in use to an alternative procedure?

Alternatives to majority voting

- Let us evaluate the performance of these voting procedures relative a different normative criterion:
- **Utilitarian efficiency** (i.e., maximizing the sum of individual utilities).
  - More simulations by Merrill (1984).
  - *General findings:*
  - Borda count performs extremelly well, relative to the other voting procedures.

Alternatives to majority voting

- Utilitarian efficiency

	# of candidates				
Voting system	3	4	5	7	10
Runoff	89.5	83.8	80.5	75.6	67.6
Plurality	83.0	75.0	69.2	62.8	53.3
Hare	89.5	84.7	82.4	80.5	74.9
Coombs	89.7	86.7	85.1	83.1	82.4
Approval	85.4	91.1	89.1	87.7	87.0
Borda	94.8	94.1	94.4	95.4	95.9
Condorcet	93.1	91.9	92.0	93.1	94.3