Nash equilibrium with N players

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EconS 424 - *Strategy and Game Theory*

Games with n players

- Main reference for reading: Harrington, Chapter 5.
- Both symmetric (remember the definition) or asymmetric games.
    - We will start with symmetric games, then move to asymmetric games.
- Two distinct classes of games:
    - In some, we will talk about network effects (or tipping points).
    - In others, we will talk about congestion effects.
- This suggests that we might find *asymmetric* equilibria even in games where players are *symmetric*.

Symmetric vs. Asymmetric Games

- A game is *symmetric* if
  - all players share the same set of available strategies, and
  - when all players choose the same strategy, $s_1 = s_2 = s$, their payoffs coincide, i.e., $u_1 = u_2$.If we switch strategies, then their payoffs switch as well, i.e.,$u_1(s', s") = u_2(s", s')$
- Intuitively, this implies that players' preferences over outcomes coincide.
  - That is, players have the same ranking of the different outcomes that can emerge in the game.
- Example:

		Player 2		
		Low	Moderate	High
	Low	1, 1	$\underline{3}$, $\underline{2}$	1, $\underline{2}$
Player 1	Moderate	$\underline{2}$, $\underline{3}$	2, 2	2, 1
	High	$\underline{2}$, 1	1, 2	$\underline{3}$, $\underline{3}$

Symmetric vs. Asymmetric Games

- **Symmetric NE:**
  - All players use the same strategy.
- **Asymmetric NE:**
  - Not all players use the same strategy.
- Note that we can have an asymmetric NE in a symmetric game if, for instance, congestion effects exist.
  - *Example:* Consider a **symmetric** game where all drivers assign the same value to their time, and they all have only two modes of transportation (car vs. train).
  - When the number of drivers using the same route is sufficiently high (congestion effects are large), additional drivers who consider which mode of transportation to use will NOT use the car, leading to an **asymmetric** NE where a set of drivers use their cars and another set use the train.

Symmetric vs. Asymmetric Games

- Similarly, we can have symmetric NE in asymmetric games if network effects (also referred to as tipping points) are strong enough.
  - *Example:* Consider an **asymmetric** game where a group of consumers assign different values to two technologies A and B (e.g., software packages).
  - If the number of customers who own technology A is sufficiently high, even the individual with the lowest valuation for A might be attracted to acquire A rather than B, leading to a **symmetric** NE where all customers acquire the same technology A.

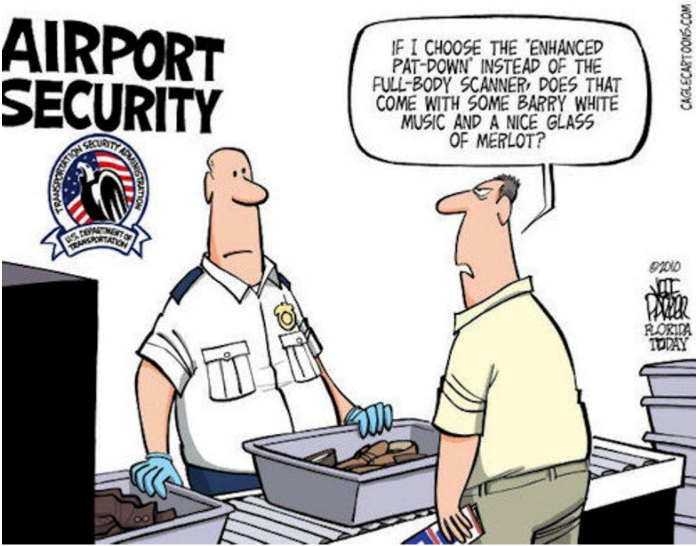
Symmetric vs. Asymmetric Games

- *Very useful property:*
  - Consider a symmetric game, and suppose you find an asymmetric NE, meaning that not all players use the same strategy.
  - Then, there are other asymmetric NE in this game that have players swap strategies.
- Example:
  - In a two-player symmetric game, if (s', s'') is a NE, then so is (s'', s').
  - In a three-player symmetric game, if (s', s'', s''') is a NE, then so are (s', s''', s''), (s'', s', s'''), (s'', s''', s'), (s''', s', s''), and (s''', s'', s').
- That is, if (Low, Moderate) was an asymmetric NE in the previous payoff matrix, so is (Moderate, Low)

The airline security game

- An airline's security is dependent not just on what security measures it takes, but also on the measures taken by other airlines since bags are transferred
- Example:
  - The suitcase that blew up the Pam Am flight over Lockerbie, Scotland, had been checked in Malta, transferred in Frankfurt and then in London.

The airline security game



The airline security game

- Players: $n \geq 2$ symmetric airlines.
- Each selects a security level $s_i = \{1, 2, ..., 7\}$
- Payoff for airline/airport i is

$$50 + 20 \min\{s_1, s_2, ..., s_n\} - 10 s_i$$

- *Intuition*: the overall security level is as high as its weakest link. Hence, this game serves as an illustration of the more general "weakest link coordination game."

The airline security game

- Note that if airline i selects $s_i > \min\{s_1, s_2, ..., s_n\}$ it can increase its payoff by reducing security without altering the overall security level.
- Hence, $s_i > \min\{s_1, s_2, ..., s_n\}$ cannot be an equilibrium.
- Since this argument can be extended to all airlines, no asymmetric equilibruim can be sustained.
- Only symmetric equilibria exist.

The airline security game

- Assume that, in a symmetric equilibrium, all airlines select $s_i = s'$ for all i.
- Any airline's payoff from selecting s' is

$$50 + 20 \underbrace{\min\{s', s', ..., s'\}}_{s'} - 10s' = 50 + 20s' - 10s' = 50 + 10s'$$

The airline security game

- Is there a profitable deviation?
- Let us first check the payoff from deviating to $s'' > s'$

$$\begin{aligned} 50 + 20\underbrace{\min\{s'', s', ..., s'\}}_{s'} - 10s'' &= 50 + 20s' - 10s'' \\ &= 50 + 10s' + 10s' - 10s'' \\ &= 50 + 10s' - 10\underbrace{(s'' - s')}_{+ \text{ since } s'' > s'} \end{aligned}$$

which is lower than $50 + 10s'$ (payoff from selecting s').

- *Intuition:* Security is not improved, but the airline's costs go up.

The airline security game

- What if, instead, the airport deviates to $s^0 < s'$? Then its payoffs is

$$
\begin{aligned}
50 + 20\underbrace{\min\{s^0, s', ..., s'\}}_{s^0} - 10s^0 &= 50 + 20s^0 - 10s^0 \\
&= 50 + 10s^0
\end{aligned}
$$

  which is lower than $50 + 10s'$ (payoff from selecting s').
- *Intuition:* reduction in actual security swamps any cost savings.

The airline security game

- Hence, $s_i = s'$ for all airports is the symmetric NE of this game.
- Seven symmetric NE, one for each security level.
- Airports are, however, not indifferent among these equilibria.

$$
\begin{aligned}
50 + 10s' &= 50 + 10 * 1 = 60 \text{ if } s' = 1 \\
&= 50 + 10 * 7 = 120 \text{ if } s' = 7
\end{aligned}
$$

- Hence, this game resembles a Pareto coordination game, since in each NE players select the same action (same security level), but some NE Pareto dominate others, i.e., payoffs are larger for all airports if they all select $s' = 7$ than if they all choose $s' = 1$.

The airline security game

- Which equilibrium emerges among the 7 possible NEs?
- Let us check that with an experiment replicating this game, using undergrads in Texas A&M as players.
- The following table reports the percentage of students choosing action $s' = 7$, $s' = 6$, ..., $s' = 1$ (in rows).
- Let us first look at their first round of interaction (second column)...
- then at their last round of interaction (third column).

Action	Round 1, Percent of Subjects Choosing that Action	Round 10, Percent of Subjects Choosing that Action
7	31%	7%
6	9%	0%
5	32%	1%
4	16%	2%
3	5%	2%
2	5%	16%
1	2%	72%

The airline security game

- That is, a "race to the bottom" is observed as subjects interact for more and more periods.
- Before you cancel your airline reservation...
  - note that pre-play communication wasn't allowed among students, whereas it is common among airports.

Check your understanding - Exercise

- "Check your understanding 5.2" in Harrington, page 125
  - (Answer at the end of the book).
- Assume that the effective security level is now determined by the highest (not the lowest) security measures chosen by airlines.
- Airline i's payoff is now:

$$50 + 20 \max\{s_1, s_2, ..., s_n\} - 10s_i$$

- Find all Nash Equilibria.

Mac versus Windows game

- Strategy set: either buy a Mac or buy a PC.
- $n \geq 2$ symmetric players.
- Payoff from buying a Mac: $100 + 10m$
- Payoff from buying a PC (we denote it as w, for Windows): $10w = 10(n - m)$ since $w + m = n$.
- Mac is assumed to be superior:
  - Indeed, if the same number of buyers purchase a Mac and a PC, i.e., $m = w$, every individual's payoff is larger with the Mac, $100 + 10m > 10m$.

Mac versus Windows game

- Here we should expect **Network effects:**
  - the more people using the same operating system that you use, the more valuable it becomes to you
  - e.g., you can share files with more people, software companies design programs for that platform since their group of potential customers grows, etc.

Mac versus Windows game

- Let us first check if "extreme" equilibria exist where all consumers buy Mac or all buy PC.
- If all buy Mac, $m = n$, the payoff of any individual is $100 + 10n$ (equilibrium payoff).
- If, instead, I deviate towards PC, I obtain only $10[n - \underbrace{(n-1)}_{m}] = 10$.
- Since the game is symmetric, we can extend the same argument to all consumers.
- Therefore, there is a NE where everybody buys a Mac.

Mac versus Windows game

- What about the other extreme equilibrium?
- If all buy PC, my payoff is $10n$, since $w = n$.
- If, instead, I deviate towards Mac, I obtain $100 + 10 * 1 = 110$.
- In order for this extreme equilibrium to exist we thus need

$$10n \geq 110, \text{ that is } n \geq \frac{110}{10} = 11$$

- Hence, if the total population is larger than 11 individuals, an equilibrium where all individuals buy PC can be sustained.

Mac versus Windows game

- We have then showed that there exist two extreme equilibria:
  - One where all players choose Mac, which can be sustained for any population size n, and
  - One where all players choose PC, which can only be sustained if the population size, n, satisfies $n \geq 11$.
- But how can we more generally characterize all equilibria in this type of games?
  - We just want to be sure we didn't miss any!

Mac versus Windows game

- Generally, I will be indifferent between buying a PC and a Mac when the payoff from a PC, $10w$, coincides with that of buying a Mac, $100 + 10(n - w)$. That is

$$10w = 100 + 10(n - w)$$

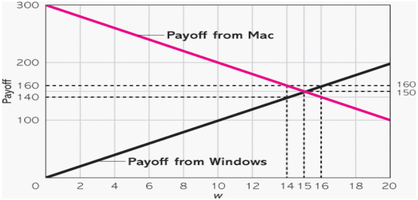
and solving for w, we obtain

$$w = 5 + \frac{1}{2}n$$

- *Example:* if, for instance, $n = 20$, payoff become $10w$ for PC, and $100 + 10(n - w) = 100 + 10(20 - w) = 300 - 10w$ for Mac.
  - The value of w that makes me indifferent becomes $w = 5 + \frac{1}{2}n = 5 + \frac{1}{2}20 = 15$.

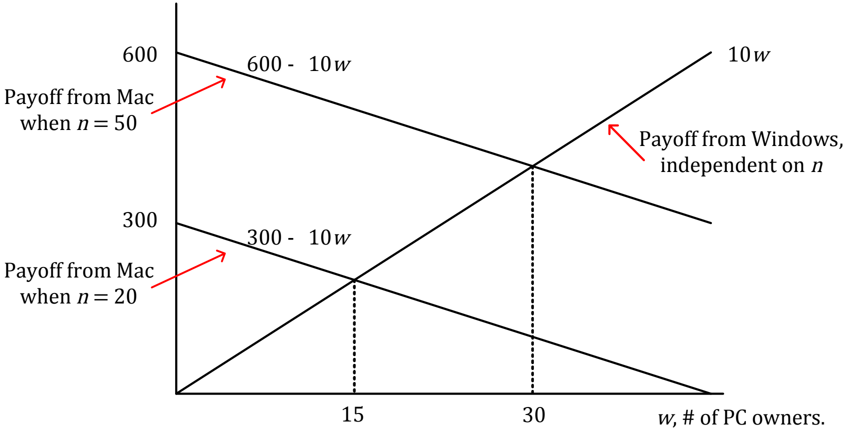
Mac versus Windows game

- The following figure depicts individual payoffs of buying a PC, $10w$, and buying a Mac, $300 - 10w$, when $n = 20$. Note that the individual is indifferent at $w = 15$.



- If $n = 50$, you can check that payoffs become $10w$ and $100 + 10(n - w) = 100 + 10(50 - w) = 600 - 10w$ for PC and Mac, respectively. (Graphically, the payoff from PC remains unaffected, but the payoff from Mac shifts upwards).
    - The value of w that makes me indifferent becomes $w = 5 + \frac{1}{2}n = 5 + \frac{1}{2}50 = 30$

Mac versus Windows game



Mac versus Windows game

- When $n > 11$, two "extreme" equilibria can therefore arise, where either all customers buy Mac or all buy PC.
- Which one actually emerges depends on customers' expectations about how many individuals will be buying/using Mac vs. Windows.
- How can a firm affect those expectations in its favor?
  - Advertising can help, but it is not sufficient:
    - A potential consumer might be swayed towards a Mac after watching the commercial, but he must be aware that many other potential buyers were swayed as well.
  - That is, it must be common knowledge that the product is compelling.
  - According to Harrington, "Perhaps the best generator of common knowledge in the U.S. is a commercial during the Super Bowl." Everyone watching it knows that almost everyone is watching it.
  - It is thus not a coincidence that the Mac was introduced in a 60-second commercial during the 1984 Super Bowl.

Applying for an internship game

- Strategy set: apply to JP Morgan (JPM) or to Legg Mason (LM).
- Players: $n \geq 2$ symmetric students.
- One available position at JPM, but 3 available in LM.
- Here there are no network effects (or tipping points).
  - Rather, we have congestion effects: the more students apply to the same internship as you do, the less likely it is that you can get it.

Applying for an internship game

- Payoffs are described in the following table with $n = 10$ students.
  - The more students that apply to JPM (first column), the lower your expected payoff becomes if you apply to JPM (second column) since you probably don't get it.
  - The more students that apply to JPM (first column), the higher your expected payoff becomes if you apply to LM instead (third column) since you probably get it.

Number of Applicants to JPM	Payoff to a JPM Applicant	Payoff to an LM Applicant
0		30
1	200	35
2	100	40
3	65	45
4	50	50
5	40	60
6	35	75
7	30	100
8	25	100
9	20	100
10	15	

Applying for an internship game

- Is there an equilibrium where no student applies to JPM? No, since you would have incentives to deviate towards JPM.
- Is there an equilibrium where only 1 student applies to JPM? No, since you would have incentives to deviate towards JPM.
- Similarly for equilibria with 2 and 3 students applying to JPM.
- Not for the case where 4 students apply to JPM (and therefore 6 apply to LM).

No student applies to JPM

Only one student applies to JPM

Number of Applicants to JPM	Payoff to a JPM Applicant	Payoff to an LM Applicant
0		30
1	200	35
2	100	40
3	65	45
4	50	50
5	40	60
6	35	75
7	30	100
8	25	100
9	20	100
10	15	

Applying for an internship game

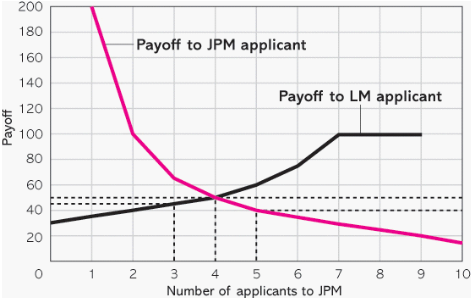
- Is there an equilibrium where all 10 students apply to JPM? No, since you would have incentives to deviate towards LM.
- Is there an equilibrium where 9 students apply to JPM? No, since you would have incentives to deviate towards LM.
- Similarly for equilibria with 8, 7, 6 and 5 students applying to JPM.

Not for the case where 4 students apply to JPM (and therefore 6 apply to LM).

Number of Applicants to JPM	Payoff to a JPM Applicant	Payoff to an LM Applicant
0		30
1	200	35
2	100	40
3	65	45
4	50	50
5	40	60
6	35	75
7	30	100
8	25	100
9	20	100
10	15	

All 10 students apply to JPM

Applying for an internship game



- Hence, although all students have the same options and preferences (i.e., the game is symmetric), an asymmetric NE exists where students make different choices: 4 choose to apply to JPM and 6 choose to apply to LM.
- Asymmetric behavior emerges from congestion effects.

Location Problem (Waston, pp. 117-118)

- 10 firms
- $S_i = \{$Locate in downtown, Locate in the suburbs$\}$.
- Payoff for every firm that locates in Downtown:

$$5n - n^2 + 50$$

  where n indicates the number of firms located in Downtown.
- Payoff for every firm that locates in the Suburbs

$$48 - m$$

  where m represents the number of firms located in the suburbs.

Location Problem

- In equilibrium, every firm cannot increase its profits by changing its location
- That is, every firm must be indifferent between locating in Downtown or in the Suburbs, as follows

$$5n - n^2 + 50 = 48 - m$$

  - and since $m + n = 10$, we have that $m = 10 - n$, which we can use in order to rewrite the above equality, as follows:

$$5n - n^2 + 50 = 48 - (10 - n)$$

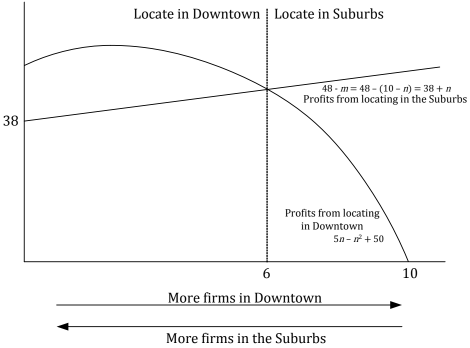
  - which simplifies to

$$-n^2 + 4n + 12 = 0$$

Location Problem

- For the quadratic equation we obtained, $-n^2 + 4n + 12 = 0$, we can find two roots:
  - $n = 6$, and
  - $n = -2$ (we can discard this solution as not meaningful)
- (See the graphical representation on the next slide).
- Threfore, the NE of this location game is that:
  - $n = 6$ firms locate in Downtown, and
  - as a consequence, the remaining $m = 10 - n = 10 - 6 = 4$ firms locate in the Suburbs.

Location Problem



- Intuitively, the location of more firms in Downtown creates congestion effects in Downtown (reducing profits), and "clears" the Suburbs, reducing congestion in the Suburbs.

Symmetric vs. Asymmetric Games

- So far, the games we have analyzed with n players were symmetric.
  - Same strategy set across players.
  - all players share the same set of available strategies, and
  - when all players choose the same strategy, $s_1 = s_2 = ...s_n = s$, their payoffs coincide, i.e., $u_1 = u_2 = ...u_n$.If we switch strategies, then their payoffs switch as well.
- What if, instead, we allow for the game to be asymmetric?
  - That is, we allow for players to be heterogeneous in their preferences?

Entry Game

- Several industries experience entry by many firms in certain periods.
  - Recent examples: MP3 players, on-line book stores, apps for mobile phones, etc.
- Understanding entry patterns can help us predict entry in other industries in the future...
  - and design policies that promote/hamper entry.
  - (We will examine entry decisions later on, when we introduce incomplete information. Hot topic in regulatory economics for decades.)
- How can we use game theory to analyze the entry decision of a potential entrant?

Entry Game

- Consider 5 potential entrants, with the following entry costs:

Company	Entry Cost
1	100
2	160
3	180
4	200
5	210

  - Lower entry costs might reflect the previous experience of the entrant in related industries, e.g., Barnes and Nobles entering the on-line business with bn.com.

- Upon entry, firms' profits are described as follows

Number of Companies	Profit per Company	Total Industry Profit
1	1,000	1,000
2	400	800
3	250	750
4	150	600
5	100	500

Entry Game

- Therefore, the net profit from entering becomes...

Company	Number of Other Companies that Enter				
	0	1	2	3	4
1	900	300	150	50	0
2	840	240	90	-10	-60
3	820	220	70	-30	-80
4	800	200	50	-50	-100
5	790	190	40	-60	-110

- How to read this table?
  - If you are in the shoes of company 2 (second row) and entered when two other companies are active (third column), your profits are $250 - 160 = 90$ (net of entry costs).

Entry Game

- Before analyzing the NEs of this game, note that there are no strictly dominated strategies:
  - If you are firm 1 (first row), entering yields a strictly higher payoff than not entering ($0) only if 3 or less firms have entered.
  - If you are firm 2 (second row), entering yields a strictly higher payoff than not entering ($0) only if 2 or less firms have entered.
    - [A similar argument is applicable to firms 3, 4 and 5.]

Company	**Number of Other Companies that Enter**				
	0	1	2	3	4
1	900	300	150	50	0
2	840	240	90	-10	-60
3	820	220	70	-30	-80
4	800	200	50	-50	-100
5	790	190	40	-60	-110

Entry Game

- But, what is the set of NEs of this game?
  - Let's start with an extreme case:
  - Checking if **zero entrants** can be a NE.
  - It cannot be, since, for instance, firm 1 would obtain \$1,000 in profits from entering, as opposed to \$0 from not entering.

Entry Game

- What is the set of NEs of this game?
  - Let us now check if **one entrant** can be a NE?
  - It cannot be. If only one firm has entered, then we are in column one (no other companies have entered, and the only entering firm is a monopoly). In this case, all firms have individual incentives to enter.

Company	Number of Other Companies that Enter				
	0	1	2	3	4
1	900	300	150	50	0
2	840	240	90	-10	-60
3	820	220	70	-30	-80
4	800	200	50	-50	-100
5	790	190	40	-60	-110

Entry Game

- What is the set of NEs of this game?
  - Let us now check if **two entrants** can be a NE?
  - It cannot be: If only two firms have entered, then we are in column two (another firm and me), and all firms have individual incentives to enter.

Company	Number of Other Companies that Enter				
	0	1	2	3	4
1	900	300	150	50	0
2	840	240	90	-10	-60
3	820	220	70	-30	-80
4	800	200	50	-50	-100
5	790	190	40	-60	-110

Entry Game

- What is the set of NEs of this game?
  - Let us now check if **three entrants** can be a NE?
  - It can be a NE. Let's see why...
  - If only three firms have entered, then we are in column three (two other firms and me).

Company	Number of Other Companies that Enter				
	0	1	2	3	4
1	900	300	150	50	0
2	840	240	90	-10	-60
3	820	220	70	-30	-80
4	800	200	50	-50	-100
5	790	190	40	-60	-110

Entry Game

- More on the NE with **three entrants:**
  - There are six possible profiles, all of them including firm 1: (1,2,3), (1,2,4), (1,2,5), (1,3,4), (1,3,5) and (1,4,5).
  - Why can't other profiles with three firms entering, such as (3,4,5) be sustained as a NE?
    - Because if firm 1 was not part of the industry, it would have incentives to enter, obtaining a net payoff of 50.

Company	Number of Other Companies that Enter				
	0	1	2	3	4
1	900	300	150 →	50	0
2	840	240	90	-10	-60
3	820	220	70	-30	-80
4	800	200	50	-50	-100
5	790	190	40	-60	-110

Entry Game

- Can we support a NE with **four entrants**? No.
  - Focus on column four (three firms and me entering): Despite firm 1 having incentives to enter a market with 3 other firms (yielding four firms in total)...
  - the profits of the other three firms (e.g., 2, 3 and 4, or any other combination) becomes negative, so they would not want to enter that market.
- A similar argument applies to the case of five entrants:four other firms and I entered, as indicated in the last column.

Company	Number of Other Companies that Enter				
	0	1	2	3	4
1	900	300	150	50	0
2	840	240	90	-10	-60
3	820	220	70	-30	-80
4	800	200	50	-50	-100
5	790	190	40	-60	-110

Entry Game

- We can hence conclude that only one "type" of NE occurs in which three firms (firm 1 and two other firms) enter the industry.
  - This implies 6 possible strategy profiles.
- If the NE is (1,2,3), then the most efficient firms are entering (those with the lowest entry costs).
- However, if the NE (1,4,5) arises, we don't have the most efficient firms entering.

Check your Understanding

- "Check your Understanding 5.3" in *Harrington*, page 134.
  - (Answer at the end of the book).
- Eliminate company 1 from the Entry game,
  - Hence, only companies 2, 3, 4 and 5 simultaneously decide whether to enter.
- Find all Nash Equilibria.

Civil Unrest Game

- Let's now have a look at an application from political science: the analysis of civil unrests and mass demonstrations in non-democratic societies that ultimately led to democracy.
  - Examples abound: Former GDR, last years of dictatorships in Spain and Portugal. More recent examples: Egypt and Hong Kong.
- We can also use game theory to analyze a citizen's individual decision to attend/not attend a demonstration.
- How to introduce asymmetric players? We do so by allowing for different costs of attending the demonstration.
  - Cost for a "radical" citizen: 6,000.
  - Cost for a "progressive" citizen: 8,000.
  - Cost for a "bourgeois" citizen: 20,000.

Civil Unrest Game

- If a citizen does not protest, his payoff is zero. If he protests, his payoff is an increasing function of the number of protesters, $50m - Cost_i$.
- Hence, a citizen protests if

$$50m - Cost_i \geq 0,$$

  where $i = \{\text{Radical, Progressive, Bourgeois}\}$.
- It is therefore easy to check that the minimal number of protesters that lead a citizen to attend the demonstration is $m \geq \frac{Cost_i}{50}$
  - This minimal cutoff (the so-called "critical mass") is $\frac{6000}{50} = 120$ for a Radical, $\frac{8000}{50} = 160$ for a Progressive and $\frac{20000}{50} = 400$ for a Bourgeois.
- In addition, let us assume that there are 100, 100, and 300 of each type of citizen, respectively.

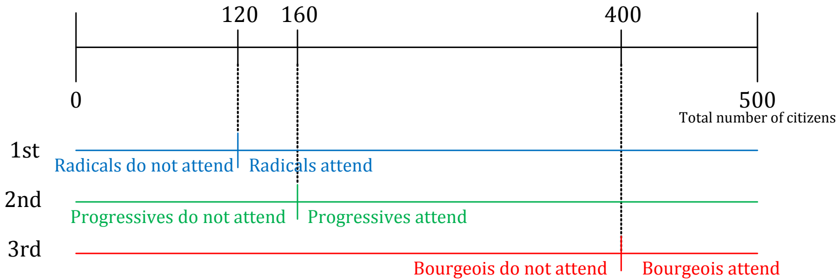
Civil Unrest Game

- The following table summarizes the initial information of this exercise.

Type of Citizen	Number of Citizens	Personal Cost	Critical Mass
Radicals	100	6,000	120
Progressives	100	8,000	160
Bourgeois	300	20,000	400

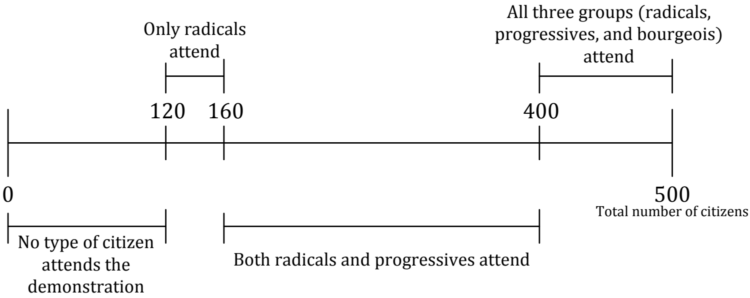
- Before we start finding the set of NEs, note a useful property of the above cutoffs (critical masses):
  - It it is optimal for a Progressive to protest, then it is also optimal for a Radical to do so.
  - If it is optimal for a Bourgeois to protest, then it is also optimal for a Radical and a Progressive to do so.
  - (see figures on next slides)

Civil Unrest Game



Civil Unrest Game

- Summarizing...



Civil Unrest Game

- Let us now find the NEs of this game.
- Starting simple, let us start with the profile in which **none of the three types protests**.
  - This can be supported as a NE: if all types of citizens believe that no one will show up in the demonstration $(m = 0)$, then the Radicals won't find it optimal to attend, nor will Progressives and Bourgeois.
- What about the profile where **only Radicals protest**?
  - This cannot be a NE: if only Radicals protest, then $m = 100$, which does not exceed the minimal cutoff for Radicals (nor does it exceed the cutoff for Progressives of Bourgeois). Hence, the Radicals won't find it optimal to attend.

Civil Unrest Game

- What about the profile where **both Radicals and Progressives protest**?
    - This can be a NE: on one hand, $m = 200$, which exceeds the minimal cutoff for Radicals and Progressives, leading both of them to attend. On the other hand, $m = 200$ is still lower than the minimal cutoff for the Bourgeois, inducing them to stay home, as prescribed by this equilibrium.
- What about the profile where **all citizens protest**?
    - This can be a NE: if $m = 500$, all types of citizens find it optimal to attend, since $m = 500$ exceeds the cutoff of all three types of citizens.

Civil Unrest Game

- Summarizing, equlibrium can involve...
  - The total absence of demonstrations, i.e., $m = 0$.
  - A modest demonstration with only Radicals and Progressives, i.e., $m = 200$.
    - We call it "modest" since these two groups only account for $40\% = \frac{100+100}{500}$ of the population.
  - A massive demonstration with full participation (all citizens), i.e., $m = 500$.

Civil Unrest Game

- Which of these multiple equilibria actually occurs depends on citizens' beliefs about participation.
  - If most people believe that the demonstration will be massive, then the last equilibrium occurs.
  - If most people believe that very few citizens will attend the demonstration, the first or second type of equilibrium occurs.
- *Example:* GDR in Sept 1989 - Feb 1990
  - Turn out started to grow from one week to the next (affecting beliefs) until massive demonstrations of 3.2 million people!
  - More details in *Harrington*, and in the article by Susanne Lohmann "The Dynamics of Informational Cascades: The Monday Demonstrations in Leipzig, East Germany," *World Politics*, 47 (1994), pp. 42-101.