Signaling games with two-sided information asymmetry

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Signaling games with two privately informed players

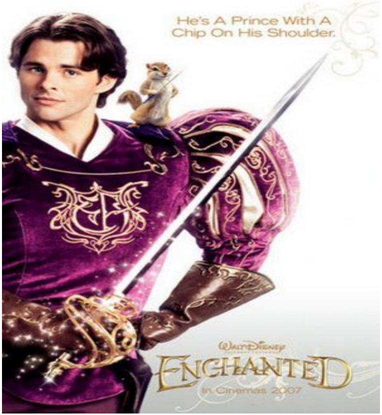
- So far we considered signaling games where only the sender was privately informed about his type.
- What if *both* the sender and the receiver are privately informed?

Courtship Game

- Harrington, pp. 337-343.
- A warning: this game is really sexist...
- To begin with, in the payoff structure we will assume that:
  - The man (Jack) wants to have sex with the woman (Rose) regardless of being in love with her,
    - "The Situation" from Jersey Shore
  - While she only wants to have sex with him if they both love each other (which leads them into marriage).
    - Not Snooky!
- We should then use this game not as a description of how things should be, but as a description of actual human relationships in certain societies (either currently or in the past, perhaps in NJ?).

Courtship Game

An example of Jack



Not Jack!!

Courtship Game

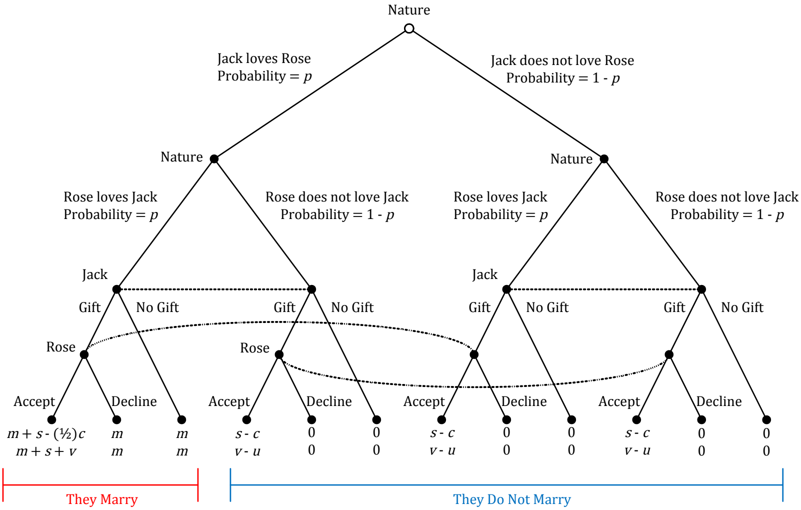
An example of Rose.

Not Rose!!

Courtship Game

- Let us first have a look at the time structure of the game by looking at the extensive form game...
    - Note that both the sender and the receiver have a private type in this game.
- and then we will explain the payoff structure.

Courtship Game



Courtship Game

- Note the position of the information sets:
  - When "Jack in love" chooses whether to make a gift to Rose, he observes his type (in love), but doesn't observe Rose's type.
    - Similarly for Jack when he is not in love.
  - When Rose must decide whether to Accept or Decline Jack's gift, she observes her own type, but doesn't observe Jack's.

Courtship Game

- If both players love each other, then they marry regardless of whether they had premarital sex.
  - Otherwise, they don't (cold feet from one of the players is enough to cancel the wedding).
- **Jack's payoffs:**
  - Jack wants to be intimate with Rose regardless of whether he loves her. The gain from having sexual relationships is $s > 0$.
  - The cost of the gift to Jack is $c > 0$, which is incurred only if it is accepted by Rose (Otherwise he can return the ring; Jack always keeps his receipts!).
  - However, the cost decreases to $\frac{c}{2}$ if he marries Rose.
    - He is so happy to see the ring on her hand...
  - The benefit from marrying the woman he loves is $m > 0$.

Courtship Game

- **Jack's payoffs:**
  - Summing up, his payoffs are
    - $m + s - \frac{c}{2}$ if he has sexual relations with Rose and they marry (because it turns out that they love each other).
    - $s - c$ if he has sexual relations, but marriage does not ensue.
    - m if he marries without "premarital intimacy."
    - 0 if he neither has sexual relations nor marries (Poor Jack!).

Courtship Game

- **Rose payoffs:**
  - The benefit of marrying the man she loves is also $m > 0$ for Rose.
  - The value of the gift for Rose is $v > 0$.
  - Rose only enjoys her "intimacy" with Jack if they both love each other, and they end up marrying.
    - In this case, if she accepts his gift and they have sexual relations, her payoff is $m + s + v$.
  - If she accepts the gift but it turns out that he didn't love her, then her payoff is $v - u$, where u is the cost of being unchaste (Think about the 1800s).
  - Her payoff is 0 if they neither have sexual relations nor marry.

Courtship Game

- Summary of Payoffs

Gift and Sex?	Jack Loves Rose?	Rose Loves Jack?	Payoff for Jack	Payoff for Rose
Yes	Yes	Yes	$m + s - (½)c$	$m + s + v$
Yes	Yes	No	$s - c$	$v - u$
Yes	No	Yes	$s - c$	$v - u$
Yes	No	No	$s - c$	$v - u$
No	Yes	Yes	m	m
No	Yes	No	0	0
No	No	Yes	0	0
No	No	No	0	0

Wedding

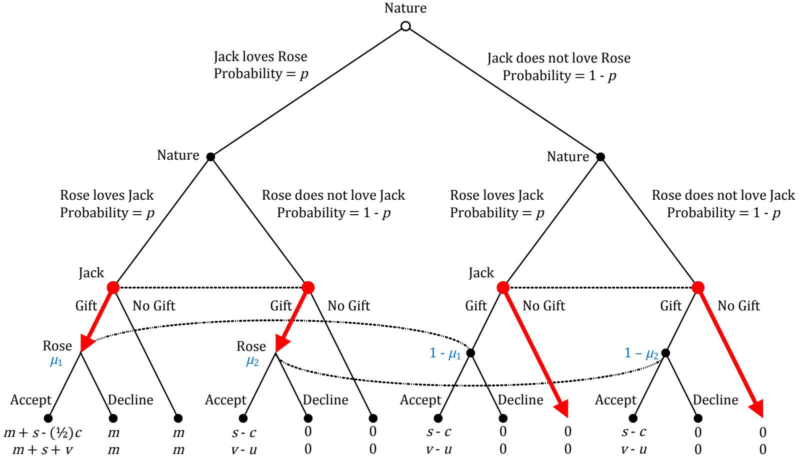
Courtship Game

- After describing the signaling game, we examine equilibrium behavior.
- As usual, we first start analyzing strategy profiles in which information is conveyed.
- In this setting, that must imply two things:
  - **SENDER. Jack's gift conveys his type (his love) to Rose:**
    - That is, Jack offers a gift to Rose if and only if he is in love with her.
  - **RECEIVER. Rose's acceptance conveys her type (her love) to Jack:**
    - That is, Rose accepts Jack's gift if and only if she is in love with him.

Courtship Game

- **We will hence consider the following separating PBE:**
  - Jack offers a gift to Rose if and only if he is in love with her.
  - Rose accepts Jack's gift if and only if she is in love with him.
  - Rose's beliefs are:
    - if Jack offers me a gift, then he loves with with probability 1.
    - If Jack doesn't offer a gift, then he doesn't love with with probability 1.
- The following figure illustrates this separating strategy profile.

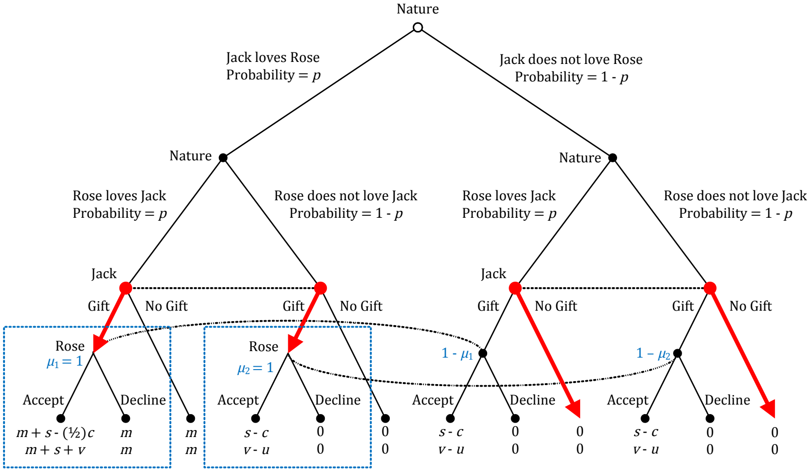
Courtship Game



Courtship Game

- **Beliefs:**
  - We denote $(\mu_1, 1 - \mu_1)$ for the case in which Rose loves Jack, and $(\mu_2, 1 - \mu_2)$ for the case in which she doesn't, with the property that $\mu_1 = 1$ and $\mu_2 = 1$.
  - Intuitively, Rose assigns full probability to Jack loving her after observing that he makes a gift, regardless of her type (i.e., regardless of her feelings for him).

Courtship Game

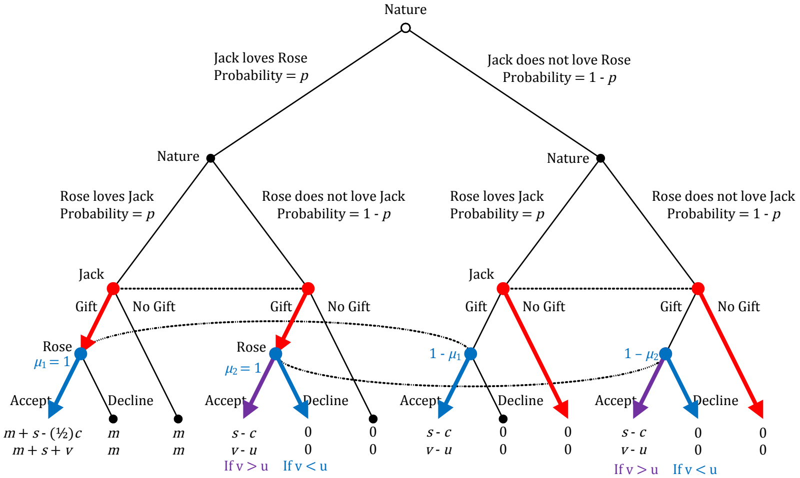


Courtship Game

- **Second mover's strategy (Rose):**
  - *When she is in love with Jack:* she accepts the gift since $m + s + v > m \iff s + v > 0$.
  - *When she is not in love with Jack:* she accepts the gift iff $v - u > 0 \iff v > u$.
    - Intuition: What a ring!
    - Otherwise, she declines the gift.
  - In this PBE we considered that Rose accepts Jack's gifts if and only if she is in love with him.
    - Therefore, we must have $v < u$.
    - Intuition for $v < u$: the social cost of being unchaste is too high.

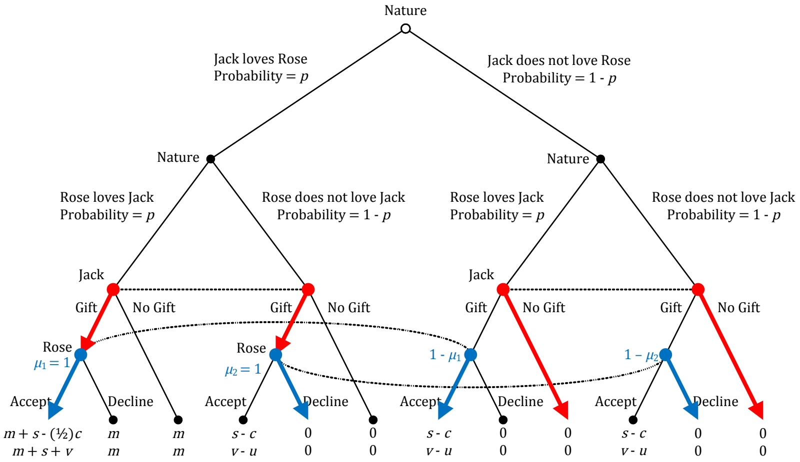
Courtship Game

- Depicting Rose's responses in the game tree



Courtship Game

- **Case 1:** where $v < u$



Courtship Game

- **First mover's strategy (Jack):**
  - When he loves Rose, he makes a gift (as prescribed in this PBE) if and only if

$$p\left(m+s-\frac{c}{2}\right)+(1-p)0 \geq pm+(1-p)0$$

$$\iff m+s-\frac{c}{2}>m \iff 2s \geq c$$

  - When he doesn't love Rose, he doesn't make a gift (as prescribed in this PBE) if and only if

$$p(s-c)+(1-p)0 \leq p0+(1-p)0$$

$$\iff s \leq c$$

- Combining both conditions, we have $2s \geq c \geq s$.

Courtship Game

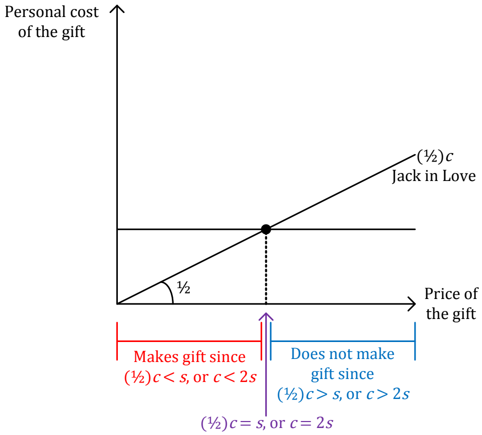
- **Summarizing**:
  - It is optimal for Jack to offer a gift to Rose when he loves her if the gift is not too expensive: $c \leq 2s$.
  - It is optimal for Jack to NOT offer a gift to Rose when he doesn't love her if the gift is too expensive: $s \leq c$.
    - Alternatively, you could interpret this as that s was relatively low (ugly Rose!).
  - Rose accepts a gift only when she is in love with Jack: $v < u$.
- The gift should then be expensive, but not too expensive (so Jack can afford it if he is in love), and
  - it must also be something that Rose doesn't value too much.

Courtship Game

- Common property in signaling games, for a separating PBE to be sustained:
  - The signal must be more costly for one type of sender than for another, e.g., education.
  - But still affordable for a type of sender (otherwise no sender chooses to send such a signal).
- Let us put the previous two conditions (costly ring, but not too costly), together in a figure.
  - We will depict the price of the gift, c, on the horizontal axis.

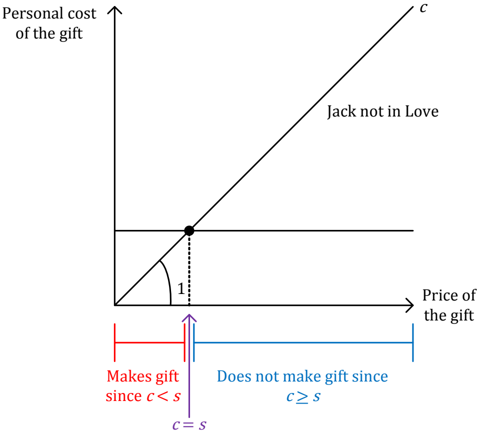
Courtship Game

- Jack "In Love" makes a gift if $c \leq 2s$. (or if $\frac{c}{2} \leq s$).



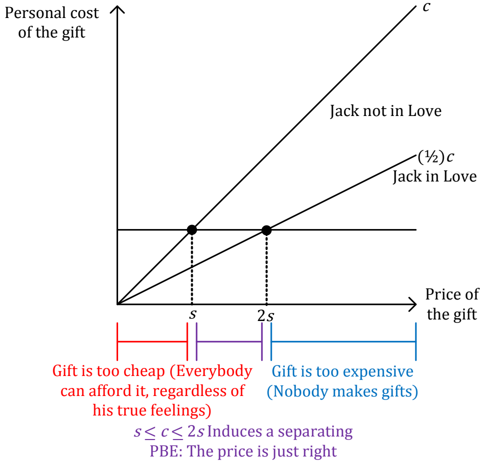
Courtship Game

- Jack "Not In Love" makes a gift if $c < s$.



Courtship Game

- Putting both conditions together:



Courtship Game

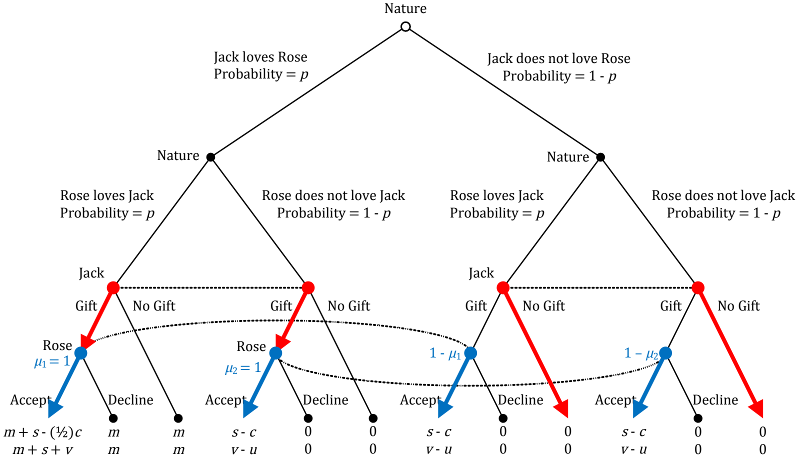
- When did the custom of offering a diamond engagement ring arose?
  - During the 1930s.
  - Before that time, many states had laws regarding "breach of promise" in which a woman could sue a fiancee who had broken off their engagement, deterring some sham engagements.
  - There laws were repealed during the 1930s.
  - Interestingly, it was during that time that the custom of offering a diamond engagement ring arose.
    - Guys needed a signal!
  - But, are they still an informative signal? Sometimes you hear things like "buy one, and get one free."
    - Too low c can inhibit the emergence of PBEs.

Courtship Game

- We have analyzed what we can refer as "Case 1," in which $v < u$.
- But, what if, instead, $v > u$?
  - Now Rose accepts the gift regardless of her true feelings. What a ring!!
- Rose's response are depicted in the following figure.

Courtship Game

- **Case 2:** where $v > u$.



- We must then go back to Jack's optimal strategy, to see if we can still support the above PBE.
  - When he **loves** Rose, he makes a gift (as prescribed in this PBE) if and only if

$$p\left(m+s-\frac{c}{2}\right)+(1-p)(s-c)\geq pm+(1-p)0$$

$$\iff \frac{2s}{2-p}\geq c$$

  - When he **doesn't love** Rose, he doesn't make a gift (as prescribed in this PBE) if and only if

$$p(s-c)+(1-p)(s-c)\leq p0+(1-p)0$$

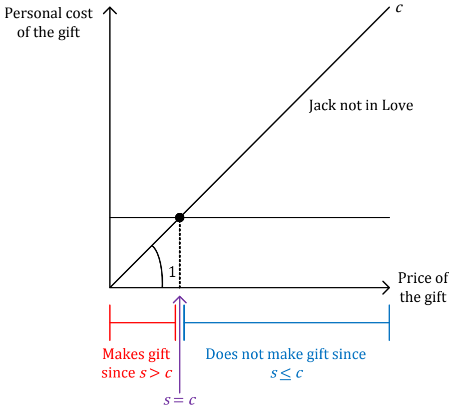
$$\iff s-c\leq 0 \implies s\leq c$$

  - Combining both conditions, $\frac{2s}{2-p}\geq c\geq s$.

Courtship Game

- We then need two conditions:

1) When Jack is "Not in Love," he doesn't make a gift (as prescribed in this equilibrium) if $s \leq c$.



Courtship Game

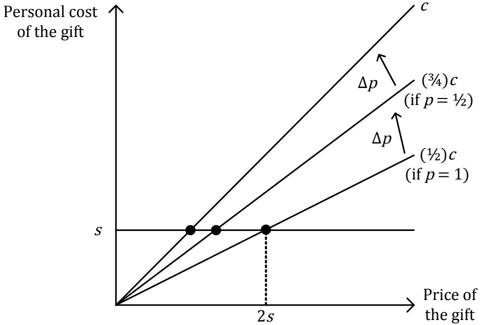
2) Jack "In Love" makes a gift (as prescribed in this equilibrium) if

$$c \leq \frac{2s}{2-p}$$

for simplicity, let's draw this cutoff for different values of p (Figure on next slide).

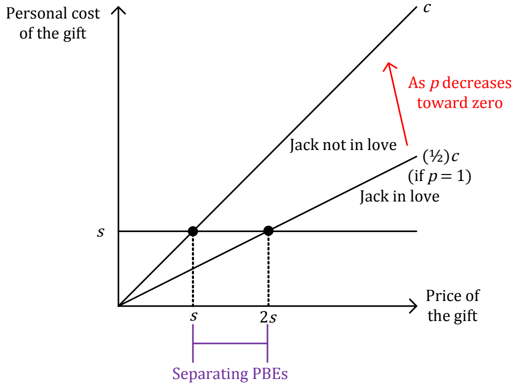
$$\text{If } p = 1,\ c \leq \frac{2s}{1} \iff c \leq 2s \iff \frac{1}{2}c \leq s$$
$$\text{If } p = \frac{1}{2},\ c \leq \frac{2s}{\frac{3}{2}} \iff c \leq \frac{4s}{3} \iff \frac{3}{4}c \leq s$$
$$\text{If } p = 1,\ c \leq \frac{2s}{2} \iff c \leq s$$

Courtship Game



Courtship Game

- Putting both conditions together



Courtship Game

- Intuitively, when $v > u$, and Rose accepts the gift regardless of her feelings...
  - the separating PBE can be supported for a larger set of prices, c, the more likely it is that Rose is in love with him.
- When the probability that Rose loves him decreases, the range of prices for which a separating PBE can be sustained shrinks.
- In the limit, when the probability that Rose loves him is really low ($p \longrightarrow 0$), there is no range of prices for which a separating PBE can be sustained.

Courtship Game

- If you prefer an algebraic approach, note that the range of c's for which the separating PBE can be sustained is:

$$c \in \left[s, \frac{2s}{2-p} \right]$$

- Hence, when $p = 1$, this range of c's becomes $c \in [s, 2s]$.
  - which coincides with the range of c's that supports the separating PBE we found for the case in which $v < u$.
- When $p = \frac{1}{2}$, this range of c's shrinks to $c \in \left[s, \frac{4s}{3}\right]$.
- When $p = 0$, this range of c's further shrinks to $c \in [s, s]$, i.e., null set.