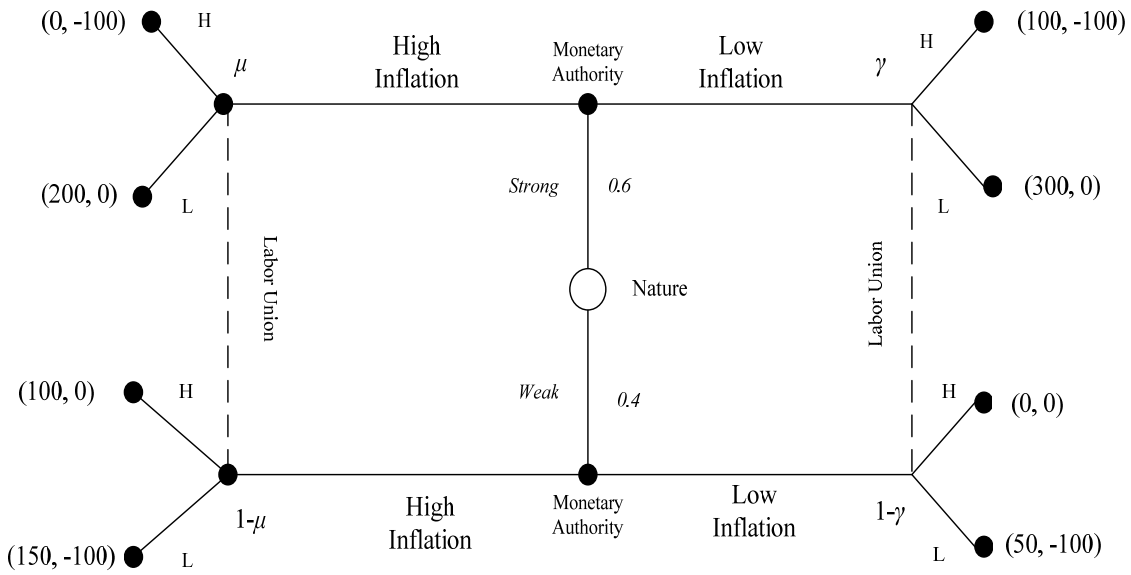
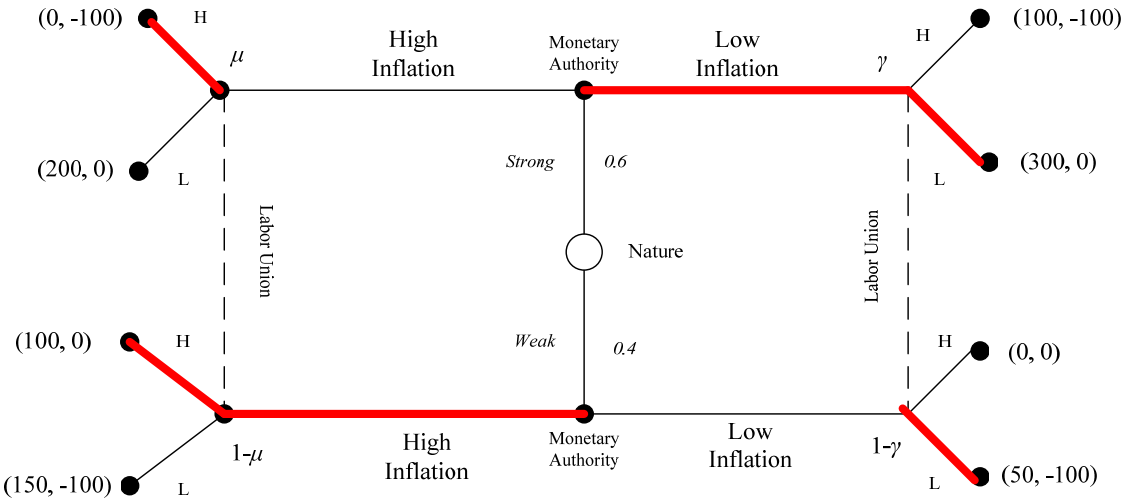
ECONS 491 – STRATEGY AND GAME THEORY
HANDOUT ON PERFECT BAYESIAN EQUILIBRIUM- II

Let us consider the following sequential game with incomplete information. A monetary authority (such as the Federal Reserve Bank, or the European Central Bank) privately observes his real degree of commitment with maintaining low inflation levels in its country or region. After knowing its type (either a Strong commitment or a Weak commitment with low inflation policies), the monetary authority decides whether to announce that the expectation for inflation during that particular period is going to be High or Low.

A labor union, observing the message sent by the monetary authority (but not its real type), decides whether to ask for high increases in the wage level during their yearly increases (denoted as H), or to go for more moderate increases in their wages (represented by the letter L).



1. ***Separating PBE with a Strong monetary authority sending Low inflation level, and a Weak monetary authority sending High: (Low, High) – Reasonable!***



a) Labor Union's beliefs (responder beliefs) in this separating PBE

- $\mu = 0$, i.e., after observing a High inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Weak.
- $\gamma = 1$, i.e., after observing a Low inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Strong.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

- After observing High inflation, the Labor Union responds with H since beliefs are $\mu = 0$ and 0>-100.
- After observing Low inflation, the Labor Union responds with L since beliefs are $\gamma = 1$, and 0>-100.

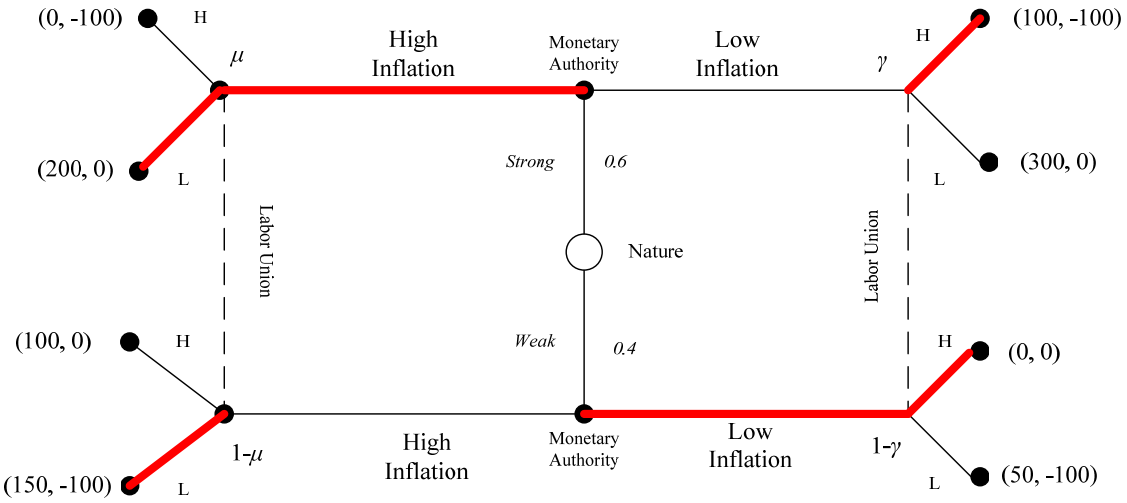
c) Given the previous points, what is the monetary authority optimal action (whether to announce High or Low inflation levels) when it is a Strongly committed central bank? What is its optimal action when it is a Weakly committed central bank?

- Strong monetary authority selects Low inflation (as prescribed in this strategy profile) since 300>0.
- Weak monetary authority selects High inflation (as prescribed in this strategy profile) since 100>50.

d) Can this separating PBE be supported from your answer in *c)*?

Yes.

2. Separating PBE with a Strong monetary authority sending High inflation level, and a Weak monetary authority sending Low: (High, Low) – Crazy!



a) Labor Union's beliefs (responder beliefs) in this separating PBE

- $\mu = 1$, i.e., after observing a High inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Strong.
- $\gamma = 0$, i.e., after observing a Low inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Weak.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

- After observing High inflation, the Labor Union responds with L since beliefs are $\mu = 1$ and 0>-100.
- After observing Low inflation, the Labor Union responds with H since beliefs are $\gamma = 0$, and 0>-100.

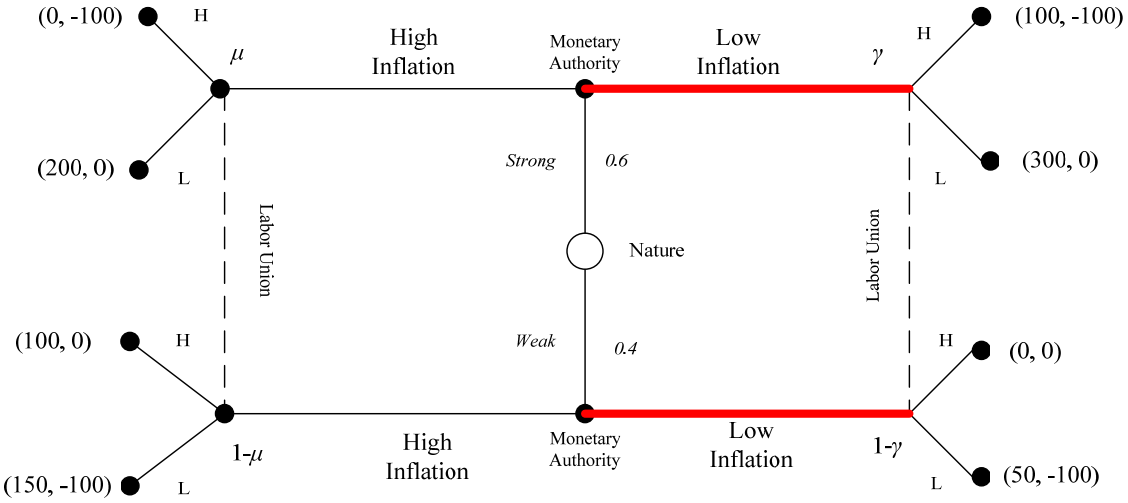
c) Given the previous points, what is the monetary authority optimal action (whether to announce High or Low inflation levels) when it is a Strongly committed central bank? What is its optimal action when it is a Weakly committed central bank?

- Strong monetary authority selects High inflation (as prescribed in this strategy profile) since 200>100.
- Weak monetary authority selects High inflation (contradicting this strategy profile) since 150>0.

d) Can this separating PBE be supported from your answer in *c)*?

No, since the Weak type of monetary authority has incentives to deviate towards High inflation.

3. *Pooling PBE with both types of Monetary authorities forecast a Low inflation : (Low, Low)*



a) Labor Union's beliefs (responder beliefs) in this pooling PBE

Using Bayes' rule, the Labor Union's beliefs after observing Low inflation (in equilibrium) are

$$\gamma = \frac{p * 1}{p * 1 + (1-p) * 1} = \frac{p}{p + 1 - p} = p = 0.6$$

And after observing High inflation (off-the-equilibrium), beliefs are

$$\mu = \frac{p * 0}{p * 0 + (1-p) * 0} = \frac{0}{0}$$

Thus, off-the-equilibrium beliefs cannot be specified using Bayes' rule, and must be left undefined in the entire range of probabilities $\mu \in [0,1]$.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

After observing **Low inflation**, let us evaluate the Labor Union's expected utility from responding with H and with L, respectively.

$$EU_{Labor}(H|Low\ Inflation) = -100 * .06 + 0 * .04 = -60$$
$$EU_{Labor}(L|Low\ Inflation) = 0 * 0.6 + (-100) * .04 = -40$$

Therefore, after observing Low inflation, the Labor Union prefers L, since it yields a higher expected utility.

Let us now move to the case in which the Labor Union observes **High inflation**. We know that this occurs off-the-equilibrium path, but we must find which the optimal response by the Labor Union is if it ever observes such announcement from the monetary authority.

$$EU_{Labor}(H|High) = -100\mu + 0 * (1-\mu) = -100\mu$$
$$EU_{Labor}(L|High) = 0 * \mu + (-100)(1-\mu) = -100 + 100 * \mu$$

Hence, the Labor union chooses H after observing a High inflation announcement if

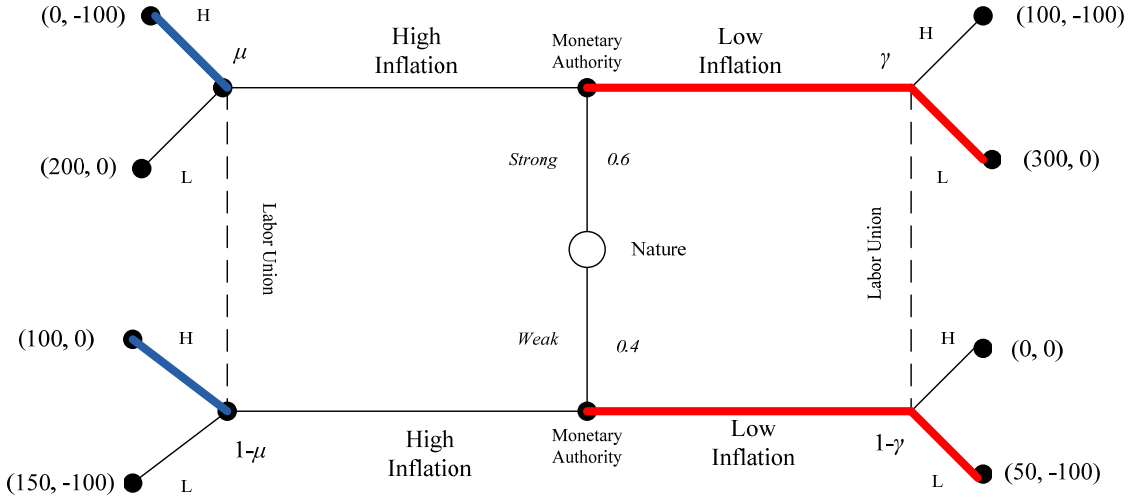
$$-\mu > -1 + \mu, \text{ or } \frac{1}{2} > \mu$$

We will then need to divide our following analysis into two cases:

- ***Case 1:*** $\mu < \frac{1}{2}$, where the Labor Union responds with H after observing High inflation.
- ***Case 2:*** $\mu > \frac{1}{2}$, where the Labor Union responds with L after observing High inflation.

CASE 1: $\boldsymbol{\mu < \frac{1}{2}}$

Hence, in the case that μ<1/2, we have that the Labor Union (responder) chooses H in the out-of-equilibrium event that the Monetary Authority announces a High Inflation forecast. The next figure helps you illustrate this case, i.e., shade the H branch for the Labor Union in the left-hand side of the figure.

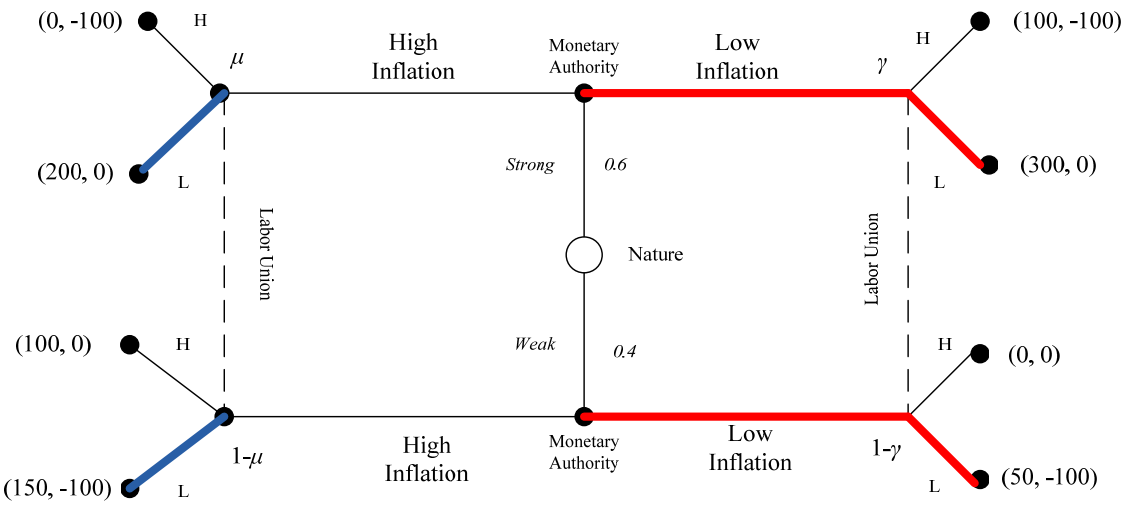


Can this pooling strategy profile be sustained as a PBE?

- **No**, since the Weak type of monetary authority prefers to deviate towards High inflation, given that 100>50.

CASE 2: $\boldsymbol{\mu > \frac{1}{2}}$

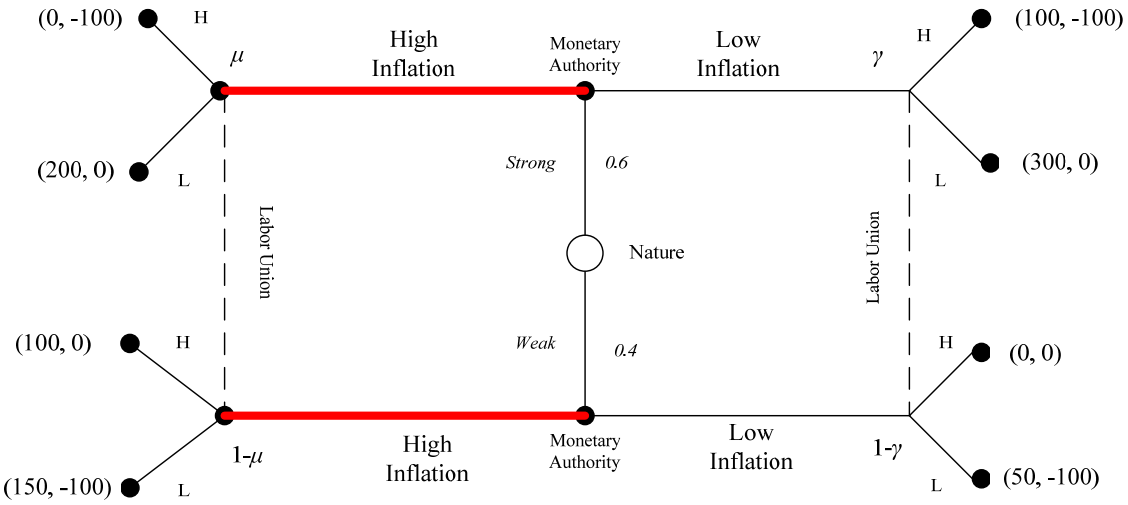
And in the case that μ>1/2, we have that the Labor Union (responder) chooses L in the out-of-equilibrium event that the Monetary Authority announces a High Inflation forecast. The next figure helps you illustrate this case, i.e., shade the L branch for the Labor Union in the left-hand side of the figure.



Can this pooling strategy profile be sustained as a PBE?

- **No**, since the Weak type of monetary authority prefers to deviate towards High inflation, given that 150>50.

4. Pooling PBE with both types of Monetary authorities forecast a High inflation : (High, High)



a) Labor Union's beliefs (responder beliefs) in this pooling PBE

Using Bayes' rule, the Labor Union's beliefs after observing High inflation (in equilibrium) are

$$\mu = \frac{p * 1}{p * 1 + (1-p) * 1} = \frac{p}{p + 1 - p} = p = 0.6$$

And after observing Low inflation (off-the-equilibrium), beliefs are

$$\gamma = \frac{p * 0}{p * 0 + (1-p) * 0} = \frac{0}{0}$$

Thus, off-the-equilibrium beliefs cannot be specified using Bayes' rule, and must be left undefined in the entire range of probabilities $\gamma \in [0,1]$.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

- After observing **High inflation**, let us evaluate the Labor Union's expected utility from responding with H and with L, respectively.

$$EU_{Labor}(H|High\ inflation) = -100 - 0.6 + 0 * 0.4 = -60$$
$$EU_{Labor}(L|High\ inflation) = 0 * 0.6 + (-100) * 0.4 = -40$$

Therefore, after observing High inflation, the Labor Union prefers L, since it yields a higher expected utility.

Let us now move to the case in which the Labor Union observes **Low inflation**. We know that this occurs off-the-equilibrium path, but we must find which the optimal response by the Labor Union is if it ever observes such announcement from the monetary authority.

$$EU_{Labor}(H|Low\ inflation) = -100\gamma + 0 * (1-\gamma) = -100\gamma$$
$$EU_{Labor}(L|Low\ inflation) = 0 * \gamma + (-100)(1-\gamma) = -100 + 100\gamma$$

Hence, the Labor union chooses H after observing a Low inflation announcement if

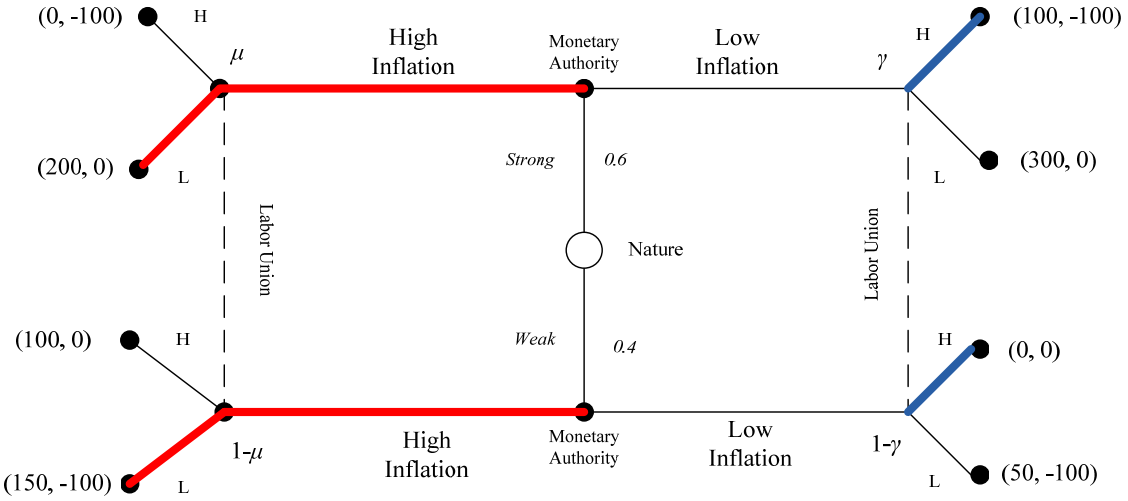
$$-\gamma > -1 + \gamma, \text{ or } \frac{1}{2} > \gamma$$

We will then need to divide our following analysis into two cases:

- ***Case 1:*** $\gamma < \frac{1}{2}$, where the Labor Union responds with H after observing Low inflation.
- ***Case 2:*** $\gamma > \frac{1}{2}$, where the Labor Union responds with L after observing Low inflation.

CASE 1: $\boldsymbol{\gamma < \frac{1}{2}}$

Hence, in the case that γ<1/2, we have that the Labor Union (responder) chooses H in the out-of-equilibrium event that the Monetary Authority announces a Low Inflation forecast. The next figure helps you illustrate this case, i.e., shade the H branch for the Labor Union in the right-hand side of the figure.



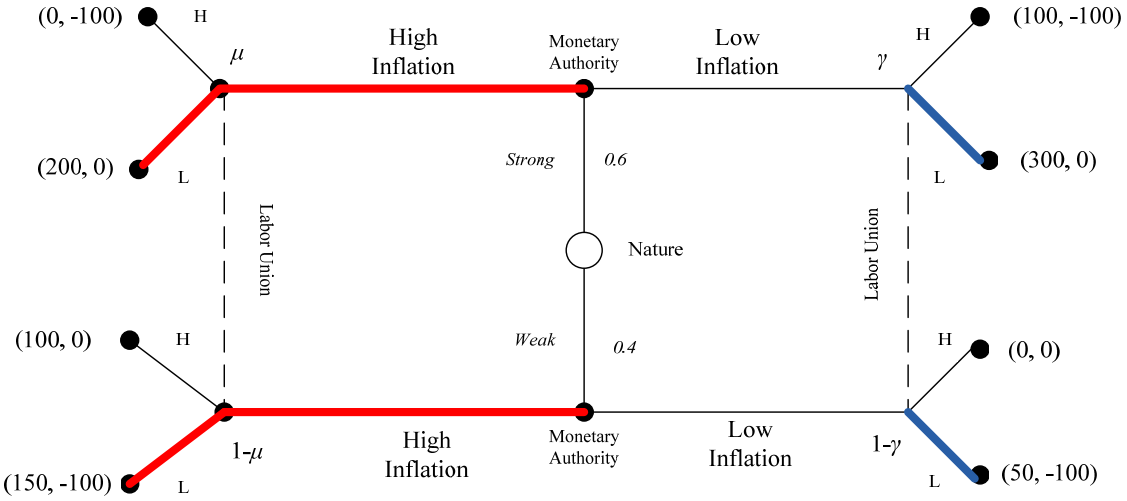
Can this pooling strategy profile be sustained as a PBE?

- **Yes:**
  - The Strong type of monetary authority obtains 200 by choosing a High inflation announcement (as prescribed in this strategy profile), but obtains only 100 if it deviates towards Low inflation (which is responded with H as indicated in the figure).
  - Similarly, the Weak type of monetary authority obtains 150 by choosing a High inflation announcement (as prescribed in this strategy profile), but obtains only 0 if it deviates towards Low inflation (which is responded with H as indicated in the figure).

  Since no type of first mover (monetary authority) has incentives to deviate from the prescribed pooling strategy, we can claim that this strategy profile can be sustained as a PBE when off-the-equilibrium beliefs satisfy $\gamma < \frac{1}{2}$.

CASE 2: $\gamma > \frac{1}{2}$

And in the case that γ>1/2, we have that the Labor Union (responder) chooses L in the out-of-equilibrium event that the Monetary Authority announces a High Inflation forecast. The next figure helps you illustrate this case, i.e., shade the L branch for the Labor Union in the right-hand side of the figure.



Can this pooling strategy profile be sustained as a PBE?

- **No**, since the Strong type of monetary authority has incentives to deviate towards Low inflation, given that 150>50.