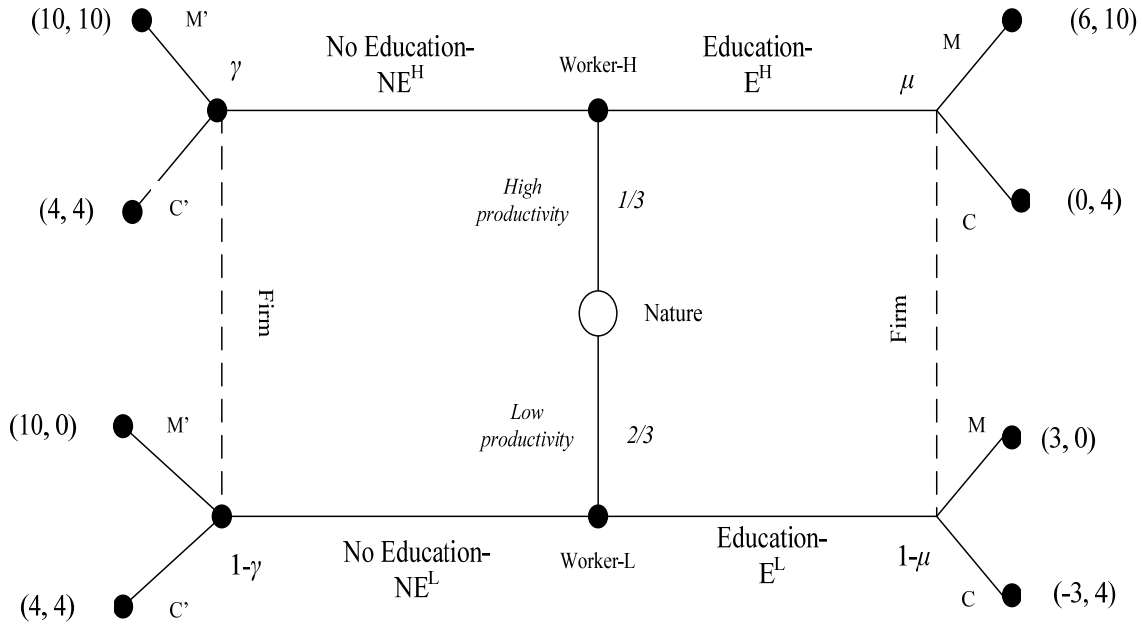
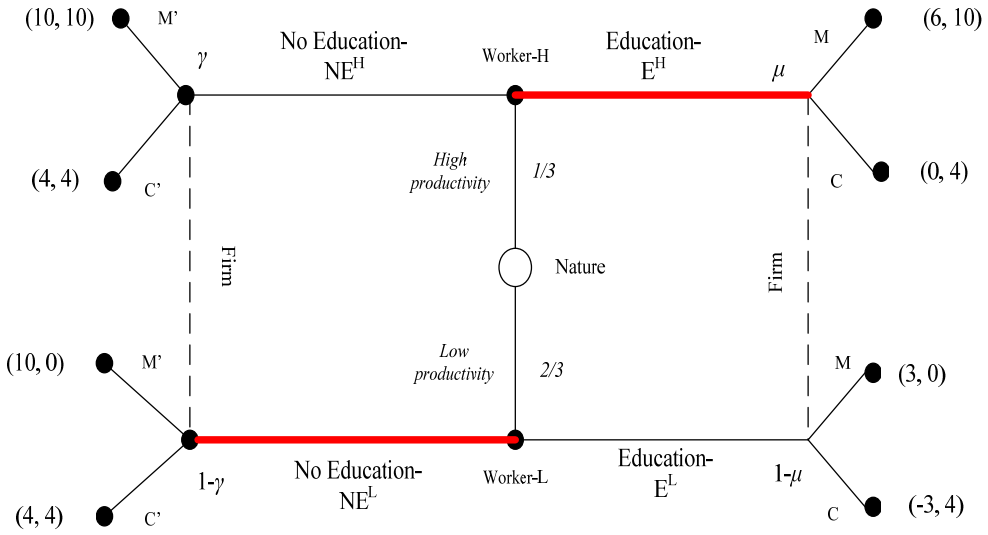
ECONS 491 – STRATEGY AND GAME THEORY[1]
SIGNALING IN THE LABOR MARKET

Let us consider the following sequential game with incomplete information. A worker privately observes whether he has a High productivity or a Low productivity, and then decides whether to acquire some education that he will be able to use as a signal about what his productivity level is. The firm that is thinking in hiring him can either hire him as a manager (M) or as a cashier (C). But the firm does not observe the real productivity level of the worker, but only how whether the worker decided to acquire college education or not.



[1] Felix Munoz-Garcia, School of Economic Sciences, Washington State University. Pullman, WA 99163, fmunoz@wsu.edu.

1. ***Separating PBE with High productivity workers acquiring education, and Low productivity workers Not acquiring education: (E^H, NE^L)***



a) Firm's beliefs (responder beliefs) in this separating PBE

$\mu = 1$, i.e., after observing education, the firm concentrates all its beliefs on the worker being highly productive
$\gamma = 0$, i.e., after observing no education, the firm concentrates all its beliefs on the worker being low production

b) Given the firm's beliefs, which is the firm's optimal action, after observing every possible education level from the worker?

M after education, since $\mu = 1$ and 10> 4
C' after no education, since $\gamma = 0$ and 4> 0

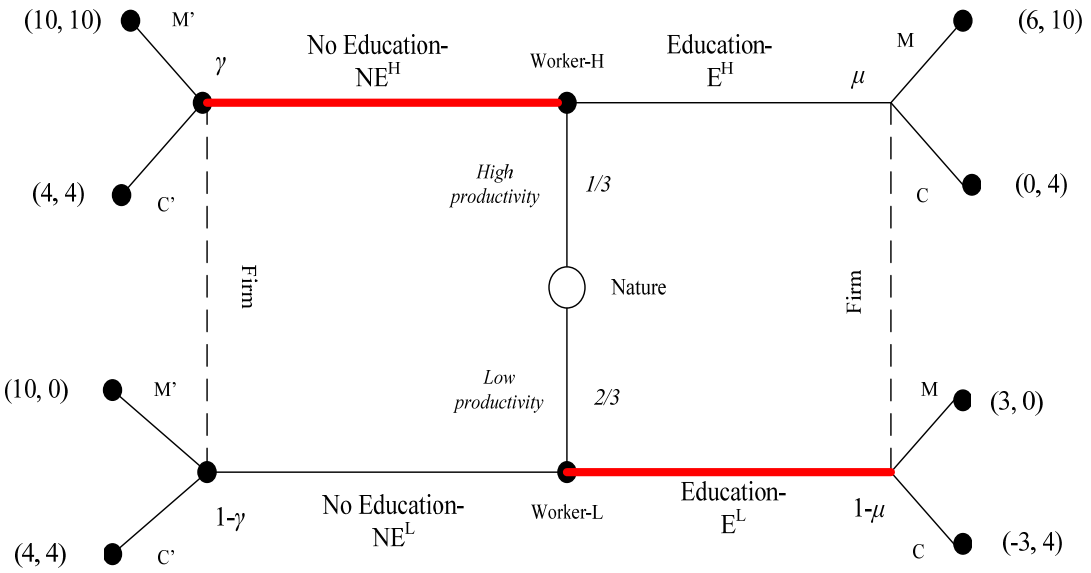
c) Given the previous points, what is the worker optimal action (whether to acquire education or not) when he is High productivity type? What is his optimal action when he is Low productivity type?

The high-productivity worker chooses E^H since 6> 4
The low-productivity worker chooses NE^L since 4> 3

d) Can this separating PBE be supported from your answer in *c)*?

Yes.

2. Separating PBE with Low productivity workers acquiring education, and High productivity workers Not acquiring education: (NE^H, E^L) – Crazy, doesn't it?



a) Firm's beliefs (responder beliefs) in this separating PBE

μ=0, i.e., after observing education, the firm concentrates all its beliefs on the worker being low productivity
γ=1, i.e., after observing no education, the firm concentrates all its beliefs on the worker being high productivity

b) Given the firm's beliefs, which is the firm's optimal action, after observing every possible education level from the worker?

C after education, since μ=0 and 4> 0
M after no education, since γ=1 and 10> 4

c) Given the previous points, what is the worker optimal action (whether to acquire education or not) when he is High productivity type? What is his optimal action when he is Low productivity type?

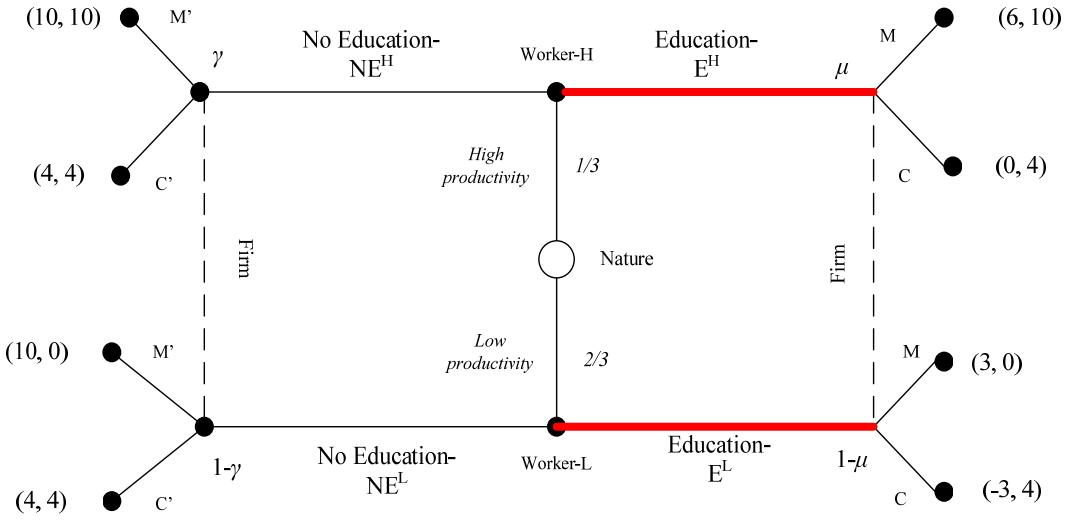
The high-productivity worker chooses NE^H since 10> 0

The low-productivity worker chooses NE^L since 10> -3. (He deviates from the prescribed strategy of E^L).

d) Can this separating PBE be supported from your answer in *c)*?

No, since the low-productivity worker has incentives to deviate to NE^L.

3. *Pooling PBE with both types of workers acquiring education: (E^H, E^L)*



a) Firm's beliefs (responder beliefs) in this pooling PBE

Using Bayes' rule,
After observing education (in equilibrium.)

$$\mu = \frac{p * 1}{p * 1 + (1-p) * 1} = \frac{p}{p + 1 - p} = p = \frac{1}{3}$$

And after observing no education (off the equilibrium), $\gamma \in [0,1]$.

b) Given the firm's beliefs, which is the firm's optimal action, after observing every possible education level from the worker?

After observing education, $EU_{Firm}(M) = 10 * \frac{1}{3} + 0 * \left(1 - \frac{1}{3}\right) = \frac{10}{3}$

$$EU_{Firm}(C) = 4 * \frac{1}{3} + 4 * \left(1 - \frac{1}{3}\right) = 4$$

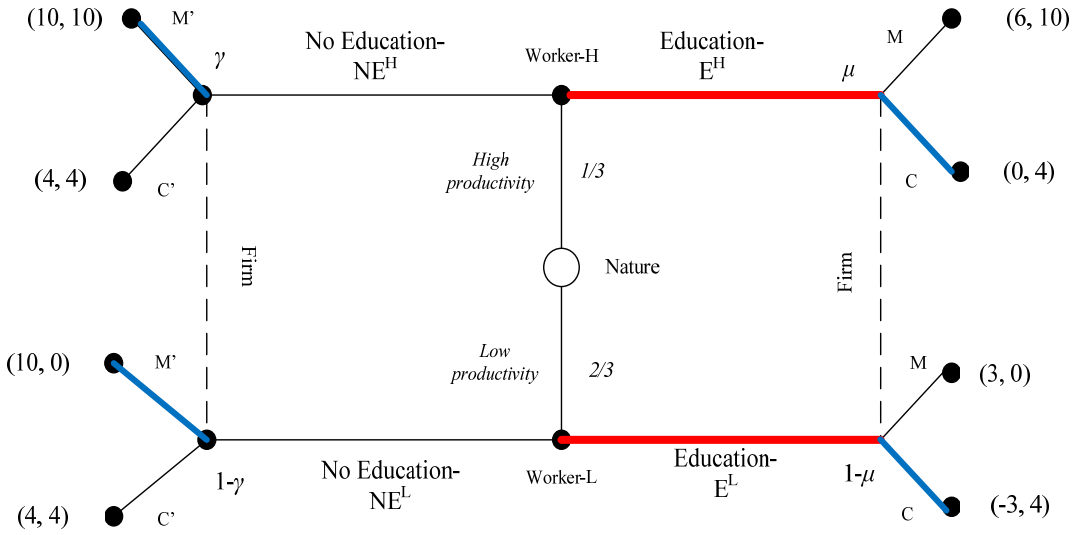
Hence, the firm hires the worker as a cashier (C) since $4 > \frac{10}{3}$.

Similarly, after observing no education, $EU_{Firm}(M') = 10 * \gamma + 0 * (1-\gamma) = 10\gamma$

$$EU_{Firm}(C') = 4 * \gamma + 4 * (1-\gamma) = 4$$

Hence, the firm hires the worker as a manager (M') if $10\gamma > 4 \; or \; \gamma > 2/5$.

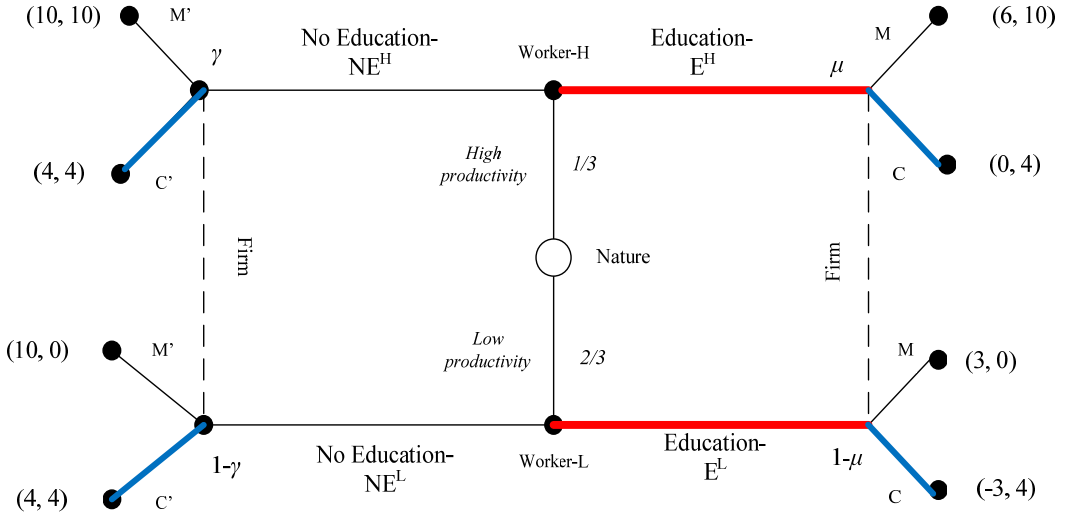
Hence, in the case that γ>2/5, we have that the firm chooses to hire the worker as a manager in the out-of-equilibrium event that the worker does not acquire education. The next figure illustrates this case.



Can this pooling strategy profile be sustained as a PBE?

No, the high-productivity worker has incentives to deviate towards NE^H since 10> 0. (Note that the low-productivity worker also has incentives to deviate to NE^L since 10> -3.)

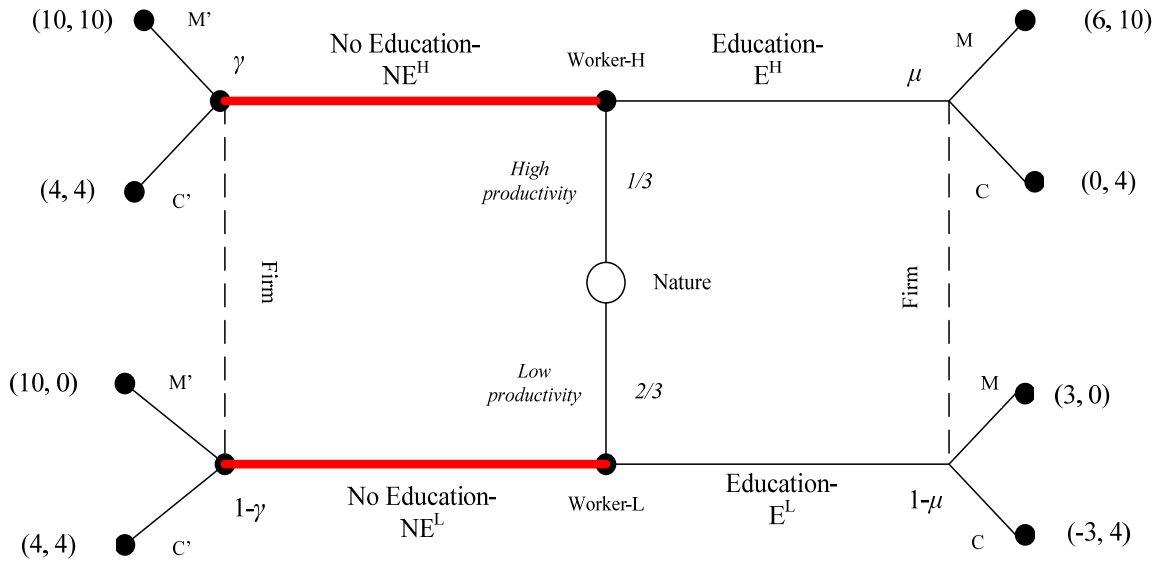
And in the case that γ<2/5, we have that the firm chooses to hire the worker as a cashier in the out-of-equilibrium event that the worker does not acquire education. The next figure illustrates this case.



Can this pooling strategy profile be sustained as a PBE?

No, the high-productivity worker has incentives to deviate towards NE^H since 4> 0. (Note that the low-productivity worker also has incentives to deviate since 4> -3.)

4. Pooling PBE with both types of workers NOT acquiring education: (NE, NE')



a) Firm's beliefs (responder beliefs) in this pooling PBE

Using Bayes' rule, after observing no education (in equilibrium), the firm's beliefs are:

$$\gamma = \frac{p * 1}{p * 1 + (1-p) * 1} = \frac{p}{1} = p$$

Whereas after observing education (off the equilibrium), the firm's beliefs are $\mu \in [0,1]$.

b) Given the firm's beliefs, which is the firm's optimal action, after observing every possible education level from the worker?

After observing no education, $EU_{Firm}(M') = 10 * \frac{1}{3} + 0 * \left(\frac{2}{3}\right) = \frac{10}{3}$

$$EU_{Firm}(C') = 4 * \frac{1}{3} + 4 * \left(\frac{2}{3}\right) = 4$$

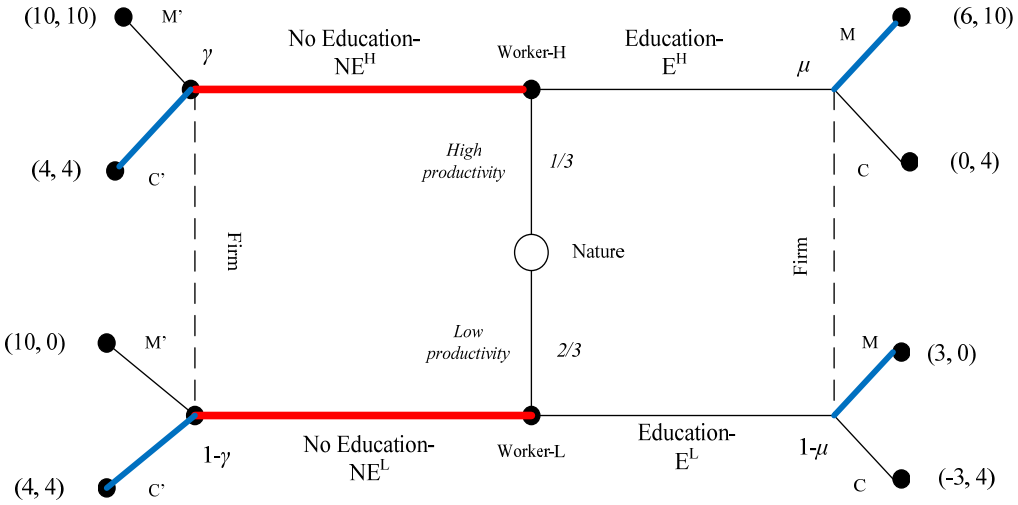
Hence, the firm hires the worker as a cashier (C') after observing no education.

Similarly, after observing no education, $EU_{Firm}(M) = 10 * \mu + 0 * (1-\mu) = 10\mu$

$$EU_{Firm}(M) = 4 * \mu + 4 * (1-\mu) = 4$$

Hence, the firm hires the worker as a manager (M) only if $10\mu > 4 \; or \; \mu > 2/5$ after observing education.

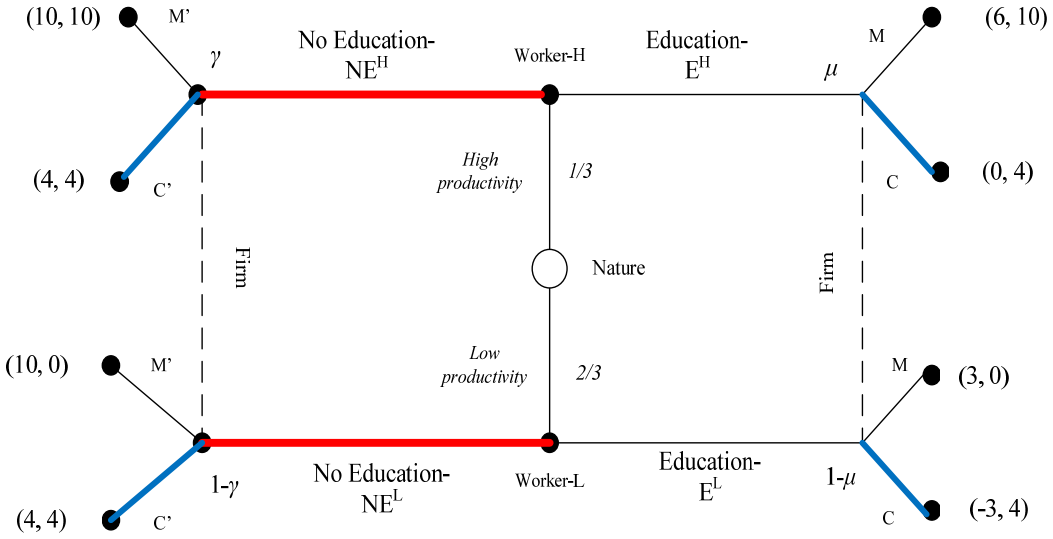
Hence, in the case that μ>2/5, we have that the firm chooses to hire the worker as a manager in the out-of-equilibrium event that the worker acquires education. The next figure illustrates this case.



Can this pooling strategy profile be sustained as a PBE?

No, the high-productivity worker has incentives to deviate towards E^H since 6> 4.

Hence, in the case that μ<2/5, we have that the firm chooses to hire the worker as a cashier in the out-of-equilibrium event that the worker acquires education. The next figure illustrates this case.



Can this pooling strategy profile be sustained as a PBE?

Yes: The high-productivity worker does not want to deviate towards E^H since his payoff from NE^H, 4, is larger than that from deviating towards E^E, 0.

Similarly, the low-productivity worker does not want to deviate towards E^L since his payoff from selecting NE^L, 4, is larger than that from deviating to E^L, -3.

Hence, the pooling strategy profile where no type of worker acquires education can be sustained as a PBE when off-the-equilibrium beliefs are $\mu < \frac{2}{5}$.