Applications of BNE:
Information aggregation among several players.

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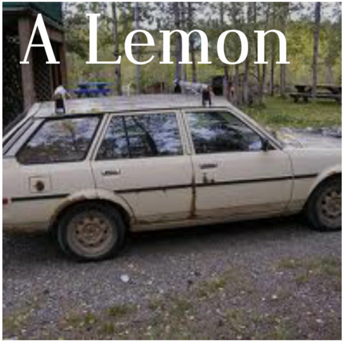
The Lemons Problem

- **Watson**: Ch. 27
- You go to buy a used car.
- Of course, the seller tells you that the car is in very good condition.
  - "An old lady owned it for 10 years, and took great care of it" (sounds familiar?)
  - According to the amount of miles on my car, the "old lady" who owned my car was driving to Seattle every weekend...
- Price of the car coincides with that in Kelley Blue Book.
- But is it really a Peach or a Lemon?

The Lemons Problem

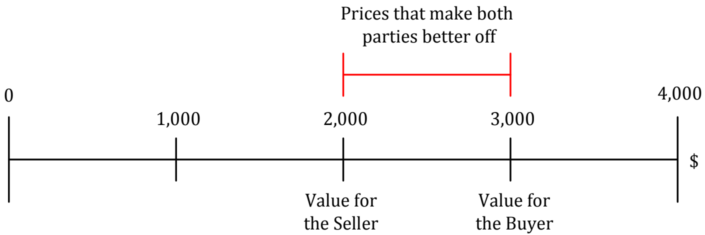


The Lemons Problem

- If the car is a peach, it is worth \$3,000 to the buyer and \$2,000 to the seller.
- If the car is a lemon, it is worth \$1,000 to the buyer and \$0 to the seller.
- Note that, if there was complete information about the true quality of the car, in both cases, the buyer values the car more than the seller does.
  - Hence, there is **room for trade**
  - That is, trade is welfare improving for both parties.
  - (Figure)$\longrightarrow$

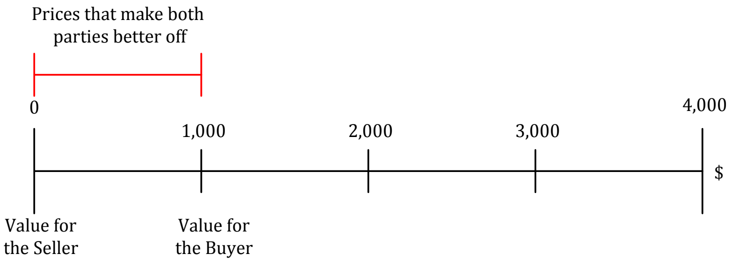
The Lemons Problem

- Peach (High Quality) Car:



The Lemons Problem

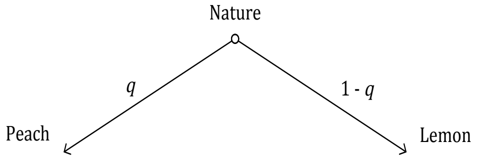
- Lemon (Low Quality) Car:



The Lemons Problem

- But what if there is incomplete information?
  - The seller observes nature's choice (or how well the previous owner was taking care of the car, e.g., a detailed mechanical inspection).
  - The buyer knows only that the car is a peach with probability q and a lemon with probability $1 - q$.(For instance, reading reports about the proportion of good and bad cars in the used cars market.)
- Then the players decide whether to trade or not trade at the market price p (Kelley Blue Book's price).
- If they both choose to trade, then the trade takes place. Otherwise, the seller keeps the car.

The Lemons Problem



$Seller^{Peach}$

Buyer	T^{Peach}	NT^{Peach}
T	3000 - p, p-2000	0, 2000
NT	0, 2000	0, 2000

$Seller^{Lemon}$

Buyer	T^{Lemon}	NT^{Lemon}
T	1000 - p, p	0, 0
NT	0, 0	0, 0

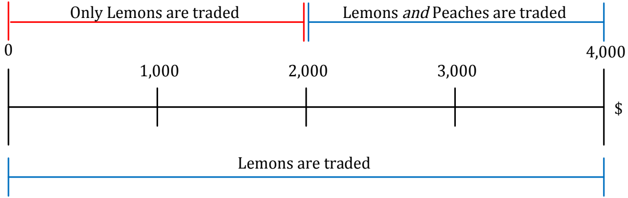
- Note that these matrices are not representing a simultaneous-move game between the seller and the buyer.
- They just summarize payoffs.

The Lemons Problem

- As usual, let us first focus on the informed player:
  - Seller with a Peach
  - Seller with a Lemon
- We can afterwards analyze the uninformed player (Buyer).

The Lemons Problem

- **Informed player (Seller):**
  - When the car is a peach, he trades if the price is $p > \$2,000$.
  - When the car is a lemon, he trades if the price is $p > \$0$.
  - Summarizing this in a figure...



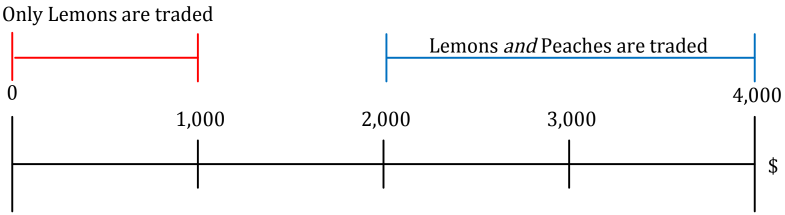
  - When examining the (uninformed) buyer, we will separately analyze each of these two intervals. $\longrightarrow$

The Lemons Problem

- **Uninformed player (Buyer)**
  - **First case:** if the buyer observes a price $p \in [0, 2000]$, he can anticipate that only lemons are being offered by the seller.
  - Then the buyer accepts the trade if
-
$$1000 - p \geq \$0$$
  - and solving for p, this implies that the price must satisfy $p \leq \$1,000$.

The Lemons Problem

- This further restricts the set of admissible prices under which only lemons are traded, from $p \in [0, 2000]$ to $p \in [0, 1000]$.



The Lemons Problem

- **Uninformed player (Buyer)**
  - **Second case:** if the buyer instead observes the price $p > \$2,000$. then both lemons and peaches are offered by the seller.
  - Then the buyer accepts such a price p if:

$$\overbrace{q}^{\text{Prob of Peach}}(3000 - p) + \overbrace{(1-q)}^{\text{Prob of Lemon}}(1000 - p) \geq 0$$
$$\iff 3000q + 1000(1-q) \geq p$$
$$\iff 1000 + 2000q \geq p$$

The Lemons Problem

- **Uninformed player (Buyer)**
  - **Second case (cont'd):**
  - Hence, we need that

$$\begin{aligned} 1000 + 2000q &\geq p > 2000 \\ 1000 + 2000q &> 2000 \implies q > \frac{1}{2} \end{aligned}$$

  - **Intuition:** if there are a lot of peaches in the market, $q > \frac{1}{2}$, then I will accept paying more than \$2,000 for a used car. (Between \$2,000 and \$3,000).

The Lemons Problem

- However, if $q < \frac{1}{2}$, then only the first type of BNE can be supported, where only lemons are traded (at prices below \$1,000).
- **But that equilibrium was inefficient!**:
  - Indeed, trading a peach creates value for the seller and the buyer (trading the peach for a price between 2,000 and 3,000 was beneficial both for the seller and the buyer).
- Hence, asymmetric information might cause some markets to malfunction.
  - When $q < \frac{1}{2}$ there is, literally, *no market for good cars!*

The Lemons Problem

- How can we avoid incomplete information in these markets, and therefore avoid market breakdowns?

Information Aggregation

- Watson, Ch. 27 (pp. 327-332 only)
- Many situations involve many players, each with his/her own private information, who must make a decision affecting the welfare of all members in the group.
- *Examples:*
  - Voting about a public project (highway): Personal costs and benefits of the project.
  - Re-elect a president: personal political preferences.
  - Convicting an accused felon: collecting the pieces of information from a jury.

Voting in a Jury Game

- Jury of two people.
- During the trial, each juror obtains a signal about whether the defendant is guilty or innocent.
- Signal received by juror i as a result of the entire trial is denoted as $s_i = \{I, G\}$
- Signal s_1 is assumed to be independent of signal s_2
    - *Intuition*: different degrees of expertise between each juror, different sleep patterns...

Voting in a Jury Game

- If the defendant is innocent, the signal...
  - $s_i = I$ will be received with probability $\frac{3}{4}$
  - $s_i = G$ will be received with probability $\frac{1}{4}$
- If the defendant is guilty, the signal...
  - $s_i = I$ will be received with probability $\frac{1}{4}$
  - $s_i = G$ will be received with probability $\frac{3}{4}$
- Thus, signal I is an indication of innocence, and signal G is an indication of guilt (but neither signal is an absolute indication about the defendant's guilt or innocence).

Voting in a Jury Game

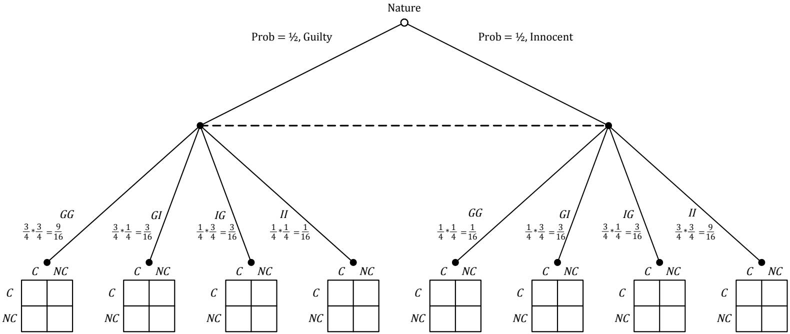
- From the above information, we can compute some conditional probabilities.
- For example, the probability that we both receive a signal of G, conditional on the defendant being guilty is

$$prob(GG|\text{guilty}) = \frac{3}{4}\frac{3}{4} = \frac{9}{16} \quad \text{Figure} \longrightarrow$$

but the probability that we both receive such signals despite the defendant being innocent is

$$prob(GG|\text{innocent}) = \frac{1}{4}\frac{1}{4} = \frac{1}{16} \quad \text{Figure} \longrightarrow$$

Voting in a Jury Game



where C : Convict and NC : Not Convict.

Voting in a Jury Game

- **More things about conditional probabilities**:
- Let us now reverse the previous conditional probability.
- What is the probability that a defendant is guilty, conditional on us both receiving a signal of G?
  - $prob(\text{guilty}|GG)$.
  - In order to compute this conditional probability we need to use Bayes' Rule.
  - (You probably encountered this in some stats course, for a review see pp. 354-357 in Harrington, or pp. 375-376 in Watson).

Voting in a Jury Game

- What is the probability that a defendant is guilty, conditional on us both receiving a signal of G? $prob(\text{guilty}|GG)$.

$$prob(\text{guilty}|GG) = \frac{prob(\text{guilty})prob(GG|\text{guilty})}{prob(GG)}$$

  where $prob(GG) = prob(\text{guilty}) \times prob(GG|\text{guilty}) + prob(\text{innocent}) \times prob(GG|\text{innocent})$.

- Hence,

$$prob(\text{guilty}|GG) = \frac{\frac{1}{2}\frac{9}{16}}{\frac{1}{2}\frac{9}{16} + \frac{1}{2}\frac{1}{16}} = \frac{9}{10}$$

- How to interpret this conditional probability in words?
  - "Observing two G signals would cause the juror to believe that the defendant is guilty 90% of the time."

Voting in a Jury Game

- Let us practice another conditional probability:
  - One juror observes a signal I, and another a signal G. What is the conditional probability that the defendant is guilty?

$$prob(\text{guilty}|IG) = \frac{prob(\text{guilty})prob(IG|\text{guilty})}{prob(IG)}$$

where $prob(IG) = prob(\text{guilty}) \times prob(IG|\text{guilty}) + prob(\text{innocent}) \times prob(IG|\text{innocent})$.

- Hence,

$$prob(\text{guilty}|IG) = \frac{\frac{1}{2}\frac{3}{16}}{\frac{1}{2}\frac{3}{16} + \frac{1}{2}\frac{3}{16}} = \frac{1}{2}$$

Voting in a Jury Game

- Let us now come back to the voting game:
- Every juror simultaneously submits his/her vote to the judge.
- Voting satisfies *Unanimity Rule*:
  - Both jurors must vote "conviction," otherwise the defendant is acquitted.
- Payoffs for both jurors are symmetric. In particular,
  - 3 if the defendant is convicted when being guilty.
  - -2 if the defendant is convicted but he/she was innocent.
  - 0 if the defendant is acquitted, regardless of his true identity.

Voting in a Jury Game

- Is voting for conviction rational if and only if you get a signal G (voting "convict" if G, but "not convict" if I)?
  - Seems reasonable? Let's check why this is *not* a BNE.
- Let us put ourselves in the situation of P1, assuming P2 votes for conviction only if he gets a signal G, i.e., P2 behaves according to the above strategy.
  - If P2 votes "not convict," then it doesn't matter what you do (because of unanimity rule).
  - If P2 votes "convict," then the defendant's fate is in your hands (your vote is *pivotal*).

Voting in a Jury Game

- Your vote has a payoff consequence only when it is pivotal (when $s_2 = G$, and P2 votes "convict"). Let us analyze your expected payoff.
  - **First case:** you receive a signal of G, so signals are GG. If you vote to convict, your expected utility is:

$$prob(\text{guilty}|GG) \cdot 3 + prob(\text{innocent}|GG) \cdot (-2)$$
$$= \frac{9}{10}3 + \frac{1}{10}(-2) = \frac{25}{10}$$

    which is higher than your payoff from voting "not convict" (zero). Hence, when you receive a signal of G, you vote conviction.

Voting in a Jury Game

- **Second case:** you receive a signal of I, so signals are IG. If you vote to convict, your expected utility is:

$$prob(\text{guilty}|IG) \cdot 3 + prob(\text{innocent}|IG) \cdot (-2)$$
$$= \frac{1}{2}3 + \frac{1}{2}(-2) = \frac{1}{2}$$

  which is higher than your payoff from voting "not convict" (zero). Hence, when you receive a signal of I you also vote for conviction.

- You would always vote for conviction, even when the signal you receive is I!

Voting in a Jury Game

- We just showed that voting according to the private signal you receive...
  - namely, voting "convict" after receiving signal G, but "not convict" after I,

cannot be sustained as a BNE.

- What is the equilibrium/equilibria of this jury game, then?

Voting in a Jury Game

- This game has, in fact, multiple equilibria.
- But let's analyze the following:
  - P2 votes for conviction, regardless of the signal he/she receives.
  - P1 votes for conviction if he receives a signal of G, but he votes for acquittal if he receives a signal of I.
- Let us check if this strategy profile can be supported as a BNE.

Voting in a Jury Game

- Let us check if this strategy profile can be supported as a BNE:
  - P1 knows that his vote is pivotal regardless of the signal that P2 received (since P2 always votes for conviction, P1 becomes a "jury of one").
  - **First case:** If the signal that P1 receives is I, then

$$\begin{aligned} prob(\text{guilty}|I) &= \frac{prob(\text{guilty})prob(I|\text{guilty})}{prob(I)} \\ &= \frac{\frac{1}{2}\frac{1}{4}}{\frac{1}{2}\frac{1}{4}+\frac{1}{2}\frac{3}{4}} = \frac{1}{4} \end{aligned}$$

- Hence, convicting the defendant yields a EU of

$$prob(\text{guilty}|I)\cdot 3 + prob(\text{innocent}|I)\cdot(-2) = \frac{1}{4}3 + \frac{3}{4}(-2) = -\frac{3}{4} <$$

  - Therefore, if P1 receives a signal of I, he votes "not convict."

Voting in a Jury Game

- Let us continue checking if this strategy profile can be supported as a BNE:
  - **Second case:** If the signal that P1 receives is G, then

$$
\begin{aligned}
prob(\text{guilty}|G) &= \frac{prob(\text{guilty})prob(G|\text{guilty})}{prob(G)} \\
&= \frac{\frac{1}{2}\frac{3}{4}}{\frac{1}{2}\frac{3}{4}+\frac{1}{2}\frac{3}{4}} = \frac{3}{4}
\end{aligned}
$$

- Hence, convicting the defendant yields a EU of

$$
prob(\text{guilty}|G)\cdot 3 + prob(\text{innocent}|G)\cdot(-2) = \frac{3}{4}3+\frac{1}{4}(-2) = \frac{7}{4} >
$$

  - Therefore, if P1 receives a signal of G, he votes "convict."

Voting in a Jury Game

- Hence, this strategy profile is a BNE, where:
  - P2 votes conviction regardless of his signal, whereas
  - P1 votes conviction if and only if he receives a signal of G.

Voting in a Jury Game

- Less information (the signal of only one juror) is then transmitted from the jury to the court than society would consider ideal.

Voting in a Jury Game

- Why this unfortunate outcome can be sustained as an equillibrium.?
  - Sender (juror) and receiver (society/judge) differ in their preferences.
    - On one hand, society prefers to convict the defendant if and only if *both* signals were G (societal preferences were implicit in the voting rule: unanimity rule)
    - On the other hand, jurors have a stronger preference to vote conviction since, given the equal probabilities of guilty/innocent, the benefits from convicting a guilty defendant (3) outweigh the loss of convicting an innocent defendant (-2).

Voting in a Jury Game

- How can we achieve more information transmission from the jurors to the judge?
  - If society and jurors have the same preferences.
  - If the number of jurors increases.
  - If jurors talk, sharing the signals they received during the trial.
- Harrington, pp. 307-312, Watson pp. 368-373

Strategic Abstention

- Harrington, pp. 307-309.
- They say that "it is your civic duty to vote."
- But, can it be beneficial for both you and the society that you *abstain* from voting?

Strategic Abstention

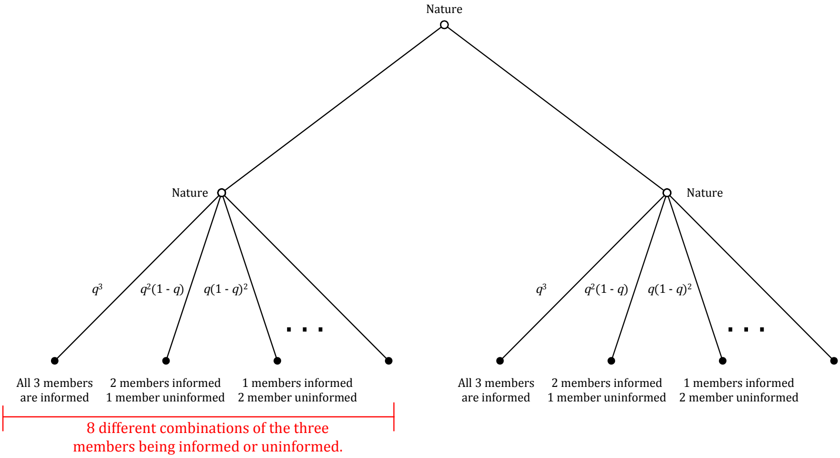
Strategic Abstention

- 3 members on a committee, voting Yes/No to a policy that changes the status quo (SQ).
- Committee members are symmetric.
  - Hence, if all members were informed about which policy is best, they would all vote Yes, or all vote No.
- But each member is privately informed about the efficacy of the new policy,
  - i.e., each member receives a private signal.

Time structure:

- First, nature determines if...
  - the policy is better than the SQ (what we refer to as "good policy"), which happens with probability $1 - p < \frac{1}{2}$; or
  - the policy is worse than the SQ ("bad policy"), which happens with probability $p > \frac{1}{2}$.
- Second, nature determines if every committee member is...
  - informed about which policy is best (with prob q), or
  - uninformed (with prob $1 - q$).
- (See figure in next slide)$\longrightarrow$

Strategic Abstention



Strategic Abstention

- 8 different combinations of the three members being informed or uninformed:

Informed Members			Uninformed Members		
A	B	C			
A	B				C
A		C		B	
	B	C	A		
A				B	C
	B		A		C
		C	A	B	
			A	B	C

Strategic Abstention

- If the committee chooses the policy that proves to be the best
  - i.e., the proposed policy when it is a "good policy," or
  - the SQ when the proposed policy was in fact a "bad policy,"

  then each member receives a payoff of $1.
- However, if they choose a wrong policy, their payoff is $0.

Strategic Abstention

- Without any additional information, the EU for the uninformed players are...

$$\begin{aligned} EU(SQ) &= \overbrace{p \cdot 1}^{\text{Bad Policy}} + \overbrace{(1-p) \cdot 0}^{\text{Good Policy}} = p, \text{ and} \\ EU(new) &= p \cdot 0 + (1-p) \cdot 1 = 1 - p \end{aligned}$$

- Then $EU(SQ)$ against $EU(new)$ implies $p > 1 - p$ implies $p > \frac{1}{2}$, which holds by definition.

Strategic Abstention

- Let us next show that there exists a BNE in which every member:
  - When he is *informed*, he votes for the policy that is best
    - i.e., vote for the proposed policy when it is a "good policy," or the SQ when the proposed policy is a "bad policy."
  - When he is *uninformed*, what will he do?

Strategic Abstention

- If **informed**, a member votes for the policy that is best:
  - His vote does not make any difference if P2 and P3 vote SQ
    - His payoff from voting in favor of the bill coincides with that from abstaining, since SQ wins regardless.
  - His vote causes the bill to pass if P2 and P3 split their votes between SQ and the new policy.
    - His payoff is higher voting in favor of the best policy (either the new bill or the SQ) than abstaining.
- Hence, if informed, it is weakly dominant to vote for the best policy.

Strategic Abstention

- What if the member is **uninformed** about which policy is best?
- Should he vote for the SQ since $EU(SQ) > EU(new)$?
  - This would imply that every player votes in favor of the best policy when he is informed, and in favor of the SQ when he is uninformed. (Symmetric BNE).
  - This implies that P2 and P3 are voting (either Y/N, but voting!).

Strategic Abstention

- Thus, P1's vote in favor of SQ when he is uninformed makes a difference only if P2 and P3's votes are split.
- Then one member (either P2 or P3) votes in favor of the new policy because he is informed about the advantages of the new policy, but...
  - P1's vote in favor of SQ (because of being uninformed) makes the SQ win!
  - A good new policy is blocked by P1's lack of information!
- It cannot be optimal for an uninformed player to participate and vote in favor of SQ.

Strategic Abstention

- What if the uninformed member simply abstains from voting?
  - This would imply that every player votes in favor of the best policy when he is informed, but...
  - Abstains when he is uninformed.
  - (Symmetric BNE).

Strategic Abstention

- Let us analyze if this strategy profile can be supported as a BNE for P1:
  - If all voters are uninformed, then they all abstain, and SQ continues yielding a expected utility of $EU = p$.
  - If either (or both) voters P2 and P3 are informed, then either (or both) go to vote, and vote for the best policy. In this case P1's payoff is 1 by abstaining.
  - P1's alternative (go to vote) yields him only $0, since his uninformed vote makes the SQ win.

Strategic Abstention

- Hence, the BNE prescribes to:
  - Vote for the best policy when informed, but
  - Abstain when uninformed.
- When a member is uninformed,
  - It is better for him to abstain, and let the informed members determine the outcome of the election.

Strategic Abstention

- Note that this result could be supported even if voting is costless.
  - If voting is costly, our results would be actually emphasized.
- Hence, if you are going to hang out at a cafe on Election Day...
  - you shouldn't be reading the *NY Times* (informed voter)!