EconS 301 – Intermediate Microeconomics
Review Session #4

1. Suppose a person's utility for leisure (L) and consumption (Y) can be expressed as $U = YL$ and this person has no non-labor income.

a) Assuming a wage rate of $10 per hour, show what happens to the person's labor supply when the person wins a lottery prize of $100 per day.

Answer

Plug in what we know to the utility function above, $U = YL$. Y should express total income earned and L should express all non-work time per day.

Rearranging yields $U = (Y^* + 10H)(24 - H) = 24Y^* + 240H - Y^*H – 10H2$.
Maximizing utility with respect to H yields $dU/dH = 240 - Y^* - 20H = 0$. (Remember that where $U=0$ it is at its maximum.)
Before winning the lottery, $Y^* = 0$, so $H = 12$.
After winning the $100 per day lottery, $Y^* = 100$, so $H = 7$.
Winning the lottery reduces this person's quantity of labor supplied by 5 hours when w = $10. Intuitively this makes sense because the more wealth they have the less desirable work becomes. L becomes more attractive with greater wealth.

b) Suppose a person's utility for leisure (L) and consumption (Y) can be expressed as $U = Y + L0.5$. Show what happens to the person's labor supply curve when the income tax is cut from 70 % to 30 %. Denote hours worked as H and wage per hour as w.

Answer

Since Y = net income, $U = w(1 - t)H + (24-H)0.5$. Note that $w(1-t)$ represents real wage.

Maximizing utility with respect to hours worked, H, yields $H = 24 - (2(1 - t)w)-2$. Any decrease in t would increase the number of hours worked. Note: This person is a workaholic. Even at a net wage of $1, this person only relaxes for 3/4 of an hour!

2. Suppose you work for a government agency that is considering removing certain agricultural subsidies. The removal of these subsidies will increase the price, thus lowering consumers' welfare. Because only aggregate market data is available, you are unable to measure the exact values for the compensated and equivalent variation by consumer. However, you are able to estimate the change in market consumer surplus. Assuming agricultural products are normal goods, how does your estimate of consumer surplus compare to the unknown EV and CV? Explain. Under what conditions will the three measures of welfare be close to one another?

Answer

For normal goods, the CS will be less than the CV and greater than the EV (in absolute value). Typically, these measures will be close for (1) small price changes, (2) small income effects/elasticity, and (3) small budget share. Remember that the CV and EV are used to measure the change of welfare (income) of the consumer after a price change and that typically the EV and the CV will not be close in magnitude. This is because most price changes have a nonzero income effect. Similarly, in the case of a quasi-linear utility function, the CV and EV will be the same because the income effect is zero.

3. Ed's utility from vacations (V) and meals (M) is given by the function $U(V,M) = V^2M$. Last year, the price of vacations was \$200 and the price of meals was \$50. This year, the price of meals rose to \$75, the price of vacations remained the same. Both years, Ed had an income of \$1500.

a) Calculate the change in consumer surplus from meals resulting from the change in meal prices. (Note: we need to compare his optimal consumption baskets before and after the price change to be able to see the change in CS)

Answer

Ed's optimization problem is

$$\text{Max } V^2M$$
$$\text{subject to } p_M M + 200V = 1500$$

where p_M is the price of meals and the price of vacations is represented by the constant 200. Using the Lagrangian (solve utility function for V and then plug V into the budget constraint and solve for M), we derive the demand for meals:

$M^* = 500/p_M$

The change in consumer surplus is found from the integral (space under curve):

$$\Delta CS = \int_{50}^{75} \frac{500}{p_M} dp_M = -500\ln(p_M)\Big|_{50}^{75} = -500\left(\ln 75 - \ln 50\right) = -202.7$$

So the change in consumer surplus is –\$202.7.

b) What is the compensating variation for the price change in meals?

Answer

Recall the CV is simply the difference in the consumer's income and the income necessary to purchase the decomposition basket at the new prices. In this case, the CV is the amount of money needed to offset a consumer's harm from a price increase (that is, CV will be additive here because the consumer will need more money to be equally happy after the price increase as she was before). So we first need to find his initial optimum basket.

$$\max L = V^2 M + \lambda\left(y - p_M M - p_V V\right)$$

$$\left.\begin{aligned} \frac{\delta L}{\delta M} &= V^2 - \lambda p_M = 0 \\ \frac{\delta L}{\delta V} &= 2VM - \lambda p_V = 0 \end{aligned}\right\} \frac{V^2}{2VM} = \frac{V}{2M} = \frac{p_M}{p_V}$$

$V = \dfrac{2 p_M M}{p_V}$ into the budget constraint

$1500 = p_M M - p_V \left(\dfrac{2 p_M M}{p_V}\right)$ solving for M,

$\dfrac{1500}{3 p_M} = \dfrac{500}{p_M} = M$ plug back into V and solve,

$$V = \frac{2 p_M \left(\dfrac{500}{p_M}\right)}{p_V} = \frac{1000}{p_V}$$

Ed's utility before the price change is based on his optimal consumption bundle where M_1 = 500/50 = 10 and V_1 = 1000/200 = 5. Thus, his initial utility is U(5,10)=(5)2(10) = 250. In order to find the decomposition basket, we need to use the MRS and the initial level of utility. From above, we know the MRS is $\dfrac{V}{2M} = \dfrac{p_M}{p_V}$.

Using this and plugging in the new prices we can solve for V in terms of M,

$$V = \frac{2(75)M}{200} = \frac{150M}{200} = \frac{3M}{4}.$$

Plug this result into the utility function when initial utility is 250 and solving for M, 250 = $(3M/4)^2M$ => M= 7.63. Solve for V = 5.72. This is our decomposition basket. The expenditure required to purchase this bundle is:

75M + 200V = 75(7.63) + 200 (5.72) = 1717.25

(we need total expenditure because CV and EV are measures of income before and after price change)

Thus the CV is \$1,500 - \$1,717.25 = \$217.25.

c) Calculate the equivalent variation for the price change in meals.

Answer

The EV is similar to the CV, except that we need to find the consumption basket that would put him on his new utility level holding prices constant (sort of the opposite of the decomposition basket where the new prices are used holding the initial utility constant). The EV is the amount of money Ed will pay to prevent the price increase. First we need to find his optimal consumption basket at new prices so we can find the new level of utility. To do this, simply use the demand equations we derived in part (b) and plug in the new prices. From above, V = 5 and M = 500/75 = 6.67. His utility from this bundle is U = $(5)^2(6.67)$=166.75. Now use the MRS and new prices to solve for V in terms of M, $V = \frac{2(50)M}{200} = \frac{100M}{200} = \frac{M}{2}$. Plug this result into the utility function with U=166.75 and we can solve for M,

$$166.75 = \left(\frac{M}{2}\right)^2 M = \frac{M^3}{4}$$
$$667 = M^3$$
$$8.74 = M$$
$$V = \frac{8.74}{2} = 4.37$$

The expenditure of this bundle is:

$$50(8.74) + 200(4.37) = 1311.$$

Ed would pay up to 1500-1311 = $189 to avoid the price change. This is the EV.

4. Linda consumes two goods, X and Y. Her utility function is $U = XY$, with $MU_X = Y$ and $MU_Y = X$. Initially, $P_X = \$18$ and $P_Y = \$2$. Linda's income is $288. Then the price of X falls to $8. [The following questions ask you to calculate a mathematical example of the income and substitution effects of a price decrease for good X.]
   a) Complete the following table.

Basket	X	Y	$U = XY$	$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$	Expenditure $P_X X + P_Y Y$
A					
B	12	48			
C					

Answer

For bundle A,

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$
$$\frac{Y}{X} = \frac{18}{2} = \frac{9}{1}$$
$$9X = Y$$
$$P_x X + P_Y Y = I$$
$$18X + 2Y = 288$$
$$18X + 18X = 288$$
$$36X = 288$$
$$X = 8$$
$$Y = 72$$

For bundle C,

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$
$$\frac{Y}{X} = \frac{8}{2} = \frac{4}{1}$$
$$4X = Y$$
$$P_x X + P_Y Y = I$$
$$8X + 2Y = 288$$
$$8X + 8X = 288$$
$$16X = 288$$
$$X = 18$$
$$Y = 72$$

Basket	X	Y	$U = XY$	$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$	Expenditure $P_X X + P_Y Y$
A	8	72	$8*72=576$	$\frac{Y}{X}=\frac{72}{8}=\frac{9}{1}=\frac{18}{2}$	$18*8+2*72=288$
B	12	48	$12*48=576$	$\frac{Y}{X}=\frac{48}{12}=\frac{4}{1}=\frac{8}{2}$	$8*12+2*48=192$
C	18	72	$18*72=1296$	$\frac{Y}{X}=\frac{72}{18}=\frac{4}{1}=\frac{8}{2}$	$8*18+2*72=288$

b) The movement from point A to point B illustrates which effect, the income effect or the substitution effect? Explain.

Answer

The movement from point A to point B illustrates the substitution effect because the consumer moves along the same indifference curve (notice that total utility remains constant) to the new tangency point between the original indifference curve and the new budget constraint (where $P_X = \$8$). That is, we are looking at the change in optimal choice induced solely by the change in the price relationship between x and y and not any change due to a change in income.

c) The movement from point B to point C illustrates which effect, the income effect or the substitution effect? Explain.

Answer

The movement from point B to point C illustrates the income effect because the consumer moves to a higher indifference curve. Point C represents the new tangency point between the new budget constraint and the new indifference curve. Here, we look solely at how the income change affects the optimal choice for this consumer. (Notice that the slope of the budget constraint does not change between points B and C but that the utility does.)

d) Is good X a normal, inferior, or Giffen good? Explain.

Answer

Good X is a normal good because as “income” increases from point B to point C, Linda consumes more X. Remember that the gap between B and C is solely measuring the effect of increased income (income effect) so we can use it to make conclusions on whether a good is giffen or normal. Note that the change between bundle A and bundle B could not be used to check this.

5. If x is an inferior good and the price of x rises

   a. The substitution effect will induce the consumer to purchase more x and the income effect will induce the consumer to purchase more x.
   b. The substitution effect will induce the consumer to purchase more x and the income effect will induce the consumer to purchase less x.
   c. The substitution effect will induce the consumer to purchase less x and the income effect will induce the consumer to purchase more x.
   d. The substitution effect will induce the consumer to purchase less x and the income effect will induce the consumer to purchase less x.

Answer

Recall that with inferior goods, the income and substitution effects move in opposite directions. So immediately you can eliminate choices A and D. Now consider the example given in figure 5.8.

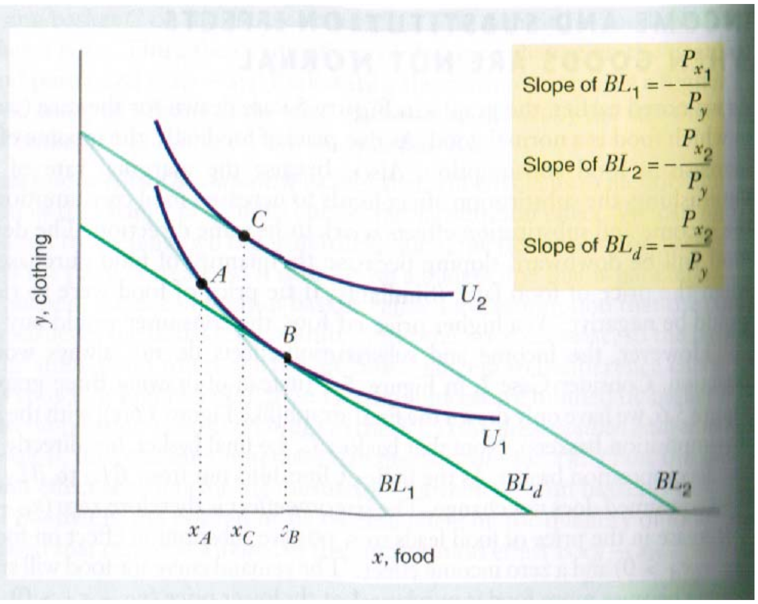


FIGURE 5.8 Income and Substitution Effects: Case 3 (x Is an Inferior Good) with a Downward-Sloping Demand Curve
As the price of food drops from P_{x_1} to P_{x_2}, the substitution effect leads to an increase in the amount of food consumed from x_A to x_B (so the substitution effect is $x_B - x_A$). The income effect on food consumption is negative ($x_C - x_B < 0$). The overall effect on food consumption is $x_C - x_A > 0$. When a good is inferior, the income and substitution effects work in opposite directions.

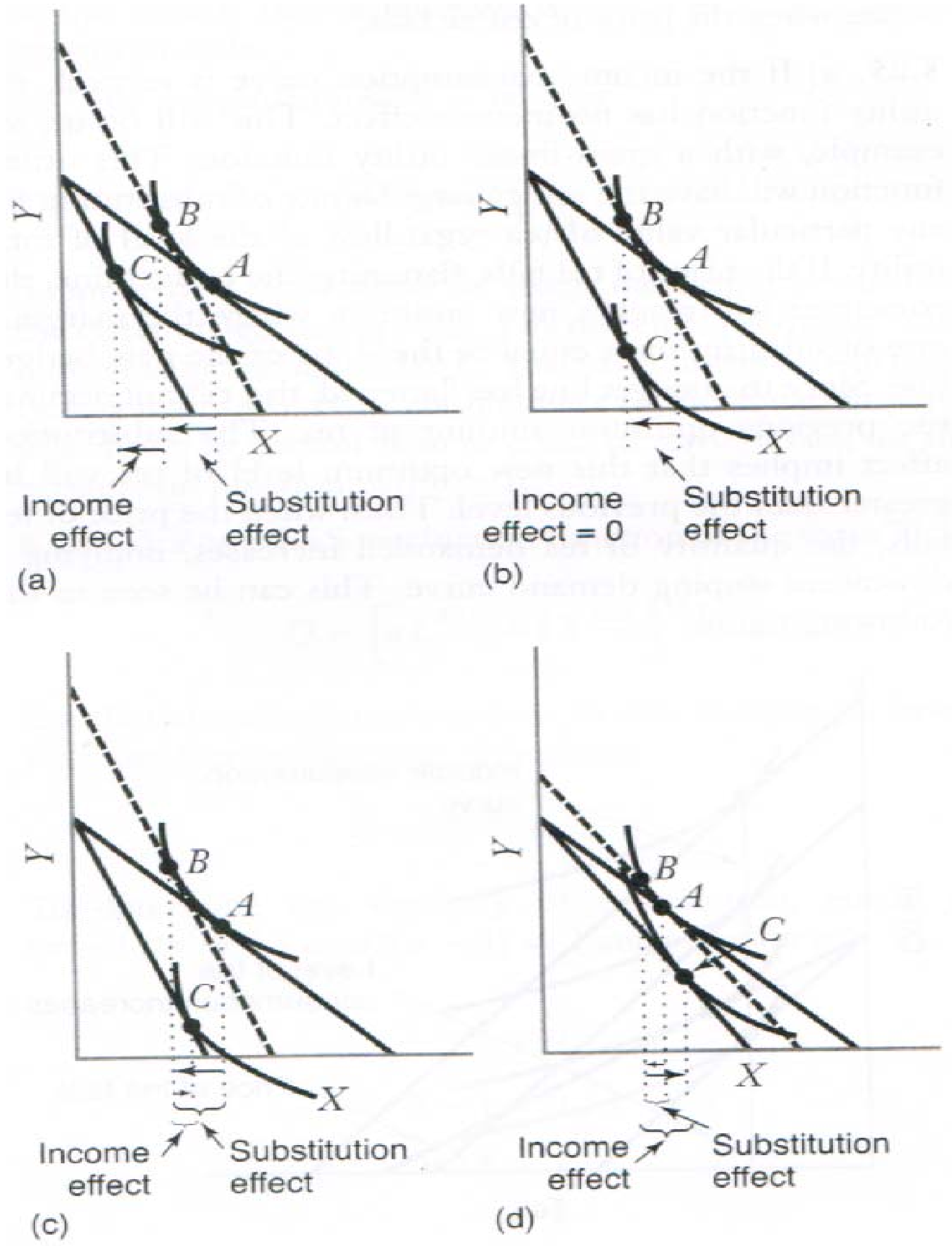
In this figure, we have a price decrease and we can see that the substitution effect (bundle A to B) causes the consumer to purchase more. Similarly, we can see the income effect (bundle B to C) causes the consumer to purchase less. Again this is for a price decrease. So for a price increase, you simply change the direction of the effects. So in our case, the SE results in less *x* and the IE results in more *x*, thus the answer is choice C.

6. Rich purchases two goods, food and clothing. He has a diminishing marginal rate of substitution of food for clothing. Let *x* denote the amount of food consumed and *y* the amount of clothing. Suppose the price of food increase from P_{x1} to P_{x2}. On a clearly labeled graph, illustrate the income and substitution effects of the price change on the consumption of food. Do so for each of the following cases:

   a. Food is a normal good.
   b. The income elasticity of demand for food is zero.

c. Food is an inferior good, but not a Giffen good.
d. Food is a Giffen good.

Answer



7. Suppose that Bart and Homer are the only people in Springfield who drink 7-UP. Moreover, their inverse demand curves for 7-UP are, respectively, $P=10-4Q_B$ and $P=25-2Q_H$, and, of course, neither one can consume negative amounts. Write down the market demand curve for 7-UP in Springfield, as a function of all possible prices.

Answer

Recall that the market demand is simply the horizontal summation (adding in x and not in y) of all the individual demand curves. That is, you sum the quantities demanded. We have the inverse demand equations, so we need to solve each for their respective quantities,

$$P = 10 - 4Q_B$$
$$4Q_B = 10 - P$$
$$Q_B = \begin{cases} 2.5 - .25P \text{ when } P < 10 \\ 0 \text{ when } P \geq 10 \end{cases}$$

and

$$P = 25 - 2Q_H$$
$$2Q_H = 25 - P$$
$$Q_H = \begin{cases} 12.5 - .5P \text{ when } P < 25 \\ 0 \text{ when } P \geq 25 \end{cases}$$

now sum Q_H and Q_B,

$$Q_{market} = Q_B + Q_H = 2.5 - .25P + 12.5 - .5P = 15 - .75P$$
$$Q_{market} = \begin{cases} 15 - .75P \text{ when } P < 10 \\ 12.5 - .5P \text{ when } 10 \leq P < 25 \\ 0 \text{ when } P \geq 10 \end{cases}$$

For a graphical representation refer to Figure 5.21. Notice the "kink" once one consumer's maximum price is reached.

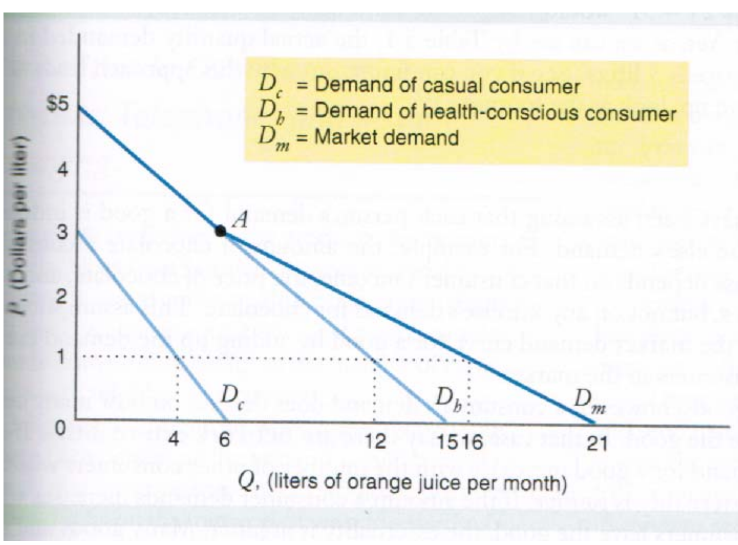


FIGURE 5.21 Market and Segment Demand Curves
The market demand curve D_m (the dark curve) is found by adding the demand curves D_h and D_c for the individual consumers horizontally.