EconS 301 – Intermediate Microeconomics

Review Session #3

1. A consumer purchases two goods, food (F) and clothing (C). Her utility function is given by $U(F,C) = FC + F$. The marginal utilities are $MU_F = C + 1$ and $MU_C = F$. The price of food is P_F, the price of clothing is P_C, and the consumer's income is I.

a) What is the equation for the demand curve for clothing?

Answer

Setting up the tangency condition implies

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$
$$\frac{C+1}{P_F} = \frac{F}{P_C}$$
$$FP_F = P_C(C+1)$$

Substituting this result into the budget line implies

$$P_C C + P_F F = I$$
$$P_C C + P_C(C+1) = I$$
$$P_C(2C+1) = I$$
$$2C+1 = \frac{I}{P_C}$$
$$C = \frac{I - P_C}{2P_C}$$

b) Is clothing a inferior good in this case?

Answer

Since the amount of clothing purchased will increase as income increases, as noted by the demand curve, clothing is a normal good, and not an inferior good.

2. Consider a consumer who purchases two goods, x and y. The consumer's utility function is $U(x,y)=xy$ with $MU_x = y$ and $MU_y = x$. In addition, the demand curve for y is given by $y = I/2P_y$. Assume initially that the consumer's income is \$160, the price of x is $P_x = \$8$, and the price of y is $P_y = \$1$.

a) From the given information determine 1) the utility maximizing amount of x, 2) the utility maximizing amount of y, and 3) the total utility at the utility maximizing bundle.

Answer

Using the demand curve for y we can find the utility maximizing amount of y.

$$y = \frac{I}{2P_y}$$

$$y = \frac{160}{2(1)}$$

$$y = 80$$

Since each unit of y costs \$1, the consumer will spend $\$1(80) = \$80$ on y. This leaves \$80 to spend on x. Since x costs \$8, the utility maximizing amount of x is therefore 10 units. At this bundle, total utility is $U = xy = 10(80) = 800$.

b) Now assume the price of y increases to \$2. Recompute the values from part a) at the new price.

Answer

Again, using the demand curve for y we can find the utility maximizing amount of y.

$$y = \frac{I}{2P_y}$$

$$y = \frac{160}{2(2)}$$

$$y = 40$$

Since each unit of y costs \$2, the consumer will spend $\$2(40) = \$80$ on y. This leaves \$80 to spend on x. Since x costs \$8, the utility maximizing amount of x is therefore 10 units. At this bundle, total utility is $U = xy = 10(40) = 400$.

c) Determine the decomposition basket that identifies the substitution and income effects as the consumer moves from the optimal basket in part a) to the optimal basket in part b).

Answer

To determine the decomposition basket we note that this basket must satisfy two conditions. First, the decomposition basket must have total utility the same as at the *initial* prices, $U(x, y) = xy = 800$. Second, the decomposition basket must satisfy the tangency condition at the *new* prices. This implies

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$
$$\frac{y}{8} = \frac{x}{2}$$
$$y = 4x$$

These two conditions give two equations in two unknowns: $xy = 800$ and $y = 4x$. Substituting the second equation into the first and solving implies $x = \sqrt{200} = 14.14$ and $y = 4\sqrt{200} = 56.56$.

d) Identify the substitution and income effects as the consumer moves from the initial consumption basket to the final consumption basket.

Answer

Initially, the consumer chose to consume 80 units of y and at the final basket the consumer chose to consume 40 units of y. Therefore, the substitution effect is the initial amount less the amount in the decomposition basket, $80 - 56.56 = 23.44$ and the income effect is the amount in the decomposition basket less the final amount, $56.56 - 40 = 16.16$. Notice that these two amount sum to the total change of 40 units.

3. Karl's preferences over hamburgers (*H*) and beer (*B*) are described by the utility function: $U(H,B) = \min(2H,3B)$. His monthly income is *I* dollars, and he only buys these two goods using his income. Denote the price of hamburgers by P_H and for beer P_B.

a) Derive Karl's demand curve for beer as a function of the exogenous variables.

Answer

Recall that goods that are compliments have utility functions in the "min" functional form, where the indifference curves are L shaped. And, from the utility function, we know Karl's optimal bundle will always be such that $2H = 3B$. That is, he will always consume in the ratio of 1 hamburger for every 2/3rds beer, or any other combination following that ratio such that $2H = 3B$ is satisfied. If this were not true then he could decrease the consumption of one of the two goods, staying at the same level of utility and reducing expenditure. Also, at the optimal bundle, it must be true that $P_H H + P_B B = I$. Solving the first condition for *H*, that is H=3/2B, and substituting into the second we get

$B(1.5P_H + P_B) = I$ which implies that the demand curve for beer is given by,

$$B = \frac{I}{(1.5P_H + P_B)}$$

4) Carina buys two goods, food F and clothing C, with the utility function $U = FC + F$. Her marginal utility of food is $MU_F = C + 1$ and her marginal utility of clothing is $MU_C = F$. She has an income of 20. The price of clothing is 4.

a) Derive the equation representing Carina's demand for food, and draw this demand curve for prices of food ranging between 1 and 6.

Answer

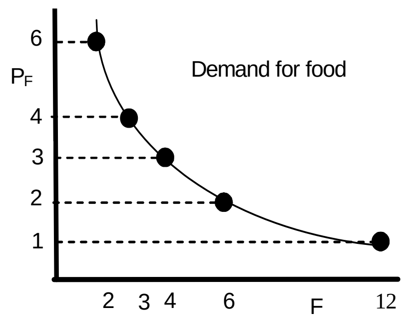
$MU_F = C + 1$ $\quad\quad\quad$ $MU_C = F$

Tangency: $MU_F/MU_C = P_F / P_C$. $\quad (C + 1)/ F = P_F/4 \quad => \quad 4C + 4 = P_FF$. (Eq 1)

Budget Line: $\quad P_FF + P_CC = I$. $\quad => \quad P_FF + 4C = 20$. (Eq 2)

Substituting (Eq 1) into (Eq 2): $4C + 4 + 4C = 20$. Thus $C = 2$, independent of P_F.

From the budget line, we see that $P_FF + 4(2) = 20$, so **the demand for F is $F = 12/P_F$.**

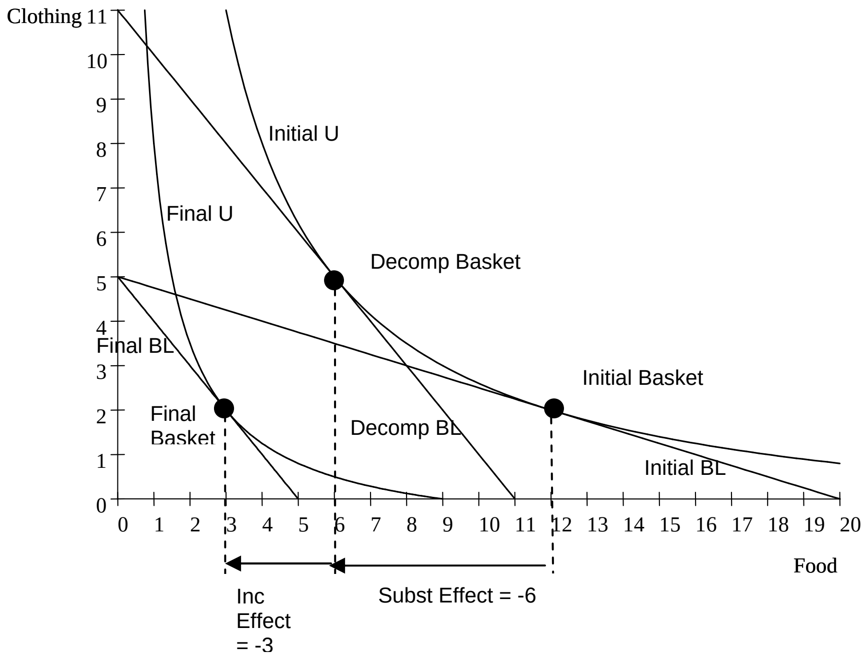


b) Calculate the income and substitution effect on Carina's consumption of food when the price of food rises from 1 to 4, and draw a graph illustrating these effects.

Answer

<u>Initial Basket:</u> From the demand for food in (a), $F = 12/1 = 12$, and $C = 2$.
Also, the initial level of utility is $U = FC + F = 12(2) + 12 = 36$.
<u>Final Basket:</u> From the demand for food in (a), we know that $F = 12/4 = 3$, and $C = 2$.
And the final level of utility is, $U = 3(2) + 3 = 9$.
<u>Decomposition Basket</u>: Must be on initial indifference curve, with $U = FC + F = 36$ (Eq 3)
Tangency condition satisfied with final price: $MU_F/MU_C = P_F / P_C$. $\quad (C + 1)/ F = 4/4$ $=> \quad C + 1 = F$. (Eq 4)
Eq 3 can be written as $F(C + 1) = 36$. Plugging Eq 4 into the rewritten Eq 3 we have $(C + 1)^2 = 36$, and thus, $C = 5$. Also, by Eq 4, $F = 6$. So the decomposition basket is $F = 6$, $C = 5$.

Income effect on F: $F_{\text{final basket}} – F_{\text{decomposition basket}} = 3 – 6 = -3$.
Substitution effect on F: $F_{\text{decomposition basket}} – F_{\text{initial basket}} = 6 – 12 = -6$.



c) Determine the numerical size of the compensating variation (in monetary terms) associated with the increase in the price of food from 1 to 4.

Answer

$P_F F + P_C C = 4(6) + 4(5) = 44$. So she would need an additional income of 24 (plus her actual income of 20). The compensating variation associated with the increase in the price of food is -24.