EconS 501 - Micro Theory I[1]
Recitation #10 - Externalities

Exercise 1

[**"Pareto irrelevant" externalities**] If the two consumers in the economy have preferences $U_1 = [x_1^1 x_2^1]^{\alpha}[x_1^2 x_2^2]^{1-\alpha}$ and $U_2 = [x_1^2 x_2^2]^{\alpha}[x_1^1 x_2^1]^{1-\alpha}$, show that the equilibrium is efficient despite the externality. Explain this conclusion.

Solution:
The marginal utility of good 1 for consumer 1 is:

$$\frac{\partial U_1}{\partial x_1^1} = \alpha[x_1^1 x_2^1]^{\alpha-1}(x_2^1)[x_1^2 x_2^2]^{1-\alpha} = \alpha\frac{[x_1^1 x_2^1]^{\alpha}[x_1^2 x_2^2]^{1-\alpha}}{x_1^1}$$

and the marginal utility of good 2 is:

$$\frac{\partial U_1}{\partial x_1^2} = (1-\alpha)[x_1^1 x_2^1]^{\alpha}[x_1^2 x_2^2]^{-\alpha}(x_2^2) = (1-\alpha)\frac{[x_1^1 x_2^1]^{\alpha}[x_1^2 x_2^2]^{1-\alpha}}{x_1^2}.$$

From these the marginal rate of substitution for consumer 1 can be calculated as:

$$MRS_{1,2}^1 = \left[\frac{\alpha}{1-\alpha}\right]\frac{x_1^2}{x_1^1}$$

Similar calculations for consumer 2 yield

$$MRS_{1,2}^2 = \left[\frac{\alpha}{1-\alpha}\right]\frac{x_2^1}{x_2^2}$$

Notice that each of the marginal rates of substitution is independent of the externality effect. Each consumer equates his or her MRS to the price ratio. Therefore the externality does not affect the fact that the equilibrium is efficient.
This conclusion holds because the externality does not affect the proportions in which the two consumers purchase the goods. (Observe that the externality effect can be factored out of the utility functions as a constant.) The same equilibrium is reached with and without the externality. As a consequence, this type of externalities are called "Pareto irrelevant".

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Exercise 2

[**Congestion as a negative externality**] There is a large number of commuters who decide to use either their car or the tube. Commuting by train takes 70 minutes whatever the number of commuters taking the train. Commuting by car takes $C(x) = 20 + 60x$ minutes, where x is the proportion of commuters taking their car, $0 \leq x \leq 1$.

(a) Plot the curves of the commuting time by car and the commuting time by train as a function of the proportion of cars users. Then determine each traveller's decision $d(x)$ on whether to use the car or the tube as a function of car time $C(x)$.

(b) What is the proportion of commuters who will take their car if everyone is taking her decision freely and independently so as to minimize her oun commuting time?

(c) What is the proportion of car users that minimizes the total commuting time?

(d) Compare this with your answer given in part b. Interpret the difference. How large is the deadweight loss from the externality?

(e) Explain how a toll could achieve the efficient allocation of commuters between train and car and the beneficial for everyone.

Solution:

(a) The commuting times for tube and cares are shown in the figure.

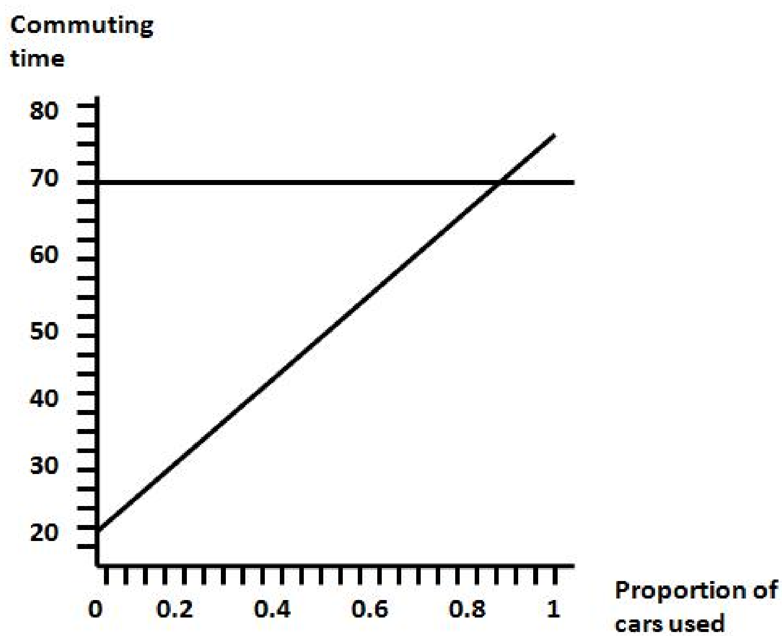

Figure 1. Commuting time.

The time by tube is constant, but the time taken by car, $C(x)$, increases as car use increases. Every traveller's decision $d(x)$ can be expressed as

$$d(x) = \begin{cases} \text{car} & \text{if } C(x) \leq 70, \text{ or} \\ \text{tube} & \text{if } C(x) > 70 \end{cases}$$

(b) The proportion of car users , if independent choices are made, will be such that the times of travel by tube and by car are equated.
Thus, $70 = 20 + 60x_m$ solving for x_m gives $x_m = 5/6 = 0.833$. This solution corresponds to the intersection point of the two commuting time courves.

(c) The total commuting time is $(20 + 60x)x + 70(1 - x)$, where x is the proportion of car users. Setting the derivative with respect to x equal to zero gives: $20 + 120x - 70 = 0$ or $120x_o - 50 = 0$ thus $x_o = 5/12 = 0.416$ is the time-minimizing car use.

(d) The free-market outcome for the proportion of car users is greater than the socially optimal outcome because the individual commuters do not take into account the negative externality generated by car travel, meaning the traffic congestion. The deadweight loss from the externality is the difference between the total commuting times. Using the earlier results obtains

$$T_m = \underbrace{\left(20 + 60\left(\frac{5}{6}\right)\right)\left(\frac{5}{6}\right)}_{\text{car}} + \underbrace{70\left(\frac{1}{6}\right)}_{\text{tube}} = \tfrac{420}{6} = 70$$

and

$$T_o = \underbrace{\left(20 + 60\left(\frac{5}{12}\right)\right)\left(\frac{5}{12}\right)}_{\text{car}} + \underbrace{70\left(\frac{7}{12}\right)}_{\text{tube}} = \tfrac{715}{12} = 59.58$$

The difference is $T_m - T_o = 70 - 59.58 = 10.41$.

(e) Suppose that the commuters attach monetary value to their travel time. It takes 45 minutes per car user, and 70 per train user. Then a toll that is worth 25 minutes of commuting time may induce car users to switch from car to tube because the amount of the toll exceeds the benefits of a shorter travel time. Given information on the monetary value of travel time, the amount of the toll can be computed so that the proportion of commuters that still find it beneficial to travel by car is exactly equal to the socially optimal level.

Exercise 3

[The tragedy of the commons.] On the island of Pago Pago there are two lakes and 20 anglers. Each angler can fish on either lake and keep the average catch on his particular lake. On lake X, the total number of fish caught is given by

$$F^x = 10l_x - \tfrac{1}{2}l_x^2$$

where l_x is the number of people fishing on the lake. For lake y the relationship is

$$F^y = 5l_y$$

(a) Under this organization of society, what will be the total number of fish caught?

(b) The chief of Pago Pago, having once read an economics book, believes it is posible to raise the total number of fish caught by restricting the number of people allowed to fish on lake X. What number should be allowed to fish on lake x in order to maximize the total catch of fish? What is the number of fish caught in this situation?

(c) Being opposed to coercion, the chief decides to require a fishing license for lake x. If the licensing procedure is to bring about the optimal allocation of labor, what should the cost of a license be (in terms of fish)?

(d) Explain how this example sheds light on the connection between property rights and externalities.

Solution:

(a) $F^x = 10l_x - 0.5l_x^2$ and $F^y = 5l_y$
First, show how total catch depends on the allocation of labor.

$$
\begin{gathered}
l_x + l_y = 20 \text{ thus } l_y = 20 - l_x \\
F^T = F^x + F^y \\
F^T = (10l_x - 0.5l_x^2) + (5l_y) = (10l_x - 0.5l_x^2) + (5\,(20 - l_x)) \\
F^T = 5l_x - 0.5l_x^2 + 100
\end{gathered}
$$

Equating the average catch on each lake gives

$$
\begin{gathered}
\frac{F^x}{l_x} = \frac{F^y}{l_y} \\
10 - 0.5l_x = 5
\end{gathered}
$$

then $l_x = 10$ and $l_y = 10$
and

$$
\begin{gathered}
F^T = 5(10) - 0.5(10)^2 + 100 \\
F^T = 100
\end{gathered}
$$

(b) The problem is to $max\, F^T = 5l_x - 0.5l_x^2 + 100$
thus the FOC wrt l_x is $\frac{dF^T}{dl_x} = 5 - l_x = 0$ then $l_x = 5$, $l_y = 15$ and then $F^T = 112.5$

(c) $F^x_{case\,1} = 10(10) - 0.5(10)^2 = 50$ average catch is $\overline{F^x_{case\,1}} = 50/10 = 5$
$F^x_{case\,2} = 10(5) - 0.5(5)^2 = 37.5$ average catch is $\overline{F^x_{case\,2}} = 37.5/5 = 7.5$
thus the license fee on lake X should be equal to 2.5.

(d) The arrival of a new fisher on lake X imposes an externality on the fishers already there in terms of a reduced average catch. Lake X is treated as a common property here. If the lake were private property, its owner would choose l_x to maximize the total catch less the opportunity cost of each fisher (the 5 fish he can catch on lake Y). So the problem is to maximize $F^x - 5l_x$ which yields $l_x = 5$ as in the optimal allocation case.

Exercise 4

[**Equilibrium and optimal extraction**] Suppose the oil industry in Utopia is perfectly competitive and that all firms draw oil from a single (and practically inexhaustable) pool. Assume that each competitor belives that it can sell all the oil it can produce at a stable world price of \$10 per barrel and that the cost of operating a well for one year is \$1,000. Total output per year (Q) of the oil field is a function of the number of wells (n) operating in the field. In particular,

$$Q = 500n - n^2$$

and the amount of oil produced by each well (q) is given by:

$$q = \frac{Q}{n} = 500 - n.$$

(a) Describe the equilibrium output and the equilibrium number of wells in this perfectly competitive case. Is there a divergence between private and social marginal cost in the industry?

(b) Suppose now that the government nationalizes the oil field. How many oil wells should it operate? What will total output be? What will the output per well be?

(c) As an alternative to nationalization, the Utopian gevernment is considering an annual license fee per well to discourage overdrilling. How large should this license fee be if it is to prompt the industry to drill the optimal number of wells?

Solution:

(a) Every firm increases q until $\pi = pq - 1000 = 0$. That is, $p(500 - n) - 1000 = 0$, implying $n = 400$.
In addition, note that revenue per well is $\frac{revenue}{well} = 5000 - 10n$, which declines in the number of wells being drilled. There is hence an externality here because drilling another well reduces output in all other wells.

(b) The social planner chooses the number of firms n in order to maximize aggregate profits

$$\max_n \; pQ - 1000n = 5000n - 10n^2 - 1000n$$

Taking FOCs with respect to n, we obtain

$$5000 - 20n - 1000 = 0$$

solving for n, $n = 200$. Hence, total output is $Q = 200 * (500 - 200) = 60,000$. So individual production is $q = 300$.
Alternatively, the social planner chooses Q where $MVP = MC$ of well. Total value:

$$5000n - 10n^2. \; MVP = 5000 - 20n = 1000. \text{ Thus } n = 200.$$

(c) Let $tax = x$. Want $\frac{revenue}{well} - x = 1000$ when $n = 200$. At $n = 200$ the average $\frac{revenue}{well} = 3000$.
So, charge $x = 2000$.