

# General Equilibrium

Felix Munoz-Garcia School of Economic Sciences  
Washington State University

EconS 503 - Advanced Microeconomics II

# General equilibrium

- NS, Chapter 13.
- Varian, Chapters 17 and 18.
- JR, Chapter 5.

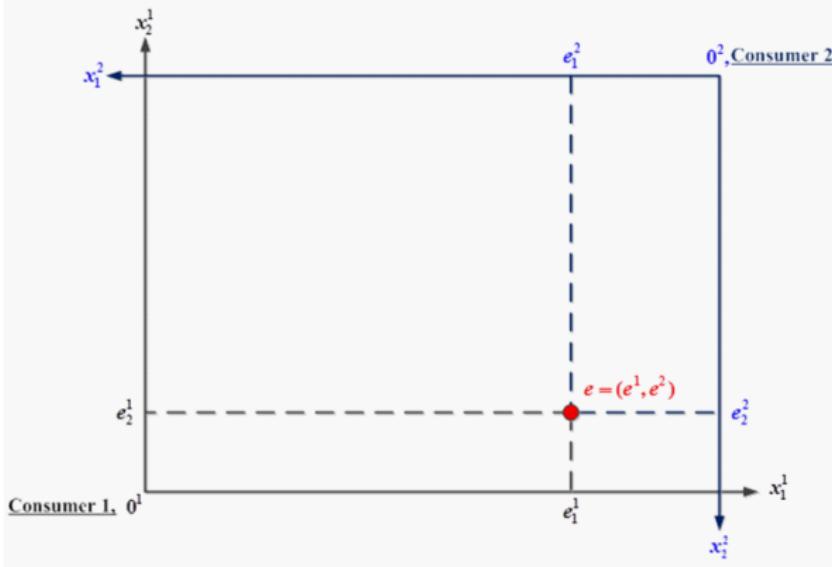
- **Additional materials:** MWG. Its treatment is probably too detailed.
- Worked-out exercises posted on EconS 503 (Angel website).
- Introductory readings about general equilibrium (undergrad level) posted on EconS 501 (Angel website).

# General equilibrium

- We will first start with equilibrium allocations in economies without production (so-called "barter equilibrium")
- Let consumer 1 be initially endowed with  $\mathbf{e}^1 \equiv (e_1^1, e_2^1)$  units of good 1 and 2, respectively.
- Similarly, let  $\mathbf{e}^2 \equiv (e_1^2, e_2^2)$  be consumer 2's endowment.
- We depict the initial endowment  $\mathbf{e} \equiv (\mathbf{e}^1, \mathbf{e}^2)$ , and any other allocation  $\mathbf{x} \equiv (\mathbf{x}^1, \mathbf{x}^2)$  with the help of the Edgeworth box.

# General equilibrium

The Edgeworth Box



# General equilibrium

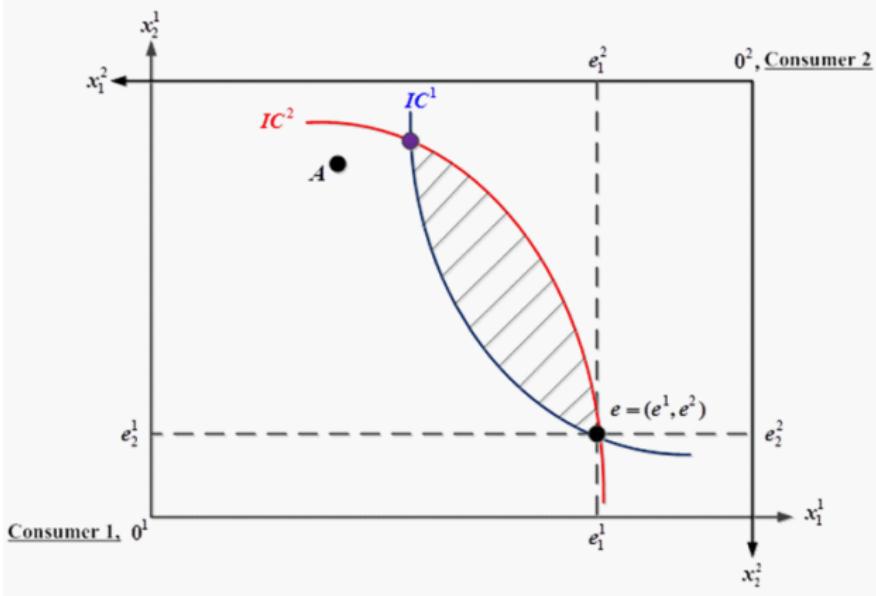
- Movement from the initial endowment  $e$  to allocation  $A$  cannot be a barter equilibrium, since consumer 1 is worse off at  $A$ , thus being able to block allocation  $A$ .
- Only points in the lens-shaped area delineated by

$$\begin{aligned}u^1(x_1^1, x_2^1) &\geq u^1(e_1^1, e_2^1) \quad \text{and} \\u^2(x_1^2, x_2^2) &\geq u^2(e_1^2, e_2^2)\end{aligned}$$

can be accepted as equilibria by both consumers.

## General equilibrium

## The Edgeworth Box

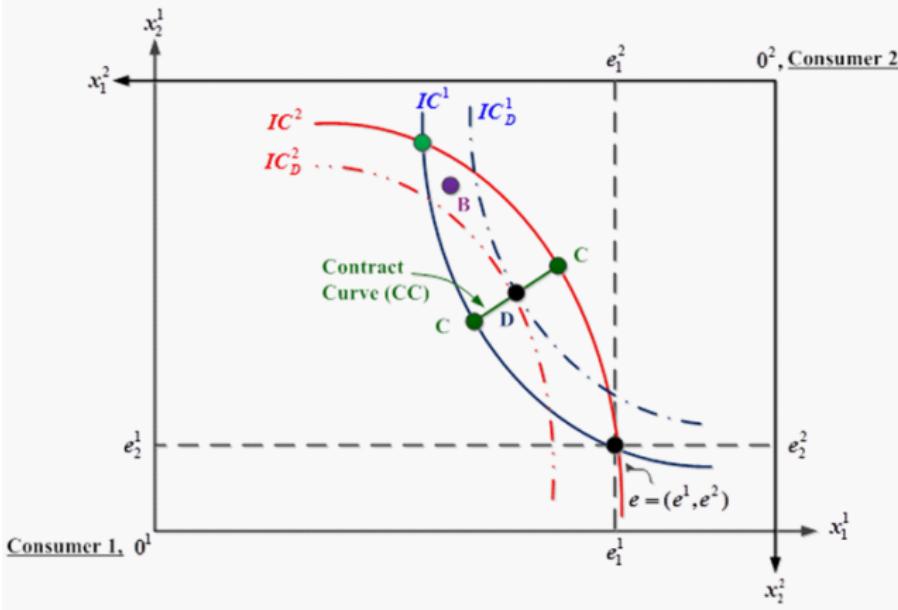


# General equilibrium

- But, is point  $B$ , which belongs to this lens-shaped area, a barter equilibrium?
  - No: other points, such as  $D$ , would still make both consumers better off than at  $B$ .
- Any point on the  $cc$  curve would be an equilibrium (no Pareto improvements are possible).
  - The  $cc$  curve is often labelled the "contract curve"

# General equilibrium

## The Edgeworth Box



# General equilibrium

- **Feasible allocation:**

- An allocation  $\mathbf{x} \equiv (\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^I)$  is feasible if satisfies

$$\sum_{i=1}^I \mathbf{x}^i \leq \sum_{i=1}^I \mathbf{e}^i.$$

- **Pareto-efficient allocations:**

- A feasible allocation is Pareto efficient if there is no other feasible allocation  $\mathbf{y}$  which is weakly preferred by all consumers,  $\mathbf{y}^i \succsim \mathbf{x}^i$  for all  $i \in I$ , and at least strictly preferred by one consumer,  $\mathbf{y}^i \succ \mathbf{x}^i$ .

# General equilibrium

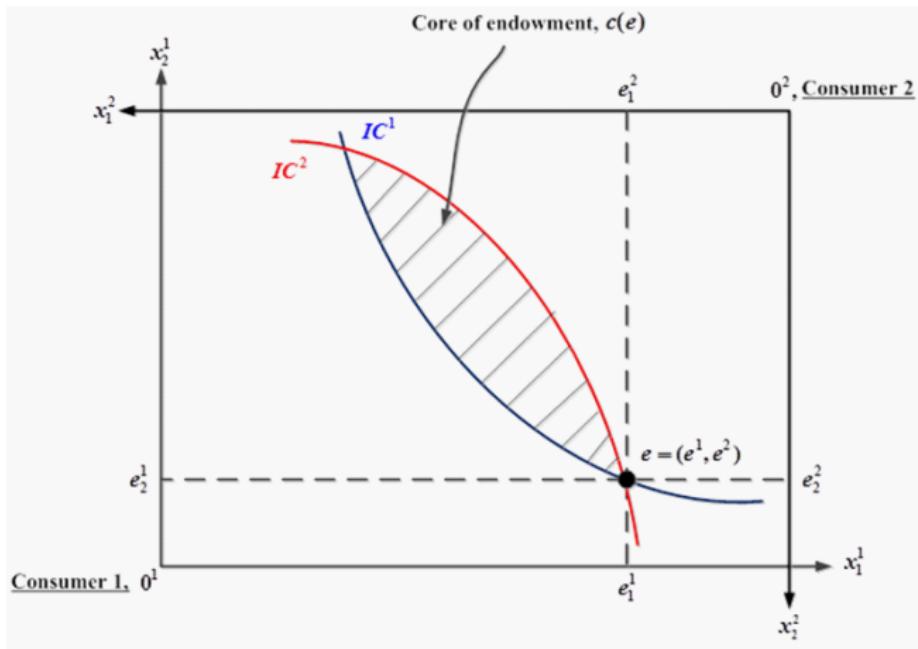
- **Blocking coalitions:**

- Let  $S \subset I$  denote a coalition of consumers. We say that  $S$  blocks the feasible allocation  $\mathbf{x}$  if there is an allocation  $\mathbf{y}$  such that
  - (1)  $\sum_{i \in S} \mathbf{y}^i = \sum_{i \in S} \mathbf{e}^i$ ; and
  - (2)  $\mathbf{y}^i \succsim \mathbf{x}^i$  for all individuals in the coalition,  $i \in S$ , with at least one individual strictly preferring  $\mathbf{y}^i$  to  $\mathbf{x}^i$ ,  $\mathbf{y}^i \succ \mathbf{x}^i$ .

# General equilibrium

- **Equilibrium:** We can, hence, summarize the requirements for an equilibrium allocation in a barter economy as follows:
  - A feasible allocation  $x$  is an equilibrium in the exchange economy with initial endowment  $e$  if  $x$  is *not blocked* by any coalition of consumers.
- **Core:**
  - The core of an exchange economy with endowment  $e$ , denoted  $C(e)$ , is the set of all unblocked feasible allocations (i.e., the set of allocations we have just identified as equilibria).

# General equilibrium



# Competitive markets

- Section 5.2 in JR.
- Road map: We will first start with some assumptions, definitions, and then we will define what we mean by a Walrasian Equilibrium Allocation (WEA), and under which conditions it exists.
- **Consumers:**
  - Consumers' utility function is continuous, strictly increasing, and strictly quasiconcave in  $\mathbb{R}_+^n$ . As a consequence, the UMP of every consumer  $i$ , when facing a budget constraint

$$\mathbf{p} \cdot \mathbf{x}^i \leq \mathbf{p} \cdot \mathbf{e}^i \quad \text{for all price vector } \mathbf{p} >> \mathbf{0}$$

has a unique solution, denoted as the Walrasian demand  $\mathbf{x}(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$ . In addition,  $\mathbf{x}(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$  is continuous in  $\mathbf{p}$ .

# Competitive markets

- **Excess demand of good  $k$ :**

$$z_k(\mathbf{p}) \equiv \sum_{i=1}^I x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - \sum_{i=1}^I e_k^i, \quad \text{where } z_k(\mathbf{p}) \in \mathbb{R}$$

- Hence, when  $z_k(\mathbf{p}) > 0$ , the aggregate demand for good  $k$  exceeds the aggregate endowment of good  $k$ . We say that there is a excess demand of good  $k$ .
- When  $z_k(\mathbf{p}) < 0$ , the opposite applies, and we say there is excess supply of good  $k$ .

# Competitive markets

- Properties of excess demand functions  $z(p)$ :

- *Walras' law:*  $p \cdot z(p) = 0$ 
  - (Cont'd) Hence,

$$\sum_{k=1}^n p_k \left[ x_k^i (p, p \cdot e^i) - e_k^i \right] = 0$$

Summing over all individuals gives

$$\sum_{i=1}^I \sum_{k=1}^n p_k \left[ x_k^i (p, p \cdot e^i) - e_k^i \right] = 0$$

# Competitive markets

- Properties of excess demand functions  $z(p)$ :

- Continuity:  $z(p)$  is continuous at  $p$ .
- Homogeneity:  $z(\lambda p) = z(p)$  for all  $\lambda > 0$ .
  - This follows from Walrasian demands being homogeneous of degree zero in prices.
- Walras' law:  $p \cdot z(p) = 0$ 
  - This follows from the property of strictly increasing utility function: the budget constraint in the UMP will be binding for every consumer  $i \in I$ . In particular, for every consumer  $i \in I$  we have

$$\sum_{k=1}^n p_k x_k^i (p, p \cdot e^i) = \sum_{k=1}^n p_k e_k^i$$

# Competitive markets

- Properties of excess demand functions  $z(\mathbf{p})$ :

- Walras' law:  $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$ 
  - (Cont'd) Since the order of summation is inconsequential, we can rewrite

$$\sum_{k=1}^n \sum_{i=1}^I p_k \left[ x_k^i (\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - e_k^i \right] = 0$$

which, in turn, is equivalent to

$$\underbrace{\sum_{k=1}^n p_k \left( \sum_{i=1}^I x_k^i (\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - \sum_{i=1}^I e_k^i \right)}_{z_k(\mathbf{p})} = 0$$

$$\iff \sum_{k=1}^n p_k z_k(\mathbf{p}) = \mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$$

# Competitive markets

- In a two-good economy, Walras' law implies

$$p_1 z_1(\mathbf{p}) = -p_2 z_2(\mathbf{p})$$

thus indicating that, if there is excess demand in market 1,  $z_1(\mathbf{p}) > 0$ , there must be excess supply in market 2,  $z_2(\mathbf{p}) < 0$ .

- Similarly, if market 1 is in equilibrium,  $z_1(\mathbf{p}) = 0$ , then so is market 2,  $z_2(\mathbf{p}) = 0$ .
  - Generally, if the markets of  $n - 1$  goods are in equilibrium, then so is the  $n$ th market.

# Competitive markets

- **Walrasian equilibrium:**

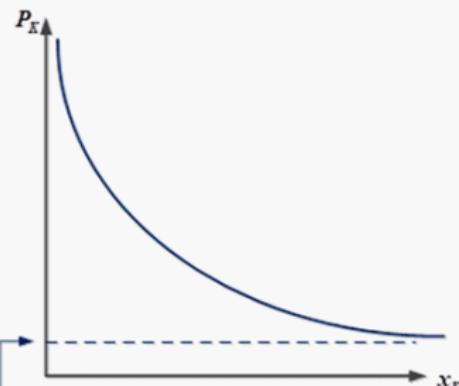
- A vector of prices  $\mathbf{p}^* \in \mathbb{R}_{++}^n$  is called a Walrasian equilibrium if aggregate excess demand is zero at that price vector,  
$$\mathbf{z}(\mathbf{p}) = 0.$$
- We next explore existence and uniqueness of a Walrasian equilibrium.

# Competitive markets

- **Existence of a Walrasian equilibrium:**
- Suppose that the excess demand function  $\mathbf{z}(\mathbf{p})$  satisfies:
  - 1)  $\mathbf{z}(\mathbf{p})$  is continuous on all strictly positive price vectors,  $\mathbf{p} \in \mathbb{R}_{++}^n$ .
  - 2) Walras' law holds, i.e.,  $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$  for all strictly positive price vectors,  $\mathbf{p} \in \mathbb{R}_{++}^n$ .
  - 3) If  $\{\mathbf{p}^m\}$  is a sequence of strictly positive price vectors,  $\mathbf{p}^m \in \mathbb{R}_{++}^n$ , converging to  $\bar{\mathbf{p}} \neq 0$ , and  $\bar{p}_k = 0$  for some good  $k$ , then for some good  $k'$  with price  $\bar{p}_{k'} = 0$ , the associated sequence of excess demands in the market for good  $k'$ ,  $\{z_{k'}(\mathbf{p}^m)\}$ , is unbounded above.
- Then, there is a price vector  $\mathbf{p}^* \in \mathbb{R}_{++}^n$  such that  $\mathbf{z}(\mathbf{p}^*) = 0$ .

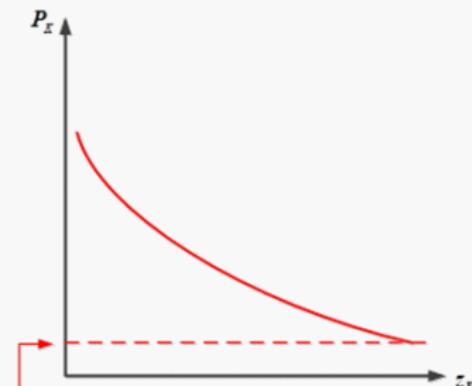
# Competitive markets

Walrasian Demand for good K



Price close to zero yields an arbitrarily high demand

Excess Demand for good K



Price close to zero yields an arbitrarily high demand

# Competitive markets

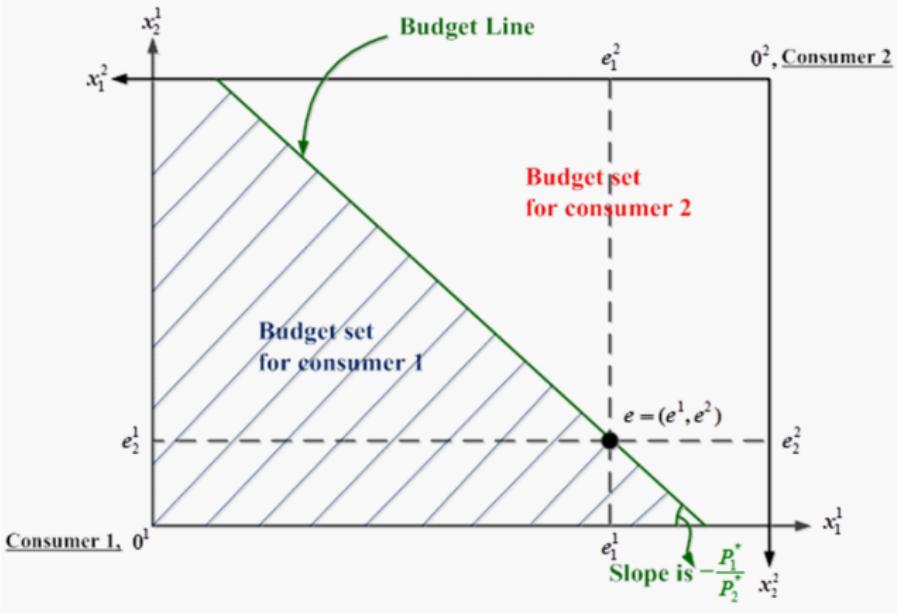
- The first two conditions for existence follow from the assumptions we imposed on consumers' utility functions.
- What about the third condition?
  - It intuitively says that, if the prices of some but not all goods are arbitrarily close to zero, then the excess demand for at least one of those goods is arbitrarily high.
  - Moreover, this condition follows from the conditions we imposed on consumers' utility functions and  $\sum_{i=1}^I \mathbf{e}^i >> 0$ . (See proof in pages 207-210 in JR.)

# Competitive markets

- Existence of a Walrasian Equilibrium Allocation (WEA) in terms of the primitives of the model
  - Hence, if utility function is continuous, strictly increasing and strictly quasiconcave, and the endowment satisfies
$$\sum_{i=1}^I \mathbf{e}^i >> 0, \text{ a price vector } \mathbf{p}^* \in \mathbb{R}_{++}^n \text{ exists such that } \mathbf{z}(\mathbf{p}^*) = 0.$$
- Worked-out example about how to find WEA:
  - Example 5.1 (pages 211-212 in JR)
  - It assumes a CES utility function  $u^i(x_1, x_2) = x_1^\rho + x_2^\rho$  for two consumers  $i = \{1, 2\}$ , where  $0 < \rho < 1$ , with endowments  $\mathbf{e}^1 = (1, 0)$  and  $\mathbf{e}^2 = (0, 1)$ .

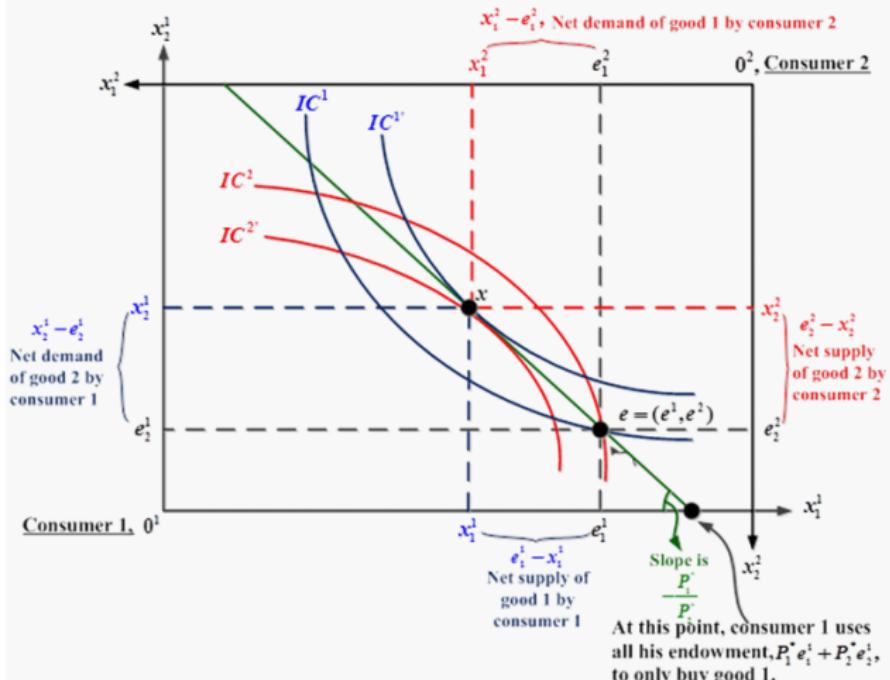
# Competitive markets

## Representing the budget line in the Edgeworth Box

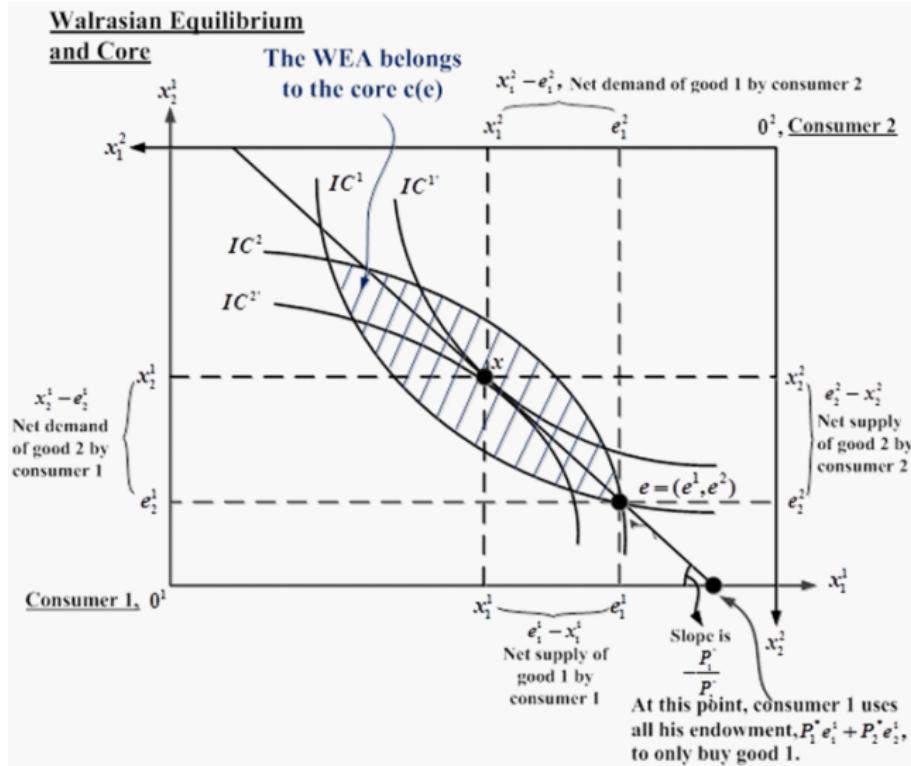


# Competitive markets

## The Edgeworth Box



# Competitive markets



# Competitive markets

- **Relationship between WEAs and the Core:**

- If each consumer's utility function is strictly increasing, then every WEA is in the Core, i.e.,  $W(\mathbf{e}) \subset C(\mathbf{e})$ .
- **Proof:** By contradiction, take a WEA,  $\mathbf{x}(\mathbf{p}^*)$  which is a WEA for equilibrium price  $\mathbf{p}^*$ , but assume that  $\mathbf{x}(\mathbf{p}^*) \notin C(\mathbf{e})$ .
- Because  $\mathbf{x}(\mathbf{p}^*)$  is a WEA, it must be feasible.
- However, if  $\mathbf{x}(\mathbf{p}^*) \notin C(\mathbf{e})$  we can find a coalition  $S$  and another allocation  $\mathbf{y}$  such that

$$u^i(\mathbf{y}^i) \geq u^i \left( \mathbf{x}^i \left( \mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i \right) \right) \quad \text{for all } i \in S$$

with strict inequality for at least one individual in the coalition, and

$$\sum_{i \in S} \mathbf{y}^i = \sum_{i \in S} \mathbf{e}^i \quad (\text{feasibility})$$

# Competitive markets

- **Relationship between WEAs and the Core:**

- We can now multiply both sides of the feasibility condition by  $\mathbf{p}^*$  to obtain

$$\mathbf{p}^* \sum_{i \in S} \mathbf{y}^i = \mathbf{p}^* \sum_{i \in S} \mathbf{e}^i$$

- However, if  $\mathbf{x}^i (\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i)$  is a WEA, the most preferable vector  $\mathbf{y}^i$  must be more costly than  $\mathbf{x}^i (\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i)$ , that is

$$\mathbf{p}^* \mathbf{y}^i \geq \mathbf{p}^* \mathbf{x}^i (\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i) = \mathbf{p}^* \mathbf{e}^i$$

with strict inequality for at least one individual. Summing over all consumers in the coalition  $S$ , we obtain

$$\mathbf{p}^* \sum_{i \in S} \mathbf{y}^i > \mathbf{p}^* \sum_{i \in S} \mathbf{x}^i (\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i) = \mathbf{p}^* \sum_{i \in S} \mathbf{e}^i$$

contradicting  $\mathbf{p}^* \sum_{i \in S} \mathbf{y}^i = \mathbf{p}^* \sum_{i \in S} \mathbf{e}^i$ .

# Competitive markets

- **Relationship between WEAs and the Core:**

- Summing over all consumers in the coalition  $S$ , we obtain

$$\mathbf{p}^* \sum_{i \in S} \mathbf{y}^i > \mathbf{p}^* \sum_{i \in S} \mathbf{x}^i \left( \mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i \right) = \mathbf{p}^* \sum_{i \in S} \mathbf{e}^i$$

contradicting  $\mathbf{p}^* \sum_{i \in S} \mathbf{y}^i = \mathbf{p}^* \sum_{i \in S} \mathbf{e}^i$ .

- Therefore,  $\mathbf{x}(\mathbf{p}^*) \in C(\mathbf{e})$ , i.e., all WEAs must be part of the Core.

- **Relationship between WEAs and the Core:**

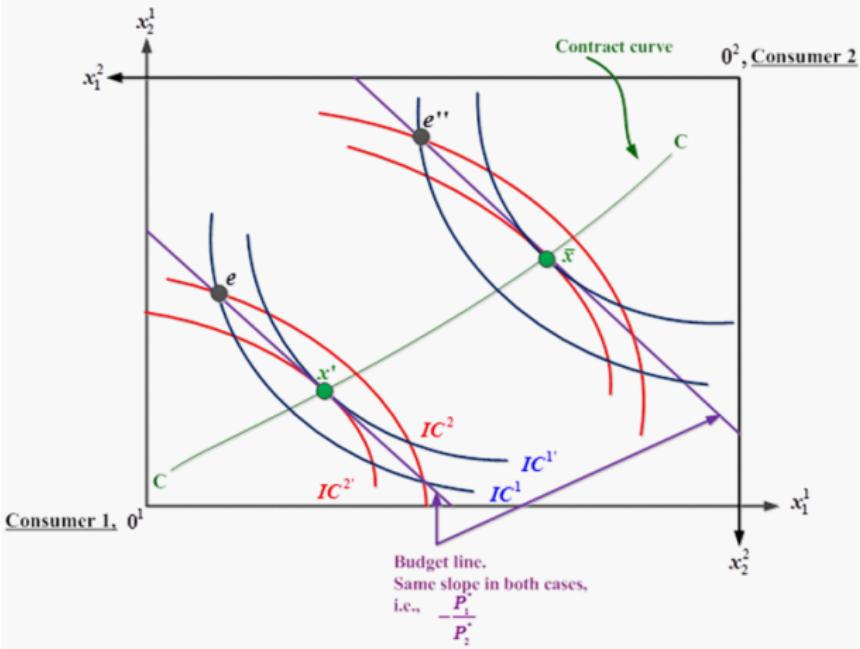
- **Corollary 1:** By the existence results we described in our last class...
  - The Core  $C(\mathbf{e})$  will be always nonempty, i.e., it will at least contain the WEAs.
- **Corollary 2:** Since all core allocations are Pareto efficient, i.e., we cannot increase the welfare of one consumer without decreases that of other consumers, then all WEAs (which are part of the Core) are also Pareto efficient.
  - This is often referred to as the First Welfare Theorem: every WEA is Pareto efficient.

# Competitive markets

- **Second Welfare Theorem:**
- Motivation:
  - In the next figure, starting from initial endowment  $e$ , consider that the WEA is  $x'$ , which also belongs to the lens describing the core  $C(e)$ .
  - But assume that society prefers allocation  $\bar{x}$  more than  $x'$ .
    - How is this aggregate preference measured? With the use of a social welfare function, which we already described in EconS 501 but we will revisit it at the end of the semester.
  - Society could alter the endowment from  $e$  to  $e''$  (or generally, to any point  $e^{*i}$  in the budget line, thus satisfying  $p^* \cdot e^{*i} = p^* \cdot \bar{x}^i$ ) and...
    - then "let the market system work" (i.e., each individual consumer solving his UMP), which would lead to the desired WEA  $\bar{x}$ .

# Competitive markets

## Walrasian Equilibrium and Core



- **Second Welfare Theorem:**
- Suppose that  $\bar{x}$  is a Pareto-efficient allocation, and that endowments are redistributed so that the new endowment vector  $e^{*i}$  lies in the budget line, thus satisfying  $p^* \cdot e^{*i} = p^* \cdot \bar{x}^i$  for every consumer  $i$ . Then, the Pareto-efficient allocation  $\bar{x}$  is a WEA given the new endowment vector  $e^*$ .