More examples of BNE: Application to the game of chicken

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Teenagers and the Game of Chicken

- Rebel Without a Cause (1955) starring James Dean
  - Two teenagers simultaneously drive their cars toward the edge of a cliff.

Teenagers and the Game of Chicken

- The Game of Chicken
  - Two teenagers are players 1 & 2
  - They drive toward each other in the middle of a street and choose to either swerve (S) to the right or drive head on (D).
- Payoffs
  - When both players choose to swerve to the right, they both receive 0
  - When one player drives head on, and the other swerves to the right, the player who drove head on receives R (respect), while the other receives 0 (Chicken!)
  - $\frac{R}{2} - K$, where K is the punishment when both players choose to drive head on (You just wrecked dad's car!)

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- Players' parents are either Harsh (H) or Lenient (L)
  - Each player knows their own parent's type, but not the type of the other player's parents.
  - If a player's parents are harsh, then they will severely punish their child for wrecking the car, which we denote as $K = H$.
  - If a player's parents are lenient, then they will only lecture their child if they wreck the car, which we denote as $K = L$.
  - Assume that $L < H$, and that the probability of being harsh is $p = 0.5$.

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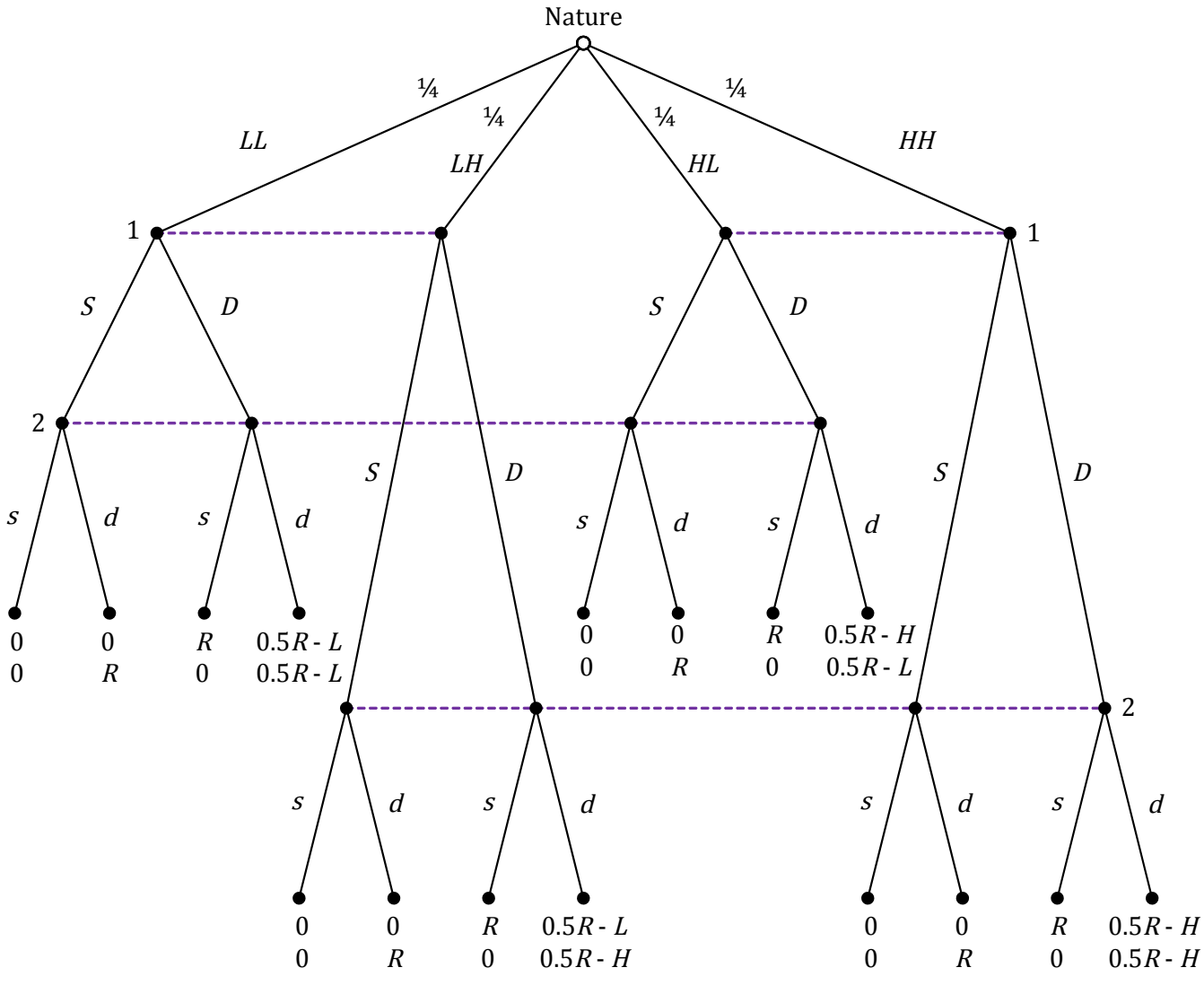

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- If player 1 plays the strategy SD (Swerve if his parents are lenient, but Drive head on if his parents are harsh), and player 2 plays dd (Drive head on regardless of his parents' type), then the expected payoffs for player 1 are

$$\begin{aligned} Ev_1(SD, dd) &= \frac{1}{4}v_1(S, d; L) + \frac{1}{4}v_1(S, d; L) \\ &\quad + \frac{1}{4}v_1(D, d; H) + \frac{1}{4}v_1(D, d; H) \\ &= \frac{1}{4} * 0 + \frac{1}{4} * 0 + \frac{1}{4} * \left(\frac{R}{2} - H\right) + \frac{1}{4} * \left(\frac{R}{2} - H\right) \\ &= \frac{R}{4} - \frac{H}{2} \end{aligned}$$

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- And the expected payoffs for player 2 are

$$\begin{aligned} Ev_2(SD, dd) &= \frac{1}{4}v_2(S, d; L) + \frac{1}{4}v_2(S, d; L) \\ &\quad + \frac{1}{4}v_2(D, d; H) + \frac{1}{4}v_2(D, d; H) \\ &= \frac{1}{4} * R + \frac{1}{4} * R + \frac{1}{4} * \left(\frac{R}{2} - L\right) + \frac{1}{4} * \left(\frac{R}{2} - H\right) \\ &= \frac{3R}{4} - \frac{L}{4} - \frac{H}{4} \end{aligned}$$

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- We can represent all of the payoffs in the following normal form game:

		Player 2			
		ss	sd	ds	dd
Player 1	SS	0, 0	0, ½R	0, ½R	0, R
	SD	½R, 0	⅜R - ¼H, ⅜R - ¼H	⅜R - ¼H, ⅜R - ¼L	¼R - ½H, ¾R - ¼L - ¼H
	DS	½R, 0	⅜R - ¼L, ⅜R - ¼H	⅜R - ¼L, ⅜R - ¼L	¼R - ½L, ¾R - ¼L - ¼H
	DD	R, 0	¾R - ¼L - ¼H, ¼R - ½H	¾R - ¼L - ¼H, ¼R - ½L	½R - ½L - ½H, ½R - ½L - ½H

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- To solve for the Bayesian Nash equilibrium, let's assume that $R = 8$, $H = 16$, and $L = 0$. Updating our normal form game,

		Player 2			
		ss	sd	ds	dd
Player 1	SS	0, 0	0, 4	0, 4	0, <u>8</u>
	SD	4, 0	-1, -1	-1, <u>3</u>	-6, 2
	DS	4, 0	<u>3</u>, -1	<u>3</u>, <u>3</u>	<u>2</u>, 2
	DD	<u>8</u>, 0	2, -6	2, <u>2</u>	-4, -4

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- This game has a unique pure-strategy Bayesian Nash equilibrium of (DS, ds).
  - The children of lenient parents will continue driving head on
  - While those of harsh parents will swerve to avoid the costly consequences.

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- However, if we assume $R = 8$, $H = 0$ and $L = 16$, this would result in the opposite prediction. ("I'm not mad. I'm disappointed.")

		Player 2			
		ss	sd	ds	dd
Player 1	SS	0, 0	0, 4	0, 4	0, <u>8</u>
	SD	4, 0	<u>3</u>, <u>3</u>	<u>3</u>, -1	<u>2</u>, 2
	DS	4, 0	-1, <u>3</u>	-1, -1	-6, 2
	DD	<u>8</u>, 0	2, <u>2</u>	2, -6	-4, -4

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- This game has a unique pure-strategy Bayesian Nash equilibrium of (SD, sd).
  - Children of lenient parents learn somehow to respect their parents' property (the car).
  - While children of harsh parents do not respect their parents' property.